May a computer be wrong?

MICHAŁ SKRZYPCZAK
Institute of Informatics

Latest Discoveries in Informatics

6th March 2024

Computers have layers...

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Hardware:

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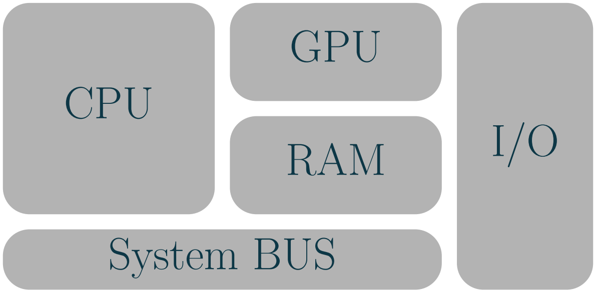
Hardware:



Logic + Physics

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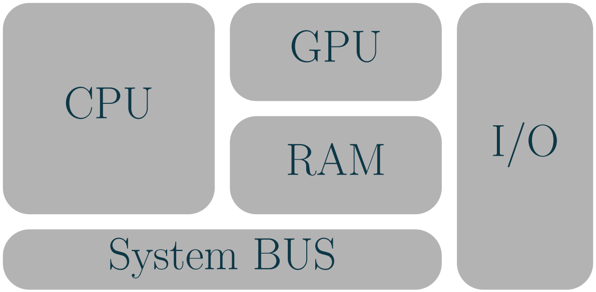

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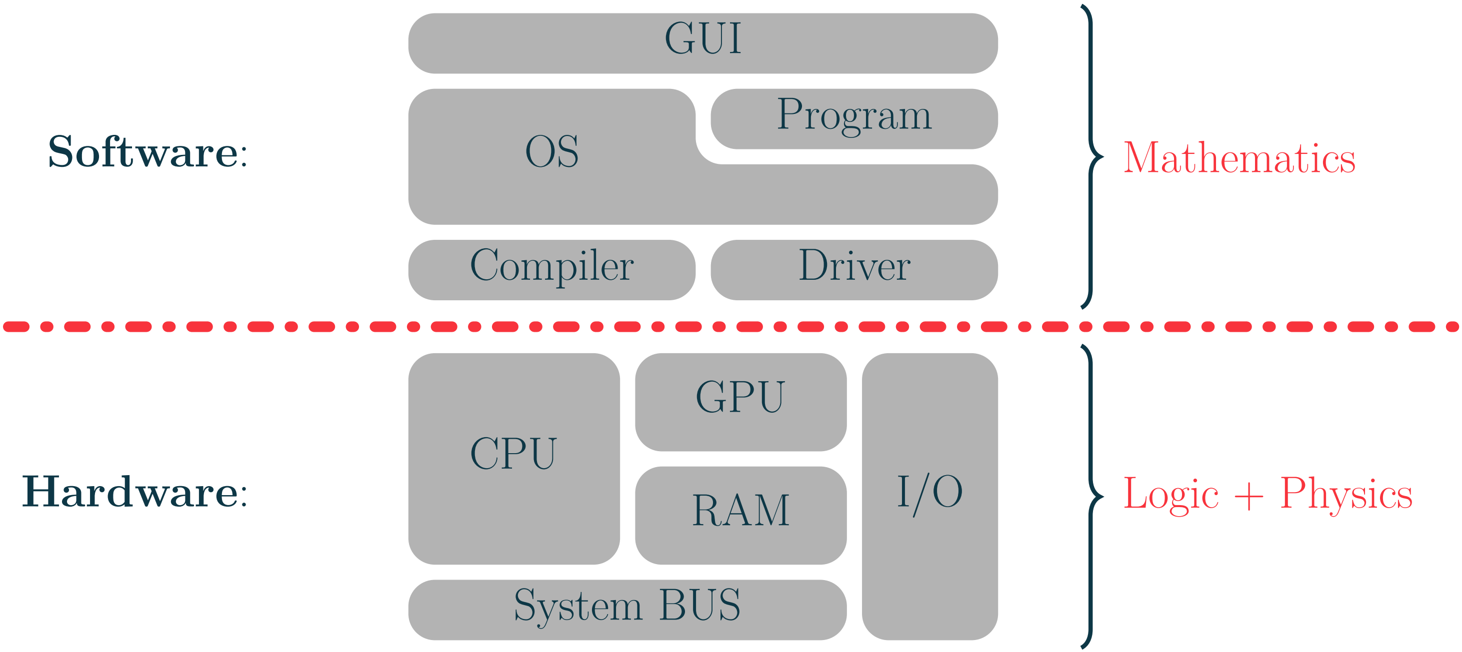

