3rd Warsaw Summer School in Probability

University of Warsaw
June 24 – 28, 2019

Wydział Matematyki, Informatyki i Mechaniki

Uniwersytet Warszawski

Venue:

University of Warsaw
Faculty of Mathematics, Informatics and Mechanics,
Banacha 2, 02-097 Warszawa
Classroom 5440 (4th (top) floor)

Schedule

Monday, June 24

- 08:00 - 08:50 – Registration
- 08:50 - 09:00 – Opening
- 09:00-10:30 – Joe Neeman, *Random processes and inference on trees and graphs (I)*
- 10:30-10:50 – coffee break
- 10:50-12:20 – Ramon van Handel, *Degenerate geometric inequalities (I)*
- 12:20-14:00 – lunch break
- 14:00 - 14:20 – Holger Sambale, *Higher order concentration of measure*
- 14:20 - 14:40 – Bartłomiej Polaczyk, *Concentration of the empirical spectral distribution of random matrices with dependent entries*
- 14:40 - 15:00 – Rafał Martynek, *Lévy-Ottaviani type inequality for Bernoulli process on an interval*
- 15:00 - 15:20 – Peter Mühlbacher, *Critical parameters of loop and Bernoulli percolation*
- 15:20 - 15:40 – coffee break
- 15:40 - 17:10 – Ivan Nourdin, *Around the Malliavin-Stein approach (I)*

Tuesday, June 25

- 09:00 - 10:30 – Ramon van Handel, *Degenerate geometric inequalities (II)*
- 10:30 - 10:50 – coffee break
- 10:50 - 12:20 – Ivan Nourdin, *Around the Malliavin-Stein approach (II)*
- 12:20 - 14:00 – lunch break
- 14:00 - 15:00 – Joscha Prochno, *Large Deviations in Asymptotic Geometric Analysis*
- 15:00 - 15:20 – coffee break
- 15:20 - 16:50 – Joe Neeman, *Random processes and inference on trees and graphs (II)*

Wednesday, June 26

- 09:00 - 10:30 – Ivan Nourdin, *Around the Malliavin-Stein approach (III)*
- 10:30 - 10:50 – coffee break
- 10:50 - 11:50 – Anna Lytova, *Anti-concentration inequalities and invertibility of random matrices*
- 11:50 - 13:30 – lunch break
- 13:30 - 14:30 – Paweł Wolff, *Brascamp-Lieb inequalities, Gaussian kernels and information theory*
- 14:30 - 14:50 – coffee break
- 14:50 - 15:10 – Alessia Caponera, *Stein-Malliavin techniques for spherical functional autoregressions*
- 15:10 - 15:30 – Anna Paola Todino, *Stein-Malliavin Approximations for Nodal Lengths of Random Spherical Eigenfunctions in Shrinking Regions*

Thursday, June 27

- 09:00 - 10:30 – Ramon van Handel, *Degenerate geometric inequalities (III)*
- 10:30 - 10:50 – coffee break
- 10:50 - 12:20 – Joe Neeman, *Random processes and inference on trees and graphs (III)*
- 12:20 - 14:00 – lunch break
- 14:00 - 15:30 – Ivan Nourdin, *Around the Malliavin-Stein approach (IV)*
- 15:30 - 15:50 – coffee break
- 15:50 - 16:10 – Giorgos Chasapis, *Affine quermassintegrals of random polytopes*
- 16:10 - 16:30 – Ben Li, *The Löwner ellipsoid function for a log-concave function*
- 16:30 - 16:50 – Konrad Krystecki, *Double finite-time ruin probability for correlated Brownian motions*
- 16:50 - 17:10 – Vladimir Fomichov, *Concentration of measure in stochastic flows*

Friday, June 28

- 09:00 - 10:30 – Ramon van Handel, *Degenerate geometric inequalities (IV)*
- 10:30 - 10:50 – coffee break
- 10:50 - 12:20 – Joe Neeman, *Random processes and inference on trees and graphs (IV)*

Abstracts

Long courses

- **Joe Neeman** (University of Texas at Austin) – *Random processes and inference on trees and graphs*

  We'll discuss random processes on trees and graphs. Of particular interest will be the notion of finite- vs. infinite-range dependence: for some processes on trees, the state of the root propagates information infinitely far, while for other processes the information effectively dies at a finite distance. The branching number of a tree will play a crucial role in this analysis.

