Weak Bisimilarity and Open Maps

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Abstract. An alternative proof (to that from [3]) is presented for weak bisimilarity of transition systems to be abstractly definable by open maps [9]. This result arises from an observation that the categories (of transition systems) well suited for studying strong and weak bisimulations are related by an adjunction (for a suitable monad), giving a link between both bisimilarities. We formulate a generalization of this result, hopefully applicable also to other equivalences of processes.

1 Introduction

Recently a categorical generalization of bisimulation was proposed, by means of *open maps* (*open morphisms*) [9, 8], enabling a uniform definition of bisimulation-like equivalences across a range of different models for parallel computations. This setting turned out appropriate for defining, among many others, strong and weak bisimilarity [12], trace equivalence [7], testing equivalence [6] and a bisimilarity of event structures [5]. Open maps can be understood as arrows witnessing a bisimulation, hence two objects A and B in a category are bisimilar if they are related by a span of open maps, representing abstractly a bisimulation.

$$\begin{tikzcd} & C \arrow[dl] \arrow[dr] & \\ A & & B \end{tikzcd} \tag{1}$$

In [3] one can find an overview of different equivalence definable by means of open maps. Open maps were also successfully applied to behavioural equivalences [1] of algebras, see [10].

In this paper we focus on weak bisimilarity of transition systems, proved already in [3] to coincide with open-maps bisimilarity in the category of transition systems. The method was similar as in the case of strong bisimilarity, that is open maps were characterized as those satisfying a suitable zig-zag condition, to be mentioned below. The only difference to strong bisimilarity is that category which turned appropriate to work in, was richer in morphisms than the usually considered category of transition systems (which is well suited for strong bisimilarity). We found an abstract categorical characterization of this category as

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the Kleisli category for a suitable monad on the category of transition systems. Both categories under consideration are moreover linked by a pair of functors forming an adjunction, which we show to be reflective. These observations lead directly to an alternative prove of coincidence of weak bisimilarity and open-maps bisimilarity, which does not require any explicit characterization of open maps. This is due to properties of open maps, proved recently in [10], which allow to "transport" openness of morphisms (and bisimilarity) via an adjunction.

On a more intuitive level, the adjunction gives an elegant connection between (abstract formulations of) strong and weak bisimilarity. There still rest an interesting question, whether this situation can be shifted to some other equivalences of processes? This is why all the properties of preservation of openness and bisimilarity, formulated in this paper as well as in [10], are intended to be re-usable and as general as possible.

We assume the reader to have some prior knowledge of category theory, in particular to be familiar with adjunctions and monads. As a reference, we propose [11].

2 Bisimulation from Open Maps

Let $\mathcal{U}$ be a category of models of computation, in which we choose a subcategory $\mathcal{P}$ (not necessarily full) of *observation objects*. $\mathcal{P}$ is also called in the sequel a *path subcategory*, as it is intended to contain „paths" for computations to follow. Any morphism $p : O \to A$ from an observation object $O \in |\mathcal{P}|$ is understood as an observable *computation* in A. A morphism $h : A \to B$ between models can be intuitively thought of as a simulation of A in B since h transforms every computation $p : O \to A$ in A to a computation $p; h : O \to B$ in B. Moreover any morphism $m : O \to O'$ in $\mathcal{P}$ making $p = m; p'$ means intuitively that a "larger" computation p' is an extension of p (via m).

In a definition below, we distinguish moreover a subcategory of models of interest $\mathcal{M} \subseteq \mathcal{U}$, and consider bisimilarity only in $\mathcal{M}$. This way we gain a notion slightly more general than usually (when $\mathcal{M} = \mathcal{U}$), following the approach of [3]. Path subcategory $\mathcal{P}$ is not required to be a subcategory of $\mathcal{M}$.

