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To whom it may concern

Review of the doctoral thesis of Adam Karczmarz

The thesis presents algorithms and data structures for planar graphs within three main areas. I will consider these three areas one by one in the following and assess the contributions of the thesis.

The first area focuses on efficiently maintaining essentially an adjacency list representation of a planar multigraph undergoing edge contractions. The thesis presents a new data structure that maintains this information in a total time that is linear in the size of the initial graph so that at any point, any basic operation such as querying for an edge between an ordered vertex pair in the current graph can be answered in constant time. This result has a number of corollaries, namely linear-time algorithms for problems related to 2- and 3-edge-connectivity and unique matchings in planar graphs.

The result is interesting and very clean. It is obtained using a well-known approach of keeping an r -division of the graph, which roughly speaking is a partition into subgraphs each of which has a small number of connections, called boundary vertices, to the rest of the graph. Due to the small size of the subgraphs, adjacency list information for these can be maintained efficiently and since the total number of boundary vertices is small, these can be handled efficiently as well. It turns out that this does not quite get down to linear total update time so the subgraphs are further partitioned into even smaller graphs for which a brute-force approach will suffice to get the desired linear time bound. This overall approach has been used in other planar graph papers but getting it to work for this problem is non-trivial and involves new ideas to handle interactions between the various subgraphs.

The second area of the thesis is decremental reachability in planar directed graphs. Here, the problem is to maintain information about which vertices can reach each other in a graph undergoing a sequence of edge deletions. More specifically, the main problem studied is that of maintaining the transitive closure which is a graph on the same vertex set as the underlying planar graph and with an edge (u, v) if and only if v can be reached from u in the current graph. It is shown how to maintain the transitive closure under edge deletions with amortized update time $\tilde{O}(n/t)$ such that a

reachability query between a vertex pair can be answered in time close to $O(\sqrt{t})$; here n is the number of vertices and t is any parameter between 1 and n . Additional related results are shown, including the maintenance of reachability from a fixed vertex in near-linear total update time.

Dynamic transitive closure is a classical algorithmic problem that has been well studied for general graphs where the decremental variant can be solved in total update time $O(mn)$ and constant worst-case query time; here m is the number of edges of the initial graph. This bound is $O(n^2)$ for planar graphs and hence gives $O(n)$ amortized update time and $O(1)$ worst-case update time. The new planarity-exploiting algorithm presented in the thesis offers an interesting and natural tradeoff for the entire range of t that essentially matches the general graph result for constant t .

The new algorithm makes use of a well-known recursive decomposition of the graph using separators but with some additional properties that were not used in earlier papers. Thus, a new proof is given to show how to ensure these properties for the decomposition. It is then shown how the transitive closure between boundary vertices of each subgraph in the decomposition can be efficiently maintained for the subgraph as well as for its complement. At a high level, this is similar to the well-known internal and external dense distance graphs which represent shortest paths but here the information maintained is reachability. It is shown how the maintenance of this information suffices to maintain the transitive closure of the entire graph within the claimed time bounds.

The reachability problems studied are important and have received considerable attention from the research community over the years. The new results are interesting and the techniques are natural and elegant.

The third and final area studied in the thesis is a static graph problem, namely the problem of efficiently computing a dense distance graph. These graphs have found numerous applications for important planar graph problems related to, e.g., shortest paths and maximum flow where an algorithm for computing dense distance graphs is used as a black box. For some of these problems, the current best time bound for this black box is the bottleneck. Hence, any improved algorithm for computing dense distance graphs will have a range of interesting corollaries. Indeed, this is what is achieved in the thesis. While the improvement in running time might seem modest, the new result is still noteworthy due to the important corollaries, one such being a faster algorithm for maximum flow in planar graphs with multiple sources and sinks. The speed-up comes from a clever optimization involving the use of rectangular rather than square Monge matrices. Given the importance of dense distance graphs and given no progress on the problem for a long time, the improvement is impressive.

The thesis is well written and does a good job of giving the high-level ideas before delving into the technical details.

In conclusion, the thesis deals with fundamental problems within the area of planar graph algorithms, makes progress on these problems, and presents the new results in a clear manner. I deem the thesis in its current form to be sufficient to grant a PhD with an honorary distinction.

Sincerely yours,

A handwritten signature in black ink that reads "Chr. Wulff-Nilsen". The signature is written in a cursive, slightly slanted style.

Christian Wulff-Nilsen, associate
professor, DIKU