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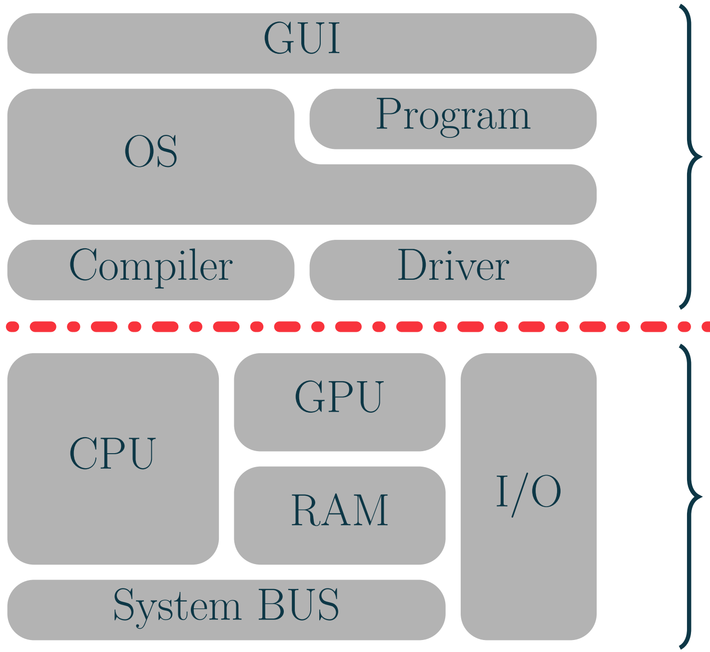

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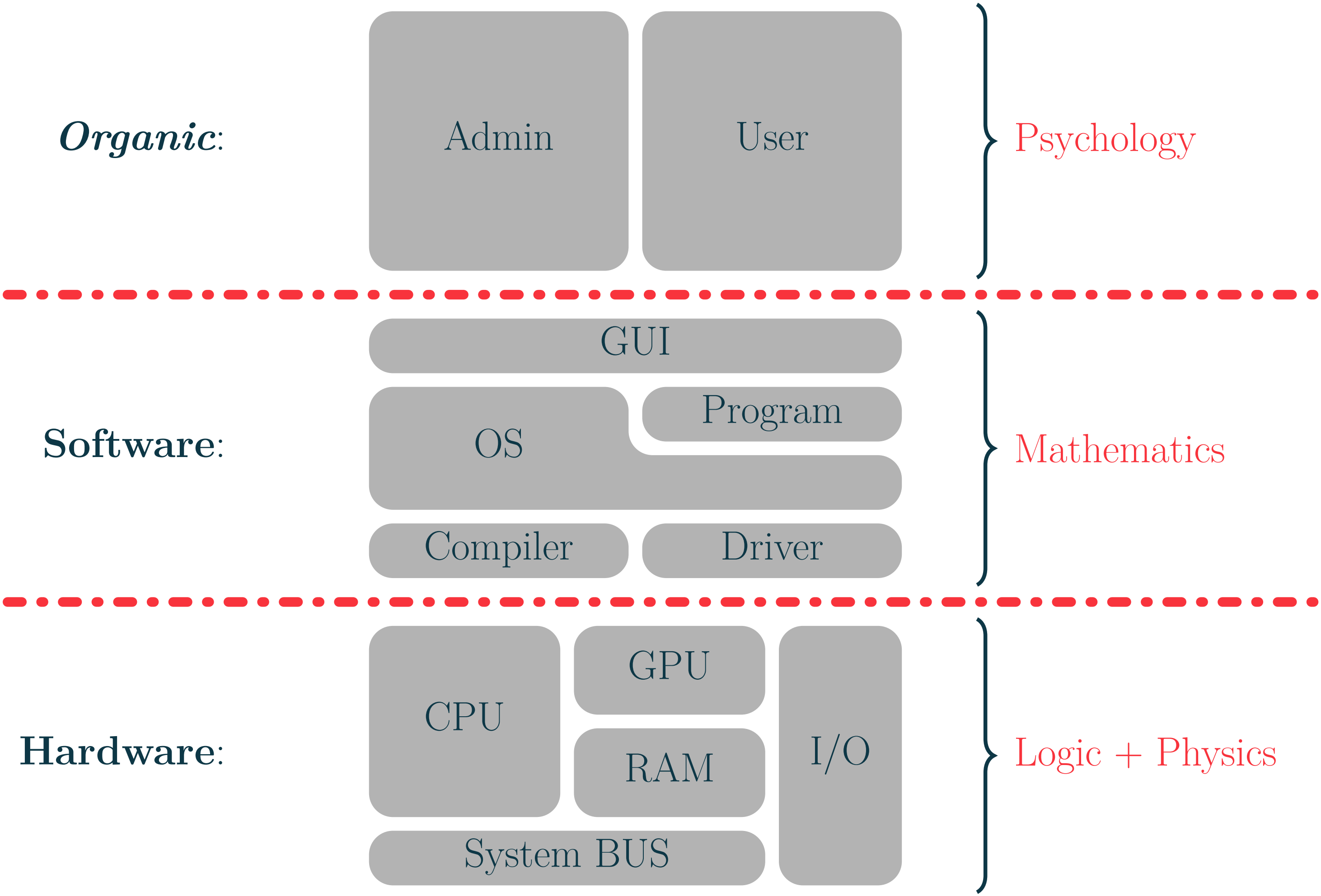