Definition 2.1 (Open maps and bisimilarity). A morphism $h : A \to B$ in $\mathcal{M}$ is $\mathcal{P}$-*open* if for any morphism $m : O \to O'$ in $\mathcal{P}$ and two computations $p : O \to A$ and $p' : O' \to B$ in $\mathcal{U}$, whenever the square

$$\begin{tikzcd} O \arrow[r, "p"] \arrow[d, "m"'] & A \arrow[d, "h"] \\ O' \arrow[r, "p'"] & B \end{tikzcd} \qquad\qquad \begin{tikzcd} O \arrow[r, "p"] \arrow[d, "m"'] & A \arrow[d, "h"] \\ O' \arrow[r, "p'"] \arrow[ur, "r"] & B \end{tikzcd}$$

commutes, i.e. $p; h = m; p'$, there exists a diagonal morphism $r : O' \to A$ in $\mathcal{U}$ making two triangles commute, i.e. $p = m; r$ and $p' = r; h$. Two objects A and B are $\mathcal{P}$-*bisimilar*, denoted by $A \sim_{\mathcal{P}} B$, if there exists in $\mathcal{M}$ a span of $\mathcal{P}$-open maps as in (1). We omit prefix $\mathcal{P}$- when obvious from a context. If not stated otherwise, we assume $\mathcal{U} = \mathcal{M}$.

The abstract notion of $\mathcal{P}$-bisimilarity is intended to generalize the strong bisimilarity between transition systems, which will serve as an illustrating example.

2.1 Strong Bisimulation

A *labelled transition system* [13] (over a set L, intended to give labels to transitions) is a triple $T = (S, i, \{ \xrightarrow{a} \}_{a \in L})$ consisting of a set of *states* S with a distinguished initial state $i \in S$ and a family of *transition relations* $\_ \xrightarrow{a} \_ \subseteq S \times S$. We write $s \xrightarrow{a} s'$ to denote that states s and s' are related by a transition labelled by a.

Morphisms between transition systems are ,,structure preserving" functions mapping states to states. Formally, a morphism from $T_1 = (S_1, i_1, \{ \xrightarrow{a} \}_{a \in L})$ to $T_2 = (S_2, i_2, \{ \xrightarrow{a} \}_{a \in L})$, is a function $\sigma : S_1 \to S_2$, such that $\sigma(i_1) = i_2$ and $s \xrightarrow{a} s'$ implies $\sigma(s) \xrightarrow{a} \sigma(s')$ (we deliberately overload here symbol $\xrightarrow{a}$, hoping that this causes no troubles). This defines a category TS_L, in which morphisms compose as functions.

Following [14, 12], we say that T_1 and T_2 are *strongly bisimilar* if there exists a strong bisimulation between them; bisimulation is defined as usual, with the only additional requirement to relate initial states. Strong bisimilarity was shown in [9] to coincide with $Bran_L$-bisimilarity in TS_L, where $Bran_L$ is the full subcategory of transition systems consisting of finite sequences of actions:

$$i \xrightarrow{a_1} s_1 \xrightarrow{a_2} \ldots \xrightarrow{a_n} s_n \tag{2}$$

$Bran_L$-open morphisms $\sigma : T_1 \to T_2$, called also *zig-zag*, are those satisfying the following *zig-zag* property: for each reachable $s \in S_1$, whenever $\sigma(s) \xrightarrow{a} s'$, then $s \xrightarrow{a} s''$, for some $s'' \in S_1$ satisfying $\sigma(s'') = s'$.

3 Weak Bisimulation

Consider transition systems over an alphabet L, fixed in the sequel. We assume L to contain a distinguished action τ, being silent or non-observable. *Weak bisimulation*, being less restrictive than strong one, allows an action a to be simulated by a sequence of the form

$$\xrightarrow{\tau} \ldots \xrightarrow{\tau} \xrightarrow{a} \xrightarrow{\tau} \ldots \xrightarrow{\tau} \tag{3}$$

that is by a preceded or followed by an arbitrary number of τ actions. Moreover, τ action need not to be simulated at all. In the formal definition below we use the following notation, proposed in [12]. First, let $s \overset{a}{\Longrightarrow} s'$ denote a sequence of transitions (3), for an arbitrary label a (including τ), that is

