Review of the doctoral thesis by D Wysocki entitled **Geometric approaches to Lie bialgebras, their classification, and applications** (University of Warsaw, Faculty of Physics, student no: 335602)

In this thesis, the author discusses the classification problem of coboundary Lie algebras from an algebraic and geometric point of view. In particular, the case of three-dimensional and indecomposable four-dimensional Lie algebras is extensively studied. An example of four-dimensional decomposable Lie algebra is also considered. The previous theory is used in the description of Hamiltonian systems on symplectic, Poisson, contact and Jacobi structures.

The thesis consists of five chapters, a section with conclusions and outlook and two appendices.

- In Chapter 1, the author reviews some definitions and basic constructions on differential geometry, Hopf algebra theory, Lie algebras and Lie systems which will be useful in the rest of the thesis.

- In Chapter 2, the problem of classification of coboundary Lie bialgebras is discussed from an algebraic point of view. For this purpose, the theory of $\mathfrak{g}$-invariant maps is used, $\mathfrak{g}$ being a Lie algebra. In fact, some methods for obtaining such invariant maps are described. For it, a gradation on the Grassmann algebra is introduced. All the previous results are applied in the problem of classification of inequivalent r-matrices for all three-dimensional Lie algebras. In addition, r-matrices for the four-dimensional Lie algebra of the general linear group of order 2 are also obtained.

- In Chapter 3, a classification of coboundary Lie bialgebras for the four-dimensional case is developed. For this purpose, the notion of a Darboux family for a Vessiot–Guldberg Lie algebra on a manifold is introduced. It is an extension of a Darboux polynomial for a polynomial vector field on a vector space. In fact, the method developed in this chapter may be applied to Lie algebras of dimension greater or equal to 4. As a byproduct, a method for obtaining matrix representations of certain Lie algebras with a non-trivial center is developed.

- In Chapter 4, foliated Lie systems (as an extension of Lie systems) are discussed. The author proves that such systems admit a foliated superposition rule. In addition, a special class of them is studied: the foliated automorphic Lie systems (a foliated version of automorphic Lie systems). Moreover, the author proves that foliated Lie systems may be used to describe a new generalisation of Ermakov systems. He also shows how Poisson structures induced by r-matrices might be applied to obtain and to study foliated Lie-Hamilton systems.

- In Chapter 5, the author extends some previous constructions of integrable deformations of Poisson Hamiltonian systems to the Jacobi and contact settings. The geometric nature of such deformations and applications of the theory are also discussed.

The thesis closes with a section on conclusions, a proposal of interesting future lines of research and two appendices. In the first one, the author presents a code written in Mathematica that has been used in Chapters 2 and 3. In the second one, he discusses an effective method to obtain matrix representation for Lie algebras with nontrivial centre. This method has been used in Chapter 3.

In my opinion, all the previous results are very interesting. Moreover, the thesis is well-written with a suitable list of references. On the other hand, a part of the results are contained in three papers written by the author of the thesis (in collaboration with his supervisor and other researchers). The papers have been published in three journals of recognized international prestige: J Lie Theory, Symmetry and J Phys A: Math and Theoret.

For all the previous reasons, **I deem the thesis as sufficient to grant a PhD. In fact, since the work is exceptionally good, the thesis could be awarded with "an honorary distinction".**