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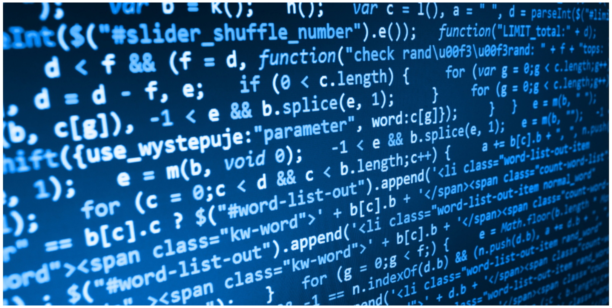

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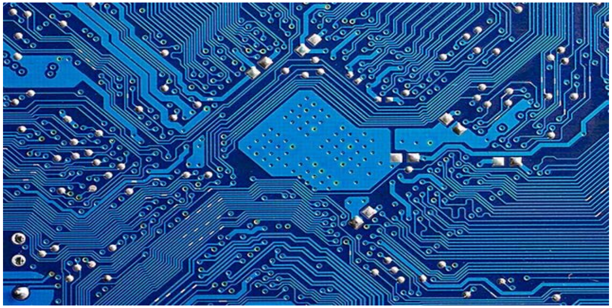

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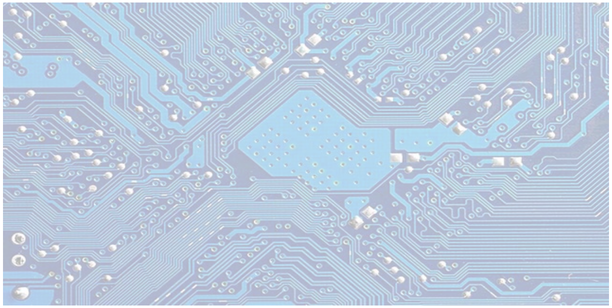

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errare humanum est

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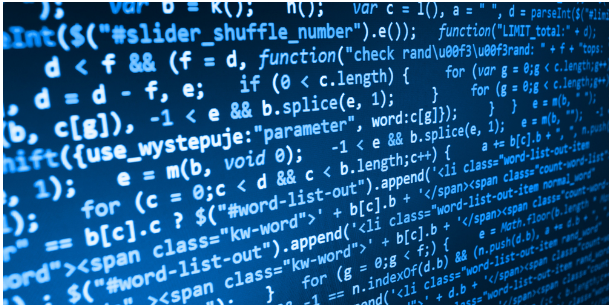

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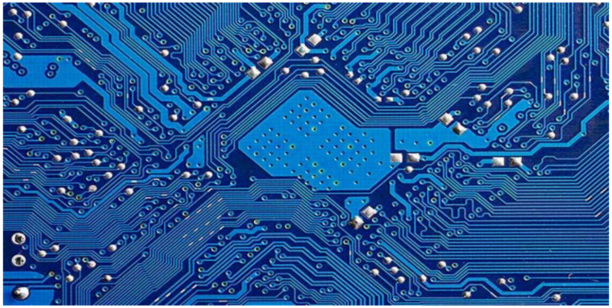