$$s \overset{a}{\Longrightarrow} s' \;\text{ iff }\; s(\xrightarrow{\tau})^* r \xrightarrow{a} r'(\xrightarrow{\tau})^* s' \text{ for some states r and r'},$$

where $(\xrightarrow{\tau})^*$ stands for the reflexive-transitive closure of $\xrightarrow{\tau}$. Second, treating an action a as a one-element sequence, we define a τ-deleting function $\widehat{\ } : L \to L^*$

by $\widehat{a} =_{def} a$, for $a \neq \tau$, and $\widehat{\tau} =_{def} \epsilon$, the empty sequence. Moreover, we assume $s \xrightarrow{\epsilon} s' \Leftrightarrow s = s'$, implicitly extending the notation $\_ \xrightarrow{a} \_$ to the set L^* of finite sequences of actions.

Definition 3.1. *A weak bisimulation* between T_1 and T_2 is any relation $\sim \subseteq S_1 \times S_2$ such that $i_1 \sim i_2$ and whenever $s_1 \sim s_2$ then for every action $a \in L$,

1. $s_1 \xrightarrow{a} s_1'$ in T_1 implies $s_2 \xRightarrow{\widehat{a}} s_2'$ in T_2 and $s_1' \sim s_2'$, for some s_2',
2. $s_2 \xrightarrow{a} s_2'$ in T_2 implies $s_1 \xRightarrow{\widehat{a}} s_1'$ in T_1 and $s_1' \sim s_2'$, for some s_1'.

If such $\sim$ exists, we say that T_1 and T_2 are *weakly bisimilar*.

To deal with weak bisimilarity we choose the appropriate category WTS_L of transition systems whose morphisms $\sigma : T_1 \to T_2$ are functions $\sigma : S_1 \to S_2$, such that $\sigma(i_1) = i_2$ and $s \xrightarrow{a} s'$ implies $\sigma(s) \xRightarrow{\widehat{a}} \sigma(s')$. These morphisms will be called *weak morphisms*; the usual morphisms of transition systems we call *strong* in the sequel. Obviously category TS_L, consisting of strong morphisms, is a subcategory of WTS_L.

As a path subcategory for weak bisimulation we take again the subcategory $Bran_L$ of finite linear transition systems as in (2) together with exclusively strong morphisms between them. Surprisingly, this is the same path subcategory as in the case of strong bisimilarity – this coincidence will be justified and explained below. Obviously $Bran_L$ is not a full subcategory of WTS_L, being full subcategory of TS_L. In [3] it was shown that weak bisimilarity coincides with $Bran_L$-bisimilarity in category of weak morphisms; it was achieved by explicit characterization of $Bran_L$-open maps as those satisfying an appropriate zig-zag property, analogously as in the case of strong bisimilarity.

4 Characterization

Our aim is to show that spans of $Bran_L$-open morphisms in category WTS_L define weak bisimilarity without referring to any explicit formulation of openness. Our considerations will be based on the observation that bisimilarity induced by open morphisms can be transported in some relevant cases via an adjunction. First, open maps are well-behaved with respect to an adjunction:

Lemma 4.1 ([10]). *For arbitrary adjunction $F \dashv G$ between categories $\mathcal{M}$ and $\mathcal{N}$ and for arbitrary subcategory $\mathcal{P}$ of $\mathcal{M}$,*

$$\text{a morphism } h \text{ in } \mathcal{N} \text{ is } F(\mathcal{P})\text{-open} \iff G(h) \text{ is } \mathcal{P}\text{-open in } \mathcal{M}.$$

Second, despite that bisimilarity itself is not transported in a similar way in general, it does when an adjunction is a reflection:

Lemma 4.2. *Assumed that the adjunction from the previous lemma is reflective (i.e. the right adjoint $G : \mathcal{N} \to \mathcal{M}$ is full and faithful), we have*

$$A \sim_{F(\mathcal{P})} B \text{ in category } \mathcal{N} \iff G(A) \sim_{\mathcal{P}} G(B) \text{ in category } G(\mathcal{N})$$

($G(\mathcal{N})$, image of G, is a subcategory of $\mathcal{M}$).

