MIT COMPUTER SCIENCE AND ARTIFICIAL INTELLIGENCE LABORATORY

Report on the PhD thesis
"Lower Bounds Under Strong Complexity Assumptions"
by Arkadiusz Socała

It is my great pleasure to provide a review of the dissertation by Arkadiusz Socała titled "Lower Bounds Under Strong Complexity Assumptions".

The thesis studies the complexity of combinatorial (mostly graph-theoretic) problems. It focuses on showing *lower bounds* on their computational complexity. Such limitations are traditionally obtained using the P≠NP hypothesis proposed in the 1970s. By showing that a problem is NP-complete, one can conclude that a polynomial-time algorithm for the problem is unlikely. Although very successful, this approach typically leaves a large gap between the (super-polynomial) lower bound and the (exponential) upper bound. In order to close this gap, over the last two decades researchers formulated stronger hypotheses that make it possible to infer stronger limitations on the complexity of NP-complete problems. Among those, perhaps the most well-studied conjectures is the Exponential Time Hypothesis (ETH), which postulates that the well-studied Satisfiability problem for formulas with n variables and O(n) clauses cannot be solved in time $2^{o(n)}$. Using this conjecture makes it possible to establish ``fine-grained'' lower bounds, often matching the runtimes of existing algorithms up to a constant factor in the exponent.

This thesis shows several results of this type, for the following problems: Subgraph Isomorphism, Channel Assignment, Rainbow k-coloring, (a:b)-coloring and Minimax Approval Voting. Most of the results establish lower bounds of the form $2^{\Omega(n \log n)}$, where n is the input size. Proving lower bounds of this type is quite challenging, as they essentially require reducing assignments over n variables to solutions consisting of n/log n elements (e.g., vertices). This thesis overcomes this difficulty by carefully encoding small ``chunks'' of the assignments as permutations. The resulting lower bounds are very strong: for Channel Assignment, (a:b)-coloring and Minimax Approval Voting, they are tight up to constant factors in the exponent.

The highlight of the thesis is a lower bound for the sub-graph isomorphism problem: given two graphs with n vertices, determine whether there is a one-to-one mapping from the vertices of the first graph to the vertices of the second graph which preserves the edge structure. This is a fundamental NP-complete problem. It is easy to solve it in $2^{O(n \log n)}$ time, by enumerating all mappings. Perhaps surprisingly, this is the fastest known algorithm for this problem. At the same time, no lower bound higher than $2^{\Omega(n)}$ was known before, and demonstrating such a lower bound was an open problem. In Chapter 4 of the thesis,

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Socała shows that the problem cannot be solved in time exponential in $O(n (\log n)^{1/2})$, making the first progress on this problem. The proof proceeds by first reducing Satisfiability to a ``colored'' version of the sub-graph isomorphism problem, and then encoding the colors by constructing appropriate graph gadgets. This result (obtained jointly with Cygan and Pachocki) was later strengthened to the optimal bound of $2^{\Omega(n \log n)}$ in another paper, also co-authored by Socała. The latter paper appeared in SODA'16 (the main conference in the field dedicated exclusively to algorithms) and its full version appeared in Journal of the ACM, the top journal in theoretical computer science.

Another important result presented in the thesis is a lower bound for the Channel Assignment problem, a natural generalization of graph coloring. Again, the problem has a $2^{O(n \log n)}$ –time solution, but no lower bound better than $2^{O(n)}$ was known. Chapter 3 provides a tight lower bound. The proof proceeds by a sequence of three reductions between intermediary problems. The single-authored paper containing these results has been published at SODA'15.

Chapter 5 is focused on another important graph-theoretic problem, Rainbow Coloring. Here the goal is to color the edges of a given graph to ensure that between any pair of vertices there is a path on which the edge colors do not repeat. Unlike before, the best known algorithm for this problem runs in time exponential in the number of edges, which could be as large as $2^{O(n^2)}$. Socała shows that the running time cannot be better than $2^{O(n^{1.5})}$. Although a gap between the upper and lower bounds still remains, the result represents a substantial progress towards understanding the complexity of this problem. The paper (joint with Kowalik and Lauri) appears in ESA'17. Chapters 6 and 7 contain hardness results for (a:b)-coloring and Minimax Approval Voting, respectively.

The thesis is well written. The proofs are illustrated by figures that explain the graph-theoretic constructions used in the reduction. The references are adequate and quite thorough.

In sum, this is a very good and well-written thesis. The material has appeared in the top conferences and journals in the field (SODA, JACM). It satisfies all dissertation requirements. Furthermore, I recommend that it be granted the title of an outstanding thesis.

Best Regards,

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