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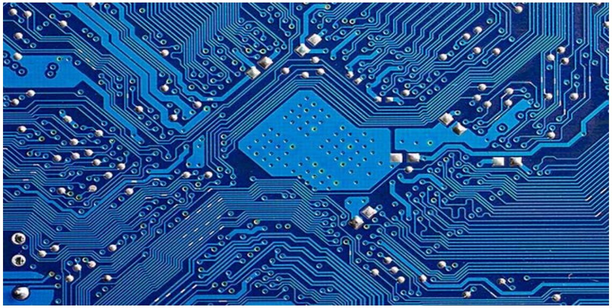

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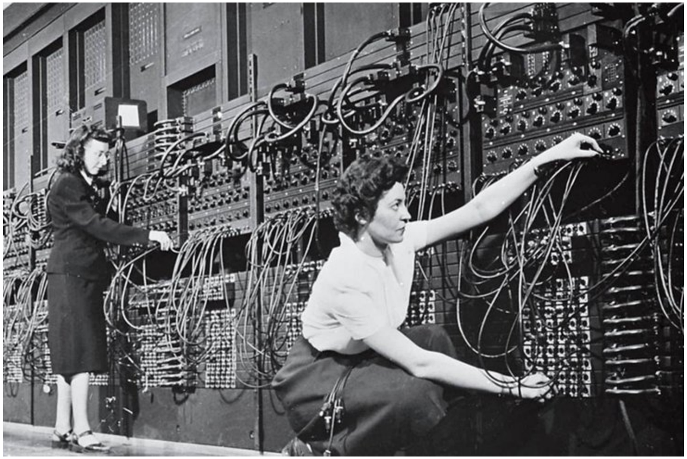

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- ∼20'000 vacuum tubes
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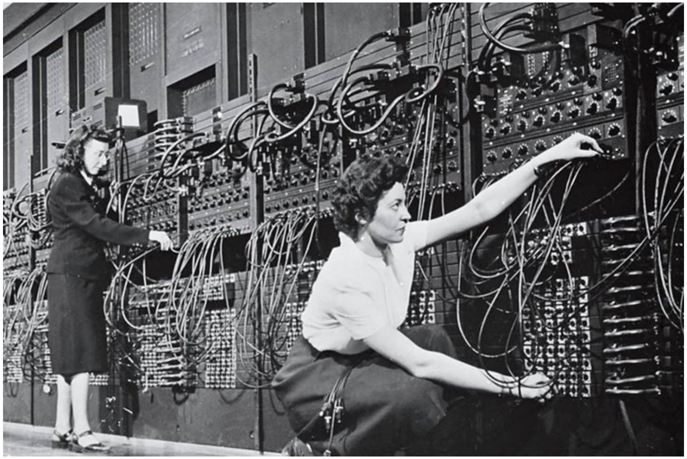

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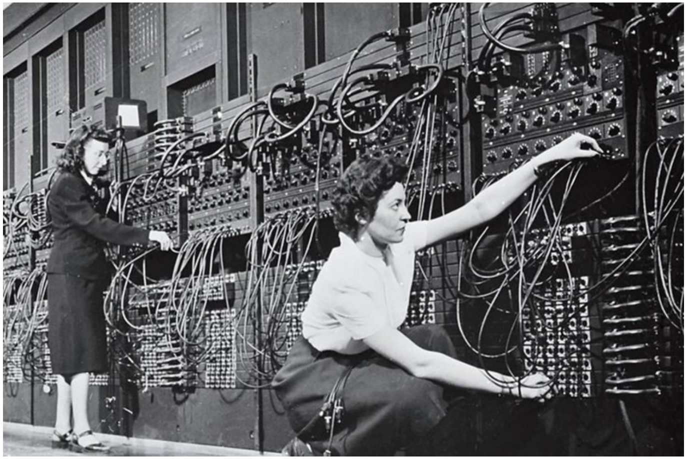