Proof: Assumed that A and B are $F(\mathcal{P})$-bisimilar in $\mathcal{N}$, that is they are related by a span $A \xleftarrow{f} C \xrightarrow{g} B$ of $F(\mathcal{P})$-open maps in $\mathcal{N}$, we can easily construct a span of $\mathcal{P}$-open maps by taking the image of G on C, f and g. By Lemma 4.1, both morphisms forming the span $G(A) \xleftarrow{G(f)} G(C) \xrightarrow{G(g)} G(B)$ are $\mathcal{P}$-open, hence $G(A)$ and $G(B)$ are $\mathcal{P}$-bisimilar.

For the opposite direction, let A and B be arbitrary objects of $\mathcal{N}$ such that $G(A)$ and $G(B)$ are $\mathcal{P}$-bisimilar in $G(\mathcal{N})$. This means that $G(A)$ and $G(B)$ are related by a span $G(A) \xleftarrow{f} C \xrightarrow{g} G(B)$ of $\mathcal{P}$-open maps and moreover there exist some morphisms in $\mathcal{N}$, say f' and g', which are mapped by G to f and g, respectively. Let C' and D' denote domains of f' and g', respectively. Since $G(C') = G(D')$ and G, being full and faithful, reflects isomorphisms, we conclude $C' \simeq D'$ and obtain a span $A \xleftarrow{f'} C' \simeq D' \xrightarrow{g'} B$. Moreover, f' and g' are $F(\mathcal{P})$-open, again by Lemma 4.1, hence $A \sim_{F(\mathcal{P})} B$. $\square$

A careful reader could have noticed that for $\mathcal{P}$-bisimilarity in category $G(\mathcal{N})$, the path subcategory $\mathcal{P}$ is not guaranteed to be a subcategory of $G(\mathcal{N})$. This motivates our general definition of bisimilarity in Section 2.

A fact similar to Lemma 4.1 was proved already in [9], in the situation when the adjunction is a coreflection. For other related results one can consult [4] and [2], where conditions are given for a functor to preserve openness.

We are going now to construct an adjunction between categories TS_L and WTS_L, to which we will apply Lemma 4.2. The adjunction will be obtained automatically by noticing that WTS_L is precisely the Kleisli category for the monad of a suitable endofunctor $\mathcal{W} : TS_L \to TS_L$, defined below. Intuitively speaking, for a transition system T, $\mathcal{W}(T)$ has the same states but more transitions; these extra ones are all of the form $s \stackrel{\widehat{a}}{\Longrightarrow} s'$. Hence $\mathcal{W}$ can be thought of as a closure on all "weak transitions" $s \stackrel{\widehat{a}}{\Longrightarrow} s'$. Formally, for $T = (S, i, \{ \xrightarrow{a} \}_{a \in L})$ we take:

$$\mathcal{W}(T) = (S, i, \{ \stackrel{\widehat{a}}{\Longrightarrow} \}_{a \in L}).$$

For a morphism $f : \mathcal{W}(T_1) \to \mathcal{W}(T_2)$ we put:

$$\mathcal{W}(f) = f : \mathcal{W}(T_1) \to \mathcal{W}(T_2)$$

i.e. $\mathcal{W}$ takes f to the same function. It can be easily checked that functor $\mathcal{W}$ equipped with two natural transformations $\eta : Id_{TS_L} \stackrel{\cdot}{\to} \mathcal{W}$ and $\mu : \mathcal{W}^2 \stackrel{\cdot}{\to} \mathcal{W}$, consisting of identity functions

$$\eta_T(s) = s, \quad \mu_T(s) = s$$

form a monad, i.e. satisfy the monad laws (cf. [11], page 133). Now consider the Kleisli category for this monad, having the same objects as TS_L and whose morphisms $T_1 \to T_2$ are all morphisms $T_1 \to \mathcal{W}(T_2)$ from TS_L. We obtain for free the canonical adjunction between TS_L and the Kleisli category; moreover