  Having dealt with trees, we'll turn to random graphs, specifically the stochastic block model. We'll show that the behavior of certain random processes on trees can be used to bound the accuracy of inference on these graphs, and by doing so we'll prove the existence of a phase transition for "detection" in the stochastic block model. To establish one direction of this phase transition, we'll discuss algorithms for detection on the stochastic block model. For certain ranges of parameters, the existence of efficient algorithms is an interesting open problem.

- **Ivan Nourdin** (Université du Luxembourg) – *Around the Malliavin-Stein approach*

  Stein's method was invented in the 1960s by the eminent statistician Charles Stein for teaching purposes. The latter was looking for a simple and accessible way to demonstrate to his students a central combinatorial limit theorem due to Wald and Wolfowitz. Totally unrelated, Malliavin calculus was developed by the great analyst Paul Malliavin in the 1970s, in his wish to provide a fully probabilistic proof of Hörmander's famous criterion of hypoellipticity for partial differential equations. Although Stein's method and Malliavin calculus have historically pursued quite different goals, they both have in common the use of integration-by-parts techniques. This last fact was recently exploited by Ivan Nourdin and Giovanni Peccati in the invention of their so-called Malliavin-Stein approach, which represents a theory of normal approximation for probabilistic objects living in possibly infinite-dimensional Gaussian spaces. The aim of this course will be to introduce participants to this new technique, and to show them its power through examples from diverse and contemporary mathematical backgrounds

- **Ramon van Handel** (Princeton University) - *Degenerate geometric inequalities*

  It is well understood that many sharp inequalities in probability and geometry can be derived from basic monotonicity arguments involving Markov semigroups (e.g., the Ornstein-Uhlenbeck semigroup in the Gaussian case). This class includes, for example, Poincaré and log-Sobolev inequalities, the Gaussian isoperimetric inequality, and many others. Because the relevant Markov semigroups are nondegenerate, this theory provides a natural approach for investigating the optimality properties of these inequalities (e.g., their extremizers and stability properties).

  However, there exist unusual geometric inequalities that fall well outside this theory. In these lectures I aim to discuss two such inequalities: the Ehrhard inequality, which captures the sharp convexity properties of Gaussian measures; and the Alexandrov-Fenchel inequality, which lies at the heart of convex geometry. What these inequalities have in common is that they arise from Markov semigroups that are highly degenerate and whose dynamics depends on the input data, both features that do not appear in the classical theory. The degenerate nature of these inequalities causes them to behave in unexpected ways that remain poorly understood. In particular, the degeneracy gives rise to a rich set of extremizers whose investigation requires the introduction of new techniques.

The aim of these lectures is to illustrate, in the context of the above inequalities, how the analysis of degenerate Markov processes gives rise to new phenomena in the theory of geometric inequalities. Along the way we will encounter notions such as support theorems and hypoellipticity, Dirichlet form methods, (elementary) analysis on manifolds, and a zoo of exotic Markov semigroups. No prior knowledge of any of the above topics will be assumed, beyond some basic background in probability and analysis.

The material in these lectures is based on joint work with Yair Shenfeld.

Educational lectures

- **Anna Lytova** (University of Opole) – *Anti-concentration inequalities and invertibility of random matrices*

  We will discuss some results on anti-concentration properties of sums of independent random variables together with applications of these results in singularity problems for random matrices.

- **Joscha Prochno** (University of Graz) – *Large Deviations in Asymptotic Geometric Analysis*

  We will present some large deviations results for quantities and geometric structures that appear in asymptotic geometric analysis.

- **Paweł Wolff** (Université Toulouse III) – Brascamp-Lieb inequalities, Gaussian kernels and information theory

  Several important inequalities in analysis and probability such as the Hölder inequality, the sharp Young convolution inequality, the Loomis-Whitney inequality, the Gaussian hypercontractivity or the Prékopa-Leindler inequality can all be viewed as particular instances of sharp bounds on multilinear functionals with a Gaussian integral kernel (also known under the name of the Brascamp-Lieb inequalities).

  During the lecture we will review classical results on the Brascamp-Lieb inequalities, describe their links with information theoretic inequalities and present examples of applications. We will also discuss some recent developments in the area.