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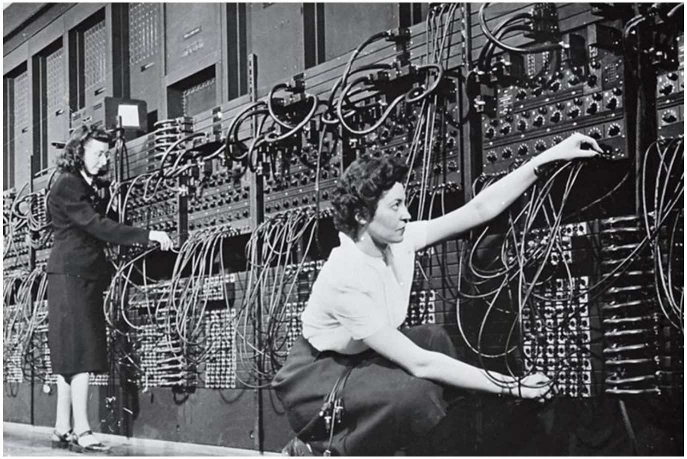

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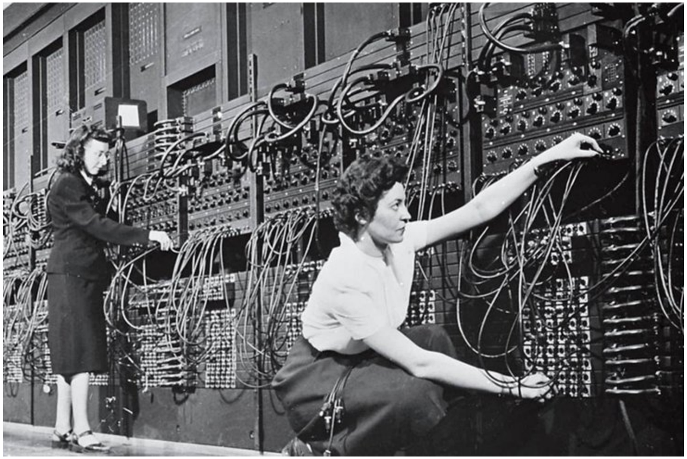

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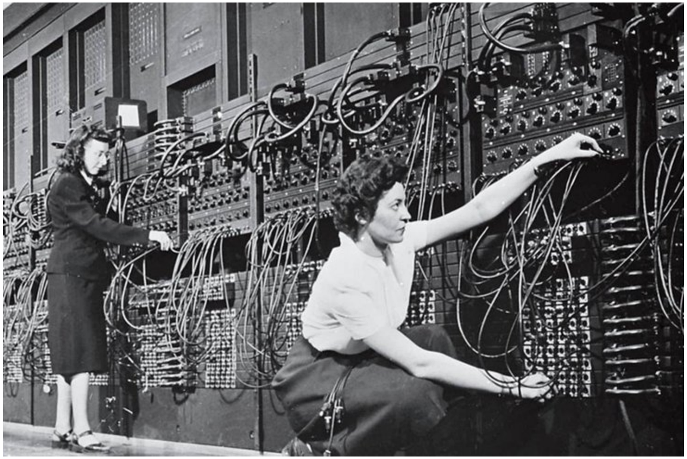

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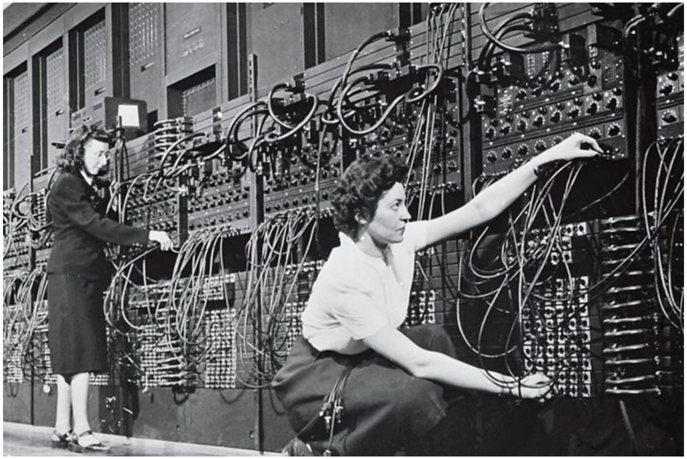