Lemma 4.3. *This adjunction is a reflection.*

Proof: We will show that counit ε is a natural isomorphism, that is $T \simeq \mathcal{W}(T)$ in the Kleisli category. To this aim we will show that the following two morphisms

$$l := \eta_T; \eta_{\mathcal{W}(T)} : T \to \mathcal{W}(T) \quad \text{and} \quad r := \varepsilon_T = id_{\mathcal{W}(T)} : \mathcal{W}(T) \to T$$

compose to identities in the Kleisli category. (Codomains of l and r above are in the Kleisli category; note that these morphisms are also morphisms in TS_L, but here they have other codomain objects: $l : T \to \mathcal{W}^2(T)$ and $r : \mathcal{W}(T) \to \mathcal{W}(T)$.) Start by observing that

$$\eta_{\mathcal{W}(T)} = \mathcal{W}(\eta_T). \tag{4}$$

Having this and applying monad laws, it is easy to show that (composing in TS_L)

$$l; \mathcal{W}(r); \mu_T = \eta_T; \eta_{\mathcal{W}(T)}; \mu_T = \eta_T \quad \text{and}$$

$$r; \mathcal{W}(l); \mu_{\mathcal{W}(T)} = id_{\mathcal{W}(T)}; \mathcal{W}(\eta_T); \mathcal{W}(\eta_{\mathcal{W}(T)}); \mu_{\mathcal{W}(T)} = \mathcal{W}(\eta_T) = \eta_{\mathcal{W}(T)}$$

i.e. both l and r compose in both cases to identities in the Kleisli category, which are as usual units of the monad. □

Finally, one can see that the Kleisli category we are considering is just another formulation of category WTS_L consisting of weak morphisms. Now, applying Lemma 4.2 to the adjunction between TS_L and WTS_L we obtain the following:

Proposition 4.4. *Weak bisimilarity coincides with $Bran_L$-bisimilarity in category WTS_L.*

Proof: Let $F_{\mathcal{W}} \dashv G_{\mathcal{W}}$ denote the functors of the adjunction, with the left adjoint $F_{\mathcal{W}} : TS_L \to WTS_L$. Observe that $F_{\mathcal{W}}(f) = f$, hence $F_{\mathcal{W}}(Bran_L) = Bran_L$. Now, by Lemma 4.2 two transition systems T and U are connected by a span of $Bran_L$-open maps in WTS_L if and only if

$$\mathcal{W}(T) \text{ and } \mathcal{W}(U) \text{ are related by a span of } Bran_L\text{-open maps in } G_{\mathcal{W}}(WTS_L). \tag{5}$$

On the other hand, notice that weak bisimulation could be defined equivalently (cf. [12]) by replacing in Definition 3.1 requirements 1 and 2 by

1. $s_1 \overset{\widehat{a}}{\Longrightarrow} s_1'$ in T_1 implies $s_2 \overset{\widehat{a}}{\Longrightarrow} s_2'$ in T_2 and $s_1' \sim s_2'$, for some s_2',
2. analogously,

which means that T and U are weakly bisimilar if and only if $\mathcal{W}(T)$ and $\mathcal{W}(U)$ are strongly bisimilar (more precisely, weak bisimulations between T and U are precisely strong bisimulations between $\mathcal{W}(T)$ and $\mathcal{W}(U)$). Hence T and U are weakly bisimilar if and only if

$$\mathcal{W}(T) \text{ and } \mathcal{W}(U) \text{ are related by a span of } Bran_L\text{-open maps in } TS_L. \tag{6}$$