Short talks

- **Alessia Caponera** (Sapienza University of Rome) – *Stein-Malliavin techniques for spherical functional autoregressions*

  We present a class of space-time processes, which can be viewed as functional autoregressions taking values in the space of square integrable functions on the sphere. We exploit some natural isotropy requirements to obtain a neat expression for the autoregressive functionals, which are then estimated by a form of frequency-domain least squares. For our estimators, we are able to show consistency and limiting distributions. We prove indeed a quantitative version of the central limit theorem, thus deriving explicit bounds (in Wasserstein metric) for the rate of convergence to the limiting Gaussian distribution; to this aim we exploit the rich machinery of Stein-Malliavin methods. Our results are then illustrated by numerical simulations.

- **Giorgos Chasapis** (Kent State University) – *Affine quermassintegrals of random polytopes*

  We study a variant of one of Lutwak's conjectures on the affine quermassintegrals of a convex body: Is it true that

  $$\frac{1}{\mathrm{vol}_n(K)^{\frac{1}{n}}}\left(\int_{G_{n,k}} \mathrm{vol}_k(P_F(K))^{-n}\, d\nu_{n,k}(F)\right)^{-\frac{1}{kn}} \leqslant c\sqrt{\frac{n}{k}}$$

  holds for every convex body K in $\mathbb{R}^n$ and all $1 \leqslant k \leqslant n$, for some absolute constant $c > 0$? Here integration is with respect to the rotation-invariant probability measure $\nu_{n,k}$ on the Grassmanian $G_{n,k}$ of all k-dimensional subspaces of $\mathbb{R}^n$, and P_F denotes the orthogonal projection onto $F \in G_{n,k}$. We establish the validity of the above for a broad class of random polytopes in $\mathbb{R}^n$, that includes the case of random convex hulls with vertices chosen independently and uniformly from the interior or the surface of a convex body. Based on joint work with Nikos Skarmogiannis.

- **Vladimir Fomichov** (National Academy of Sciences of Ukraine) – *Concentration of measure in stochastic flows*

  Let $\{x(u,t),\ u \in \mathbb{R},\ t \geqslant 0\}$ be a Harris flow with covariance function $\Gamma := \varphi * \varphi$ with $*$ standing for the usual convolution operation, where the function $\varphi \in C_K^\infty(\mathbb{R}, [0; +\infty))$ is symmetric and has a unit L_2-norm. Then with probability one for any $t \geqslant 0$ the mapping $x(\cdot,t)\colon \mathbb{R} \to \mathbb{R}$ is diffeomorphic.

  For every $t \geqslant 0$ let λ_t be the image of the Lebesgue measure λ under the action of the random mapping $x(\cdot,t)\colon \mathbb{R} \to \mathbb{R}$, i. e. set

  $$\lambda_t := \lambda \circ x^{-1}(\cdot,t),$$

  where $x^{-1}(\cdot,t)$ stands for the inverse of $x(\cdot,t)$. Then all random measures λ_t, $t \geqslant 0$, are absolutely continuous with respect to λ.

In our talk we will discuss the level-crossing intensity $\mu_t(c)$ for the stationary stochastic process $\{p_t(u),\ u \in \mathbb{R}\}$ formed by the corresponding Radon–Nikodym densities $p_t := d\lambda_t/d\lambda$. First, we will compute the exact value of $\mu_t(c)$.

Theorem 1. *For any $t > 0$ we have*

$$\mu_t(c) = \frac{\sqrt{2L''} \cdot e^{\frac{\pi^2}{2L't} - \frac{L't}{8}}}{\pi L' \sqrt{\pi t}} \cdot \frac{1}{\sqrt{c}} \cdot \int\limits_0^{+\infty} \frac{e^{-\frac{v^2}{2L't}} \sinh v \sin \frac{\pi v}{L't}}{\sqrt{1 + \frac{2\cosh v}{c} + \frac{1}{c^2}}}\, dv \qquad \text{for a. e. } c > 0, \tag{1}$$

where $L' > 0$ and $L'' > 0$ are the squared L_2-norms of φ' and φ''.

Second, by adopting a probabilistic approach, we will find the exact asymptotic behaviour of the right-hand side of equality (1) as $c \to +\infty$.