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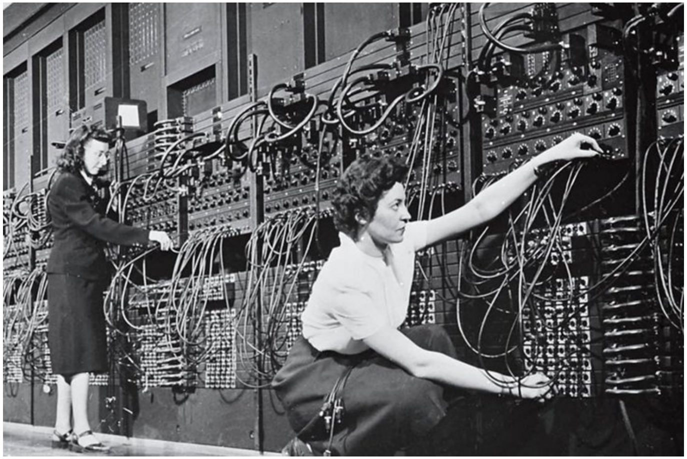

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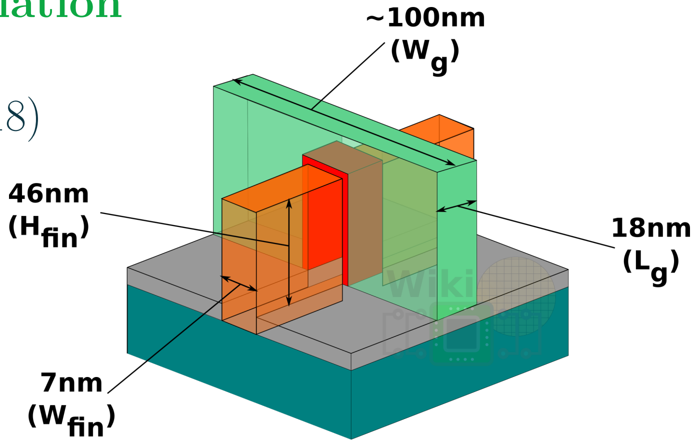
Integrated circuits in “10nm” technology (2018)



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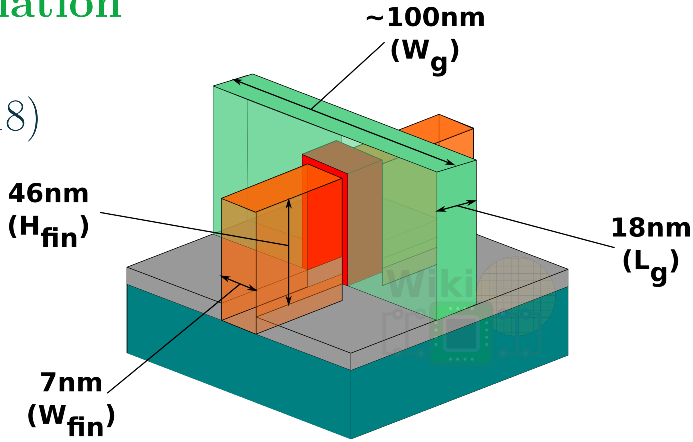

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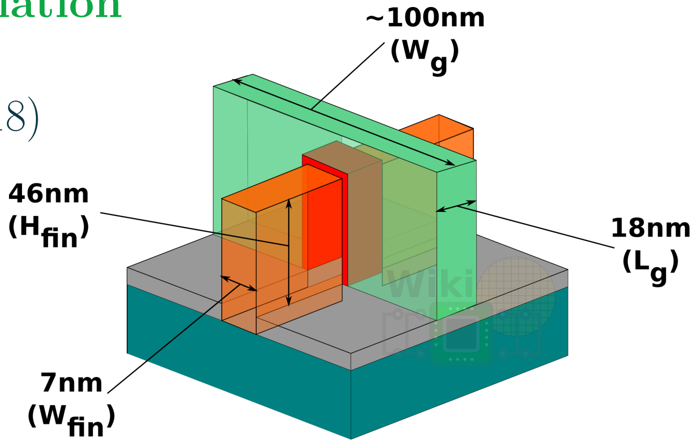

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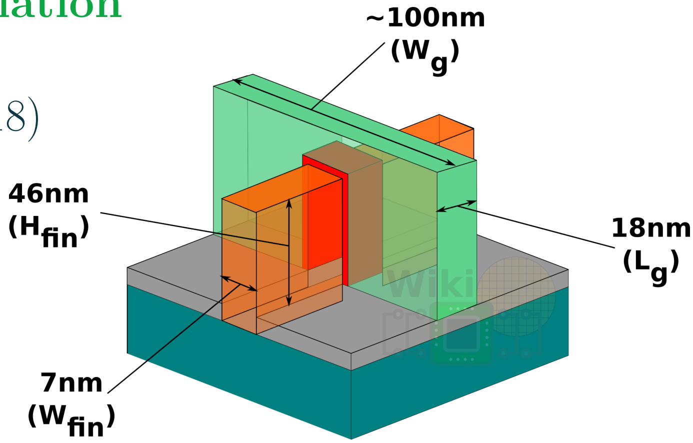

