Review of Bartłomiej Polaczyk's PhD dissertation
„Concentration of measure and functional inequalities"

The topics of the PhD Thesis by Bartłomiej Polaczyk include the concentration of measure phenomenon and its connections with functional inequalities discussed mostly in the discrete setup described in five chapters with a list of references and two appendices.
The material is presented as follows.

Chapter 1 Introduction
This chapter presents briefly the main topics and objectives of the thesis and provides a general overview of the mathematical literature on the topics of the thesis. More specifically   section 1.2 describes concentration phenomenon   and section 1.3 functional inequalities.
In subsections 1.3.3 Beckner inequalities
and 1.3.3 p-log-Sobolev inequalities the author provides a brief description of his contributions.

Section 1.4 on dependent binary random variables, after initial background subsection,   the author provides a brief description of his contributions in this area in subsection 1.4.2.

Section 1.5 concerns sampling without replacement and Hoeffding statistics.
After short background subsection,   the author provides a brief description of his contributions in this area in subsection 1.5.2.

Finally in section 1.6 the thesis structure is provided.

In the following chapters the author describes number of results obtained in collaboration as well as on his own.

Chapter 2 Beckner inequalities and moment estimates
is devoted to the modified log-Sobolev inequalities, Beckner inequalities and moment estimates.
The results of this chapter are based on a joint work [6].

Chapter 3 P-log-Sobolev inequalities
is devoted to relations between p-log-Sobolev inequalities and a solution to Mossel–Oleszkiewicz–Sen problem. These results are unpublished.

Chapter 4 Stochastic Covering Property
is devoted to concentration inequalities for certain negatively dependent binary random variables.
The results are based on a joint work [5].

Chapter 5 Sampling without replacement and Hoeffding statistics
is devoted to concentration bounds for sampling without replacement and Hoeffding statistics.
The results are based on [173].

In general the thesis is of a very good quality both in mathematical content as well as its presentation. It is clear that the author   has a broad knowledge in various directions of modern mathematical analysis/probability theory.
The thesis definitely reaches the high international standard as for example in leading institutions in France and UK.

Further Comments

One can notice following deficiencies. First of all the use and presentation of the literature, although perhaps sufficient for the pourpose of the thesis, is far from complete and frequently ahistorical. For example a property of the L_p Dirichlet form is referred to the paper [81] while it already was used in [108] and most likely it is an older piece of standard theory of Dirichlet forms.

It is a bit dissapointing why to limit the concentration estimates to the case when Dobrushin's Uniqueness theory works while a more general theory (of complete analyticity by Dobrushin and Shlosmann) exists and provides a basis for coercive inequalities in far more general situation. In fact it can be verified up to the critical point of Ising ferromagnet far beyong applicability of the Dobrushin Uniqueness.
It is clear from the list of literature and description in the thesis that the author has no idea about this and other more deeper interesting areas of statistical mechanics. This is in fact a more a general problem of the lack of expertise and interest in the Polish mathematics in the area of statistical mechanics (and theory of phase transitions in discrete models in particular) which hides a goldmine of mathematically interesting and challenging problems.

In the part on higher order estimates it is not that natural to study deviation from mean while a space of polynomials is anihillated by the higher order differential operators. It is interesting that the first order Poincare Inequality implies the higher order inequalities with some statistical polynomial arising from a simple iteration procedure. The same concerns higher order log-Sobolev inequalities which was known since works of J.Rosen (1976) and R.A. Adams (1979) for so called regular measures in non-tight form (which were tightened a couple of years ago in a form of Orlicz-Sobolev type inequalities).
Thus many tools for studying more detailed concentration exist in the literature.

Other Comments:
– p.30 „Using a Holley–Stroock type perturbation argument (cf. [118, 14]) one can also easily produce examples of non-product measures with infinite support and $p\_0 > 0$. „
    This way one can only produce relatively simple examples.

– p.32 „Let us remark......has been recently developed ”
    Some more general conditions were developed 30 years ago. That what was derived e.g. by Bauerschmid&Bodineau for spin systems is rather simple.   [They've done a beautiful work in sin-G model which is beyond the scope of the thesis.]

– p.52 – Section 2.5 works of J Rosen (1976) and R.A. Adams (1979) and recent aftermath are relevant for higher order coercive inequalities

– p.59 The statement „Arguably......interpolation between the above two" is a bit cavalerious statement. See e.g. works of J Rosen (1976) and R.A. Adams (1979) or the book E.B. Davies „Heat kernels and Spectral Theory" (1989)
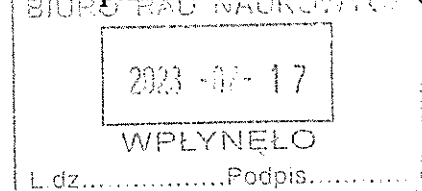
    – p. 64 Remark : There is some general recent view of Hardy Ineq in
    Electron. J. Probab. 26 (2021), article no. 142, 1–34.
https://doi.org/10.1214/21-EJP711 ,Hardy's inequality and its descendants: a probability approach, Chris A. J. Klaassen Jon A. Wellner
    – p.69 last = in (3.3.17) requires some explanation
–p. 75 line under log in (3.4.18)
    – p.124 Michal Strzelecki (break missing)

Review by Boguslaw Zegarlinski (Toulouse 15 July 2023)