Theorem 2. *Let $\overline{\mu}_t(c)$ stand for the right-hand side of equality* (1). *Then for any $t > 0$ we have*

$$\overline{\mu}_t(c) = \frac{e^{-\frac{L't}{8}} \sqrt{L''}}{\pi \sqrt{2L'}} \cdot \sqrt{\frac{c}{\ln c}} \cdot \exp\left[-\frac{(\ln c)^2}{2L't}\right] \cdot (1 + \overline{o}(1)), \quad c \to +\infty.$$

Bibliography
V. V. Fomichov, *Level-crossing intensity for the density of the image of the Lebesgue measure under the action of a Brownian stochastic flow*, Ukrainian Mathematical Journal **69**:6 (2017) 803–822. (in Russian)

- **Konrad Krystecki** (University of Wrocław) – *Double finite-time ruin probability for correlated Brownian motions*

  We focus on deriving the asymptotics of suprema of correlated Brownian motions with drift on finite time interval, i.e. we analyze the following probability

  $$\mathbb{P}\Big(\sup_{s\in[0,1]} W_1(s) - c_1 s > u, \sup_{t\in[0,1]} W_2(t) - c_2 t > au\Big)$$

  as $u \to \infty$. We derive the exact asymptotics of the probability above and study the influence of the dependence between a and the correlation of W_1 and W_2 on the results.

- **Ben Li** (Tel Aviv University) – *The Löwner ellipsoid function for a log-concave function*

  We introduce the notion of Löwner (ellipsoid) function for a log concave function and show that it is an extension of the Löwner ellipsoid for convex bodies. We investigate its duality relation to the recently defined John (ellipsoid) function. For convex bodies, John and Löwner ellipsoids are dual to each other. Interestingly, this need not be the case for the John function and the Löwner function.

- **Rafał Martynek** (University of Warsaw) – *Lévy-Ottaviani type inequality for Bernoulli process on an interval*

  In this talk I will present a tail domination result for the Bernoulli process with coefficients, which are sequences of monotone functions reaching their maxima on the interval. The approach is based on the chaining method and a special form of the concentration result. I will outline possible generalizations of the result and show a connection with the classical concentration.

- **Peter Mühlbacher** (University of Warwick)– *Critical parameters of loop and Bernoulli percolation*

  We consider a class of random loop models on graphs (including the random interchange process) that are parametrised by a time parameter $T \geq 0$. Intuitively, larger T means more randomness. In particular, at $T = 0$ we start with loops of length 1. As T increases, a phase transition for infinite cycles is conjectured for graphs with sufficiently high vertex degree, i.e. for $T > T_c$ there is a.s. at least one infinite loop while for $T < T_c$ there is a.s. none. Our random loop models admit a natural comparison to Bernoulli percolation to obtain a lower bound on T_c. If time permits we sketch a novel proof that this lower bound is not sharp on a class of graphs including $\mathbb{Z}^d$

- **Bartłomiej Polaczyk** (University of Warsaw) – *Concentration of the empirical spectral distribution of random matrices with dependent entries*

  Let X_n be a Hermitian random matrix of size $n \times n$ that can be split into independent blocks of the size at most $d_n = o(n^2)$. We prove that under some mild conditions on the distribution of the entries of X_n, the empirical spectral measure L_n of X_n concentrates around its mean.

  As a consequence, we obtain that whenever L_n converges in mean then it also converges in probability.

  Bibliography
  B.P., *Concentration of the empirical spectral distribution of random matrices with dependent entries*, ArXiv e-prints (2018).

- **Holger Sambale** (Bielefeld University) – *Higher Order Concentration of Measure*

  Abstract: We investigate higher order versions of the concentration of measure phenomenon. By this, we refer to tail estimates for functions which are typically non-Lipschitz but whose tail behaviour can be controlled by the derivatives (or differences) of order up to some natural number d. Sometimes, the functions are moreover assumed to be centered at stochastic expansions of a lower order, say $d-1$. This leads to exponential bounds for $|f|^{2/d}$ or multilevel concentration inequalities (like Hanson–Wright-type inequalities for $d = 2$). In many cases, our results are derived from log-Sobolev or modified log-Sobolev-type inequalities by means of the entropy method. A special focus is put on functions of weakly dependent random variables, where the dependence might be controlled by a Dobrushin-type condition, for instance. Examples include spin systems like the Ising model or exponential random graph models. This is joint work with S. Bobkov, F. Götze and A. Sinulis.