Now we only need to fill the gap between (5) and (6), different only in the category, the spans of open maps are to come from; it suffices to show (6) $\Rightarrow$ (5). Consider any $Bran_L$-open morphism $f : V \to \mathcal{W}(T)$ in TS_L. Note that it is necessarily its own adjunct $f^{\#} = f : V \to T$ in WTS_L. We will show that $G_{\mathcal{W}}(f^{\#}) = \mathcal{W}(f); \mu_T$ is also $Bran_L$-open. First, counit μ_V is open, being an identity function. Moreover, $\mathcal{W}(f) : \mathcal{W}(V) \to \mathcal{W}^2(T)$ is also open (i.e. zig-zag), since zig-zag property from Section 2.1 can be easily proved to hold also for sequences of the form (3). Hence, if $\mathcal{W}(T)$ and $\mathcal{W}(U)$ are connected by a span of $Bran_L$-open maps from V, then they are also connected by a span of $Bran_L$-open maps from $\mathcal{W}(V)$, laying necessarily in $G_{\mathcal{W}}(WTS_L)$. $\square$

We would like to stress on that in the proof we did not need refer to any explicit characterization of $Bran_L$-open maps in WTS_L, despite that such a characterization exists and is given by a zig-zag condition similar to that in Section 2.1 (cf. [3]): a morphism $\sigma : (S_1, i_1, t_1) \to (S_2, i_2, t_2)$ is $Bran_L$-open in WTS_L iff for each reachable $s \in S_1$, whenever $\sigma(s) \xrightarrow{a} s'$, then $s \overset{\widehat{a}}{\Longrightarrow} s''$, for some $s'' \in S_1$ satisfying $\sigma(s'') = s'$. Moreover, it is an interesting observation, that the choice of $Bran_L$ for the path subcategory for weak bisimulation seems to be the only reasonable one. For instance, one can easily check that we obtain a different equivalence when we replace $Bran_L$ by the full subcategory of WTS_L of finite linear transition systems. Surprisingly, bisimilarity induced in such a case would even not relate the following two weakly bisimilar transition systems

$$\bullet \qquad\qquad\qquad \bullet \xrightarrow{\tau} \bullet$$

one of them consisting exclusively of a single initial state.

5 Generalization

Let $(\mathcal{W}, \eta, \mu)$ be a monad on some category $\mathcal{M}$. Let $\mathcal{M}_{\mathcal{W}}$ denote its Kleisli category and $F_{\mathcal{W}} \dashv G_{\mathcal{W}}$ denote the usual adjunction between $\mathcal{M}$ and $\mathcal{M}_{\mathcal{W}}$.

Motivated by the example of weak bisimilarity and by its equivalent formulation in points 1 and 2 in the proof of Proposition 4.4, and especially by observation (6), we propose the following general definition:

Definition 5.1. Objects A and B of $\mathcal{M}$ are *$\mathcal{P}$-bisimilar w.r.t. $\mathcal{W}$*, for a subcategory $\mathcal{P}$ of $\mathcal{M}$, if $\mathcal{W}(A)$ and $\mathcal{W}(B)$ are $\mathcal{P}$-bisimilar (in $\mathcal{M}$).

Before we state a generalization of Proposition 4.4 (Theorem 5.2), let us analyze properties satisfied by $\mathcal{W}$ in the previous section. First, the monad is *idempotent* in the sense that μ is a natural isomorphism there. This is sufficient in a general situation for (4), hence implies $G_{\mathcal{W}}$ to be full and faithful. Second, open maps are preserved by $\mathcal{W}$, i.e. whenever f is open, $\mathcal{W}(f)$ is open as well. (This fact is implicitly used for proving Proposition 4.4.) Fortunately, these two properties guarantee that $\mathcal{P}$-bisimilarity w.r.t. $\mathcal{W}$ is definable by means of open maps:

Theorem 5.2. *$\mathcal{P}$-bisimilarity w.r.t. $\mathcal{W}$ coincides with $F_{\mathcal{W}}(\mathcal{P})$-bisimilarity in Kleisli category $\mathcal{M}_{\mathcal{W}}$, when monad $(\mathcal{W}, \eta, \mu)$ is idempotent and $\mathcal{W}$ preserves $\mathcal{P}$-openness.*

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