- **Anna Paola Todino** (Ruhr-Universität Bochum) – *Stein-Malliavin Approximations for Nodal Lengths of Random Spherical Eigenfunctions in Shrinking Regions*

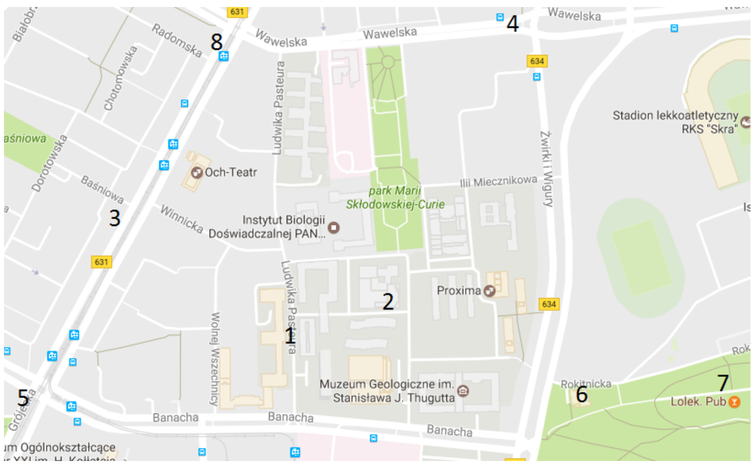
  Recently, considerable interest has been drawn by the analysis of geometric functionals (Lipschitz-Killing curvatures, hereafter LKCs) for the excursion sets of random eigenfunctions on the unit sphere (spherical harmonics). In dimension 2, LKCs correspond to the area, half of the boundary length and the Euler-Poincaré characteristic; the asymptotic behavior of their expected values and variances have been investigated and quantitative central limit theorems have been established in the high energy limits. In this talk we extend these results to local behavior; more precisely, we consider Nodal Lengths in shrinking domains and after computing their asymptotic variances, we establish a Central Limit Theorem exploiting Wiener chaos expansions and Stein-Malliavin techniques.

Practical information

Internet access

A wi-fi network is available for visitors:
SSID: wssp password: Warsaw2019.
Additionally there is an Eduroam network available in the whole building.

Where to eat



Here is a short list of lunch options within the walking distance from the conference venue

1. A cafeteria at the Faculty of Mathematics, Informatics and Mechanics - 3rd floor

2. A cafeteria at the Faculty of Biology

3. Sedo Kebab - a Turkish style grill bar. Grójecka Street

4. Aceto Balsamico - a small Italian style restaurant. Wawelska Street

5. Van Binh - a Vietnamese restaurant. Grójecka street

6. Jeff's – an American style restaurant. Pole Mokotowskie

7. Lolek Pub – a barbecue place, a little bit farther away from the conference venue. Pola Mokotowskie

8. Bar Smak - a relatively cheap diner remembering the times of the Polish People's Republic (doesn't seem to have changed much since the 1980s). For the adventurous. Grójecka Street.

Transportation

The public transportation in Warsaw works quite efficiently. During the rush hours we recommend using trams or the metro. The best way to schedule your trip is to use `http://warszawa.jakdojade.pl/?locale=en`. The closest stop from the conference venue is called "Och-teatr". The most common types of tickets are: valid for 20 mins for any number of means of transport (costs 3.4PLN), 75 mins (costs 4.4PLN), 24 hour (costs 15PLN). The tickets can be purchased in kiosks, shops and in a number of vending machines located at bus or tram stops. The tickets have to be **validated** during the first journey after boarding the bus or tram. On the metro one should validate the ticket before entering the platform.
There are multiple companies offering taxi services, you can hail them on the street. The prices vary from 1.6PLN-2.4PLN/km (day) and 2.4PLN-3.6PLN/km (night) plus the entrance fee. They are typically reliable and safe. Arguably best (and most expensive) are Ele-Taxi, MPT, Sawa. Another option is using applications such as Uber, Bolt or mytaxi. Close to the conference venue you can also find one of many public bike rental stations, to rent a bike you need to register on the web page `https://www.veturilo.waw.pl/en/`.

Money

The currency in Poland is złoty (pronounced "zwoty"). The approximate exchange rate is:

- 1EUR=4.26PLN,
- 1USD=3.75PLN,
- 1CHF=3.80PLN,

There are a number of ATMs on Grójecka Street, close to the conference venue.