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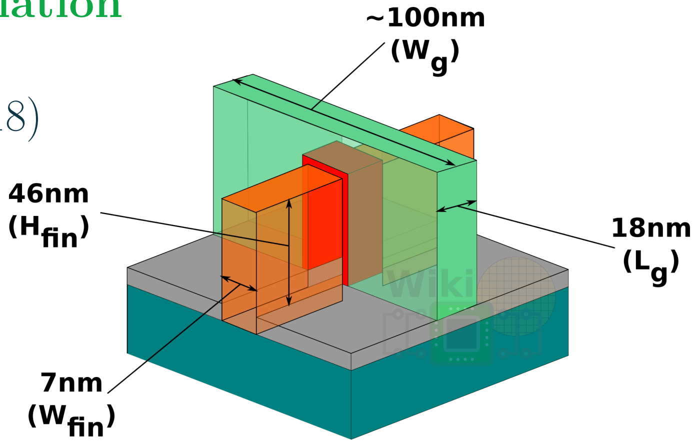

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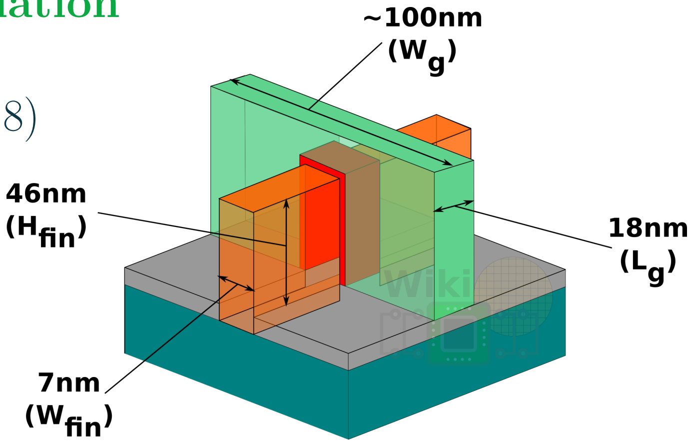

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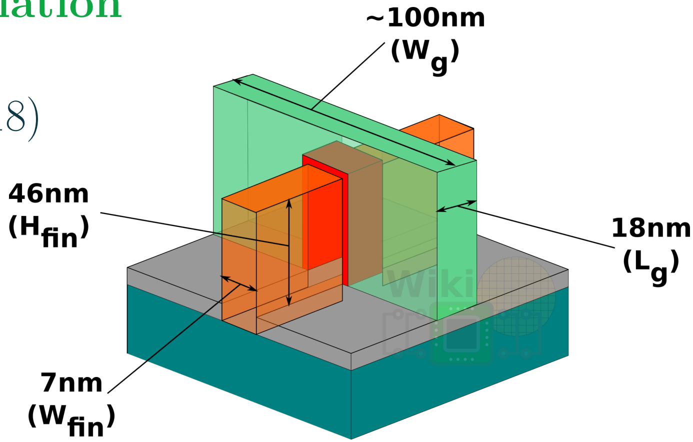

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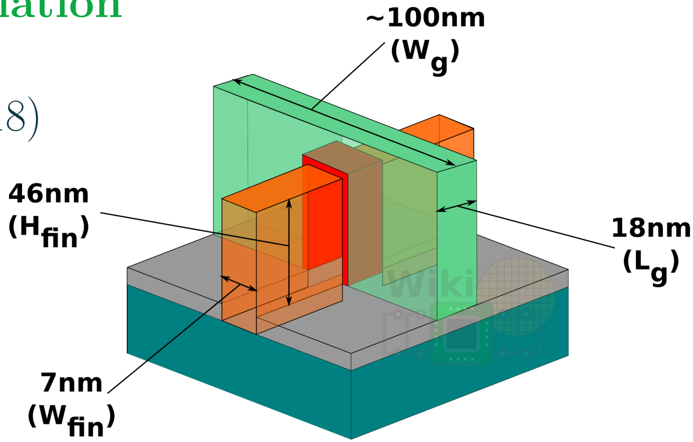

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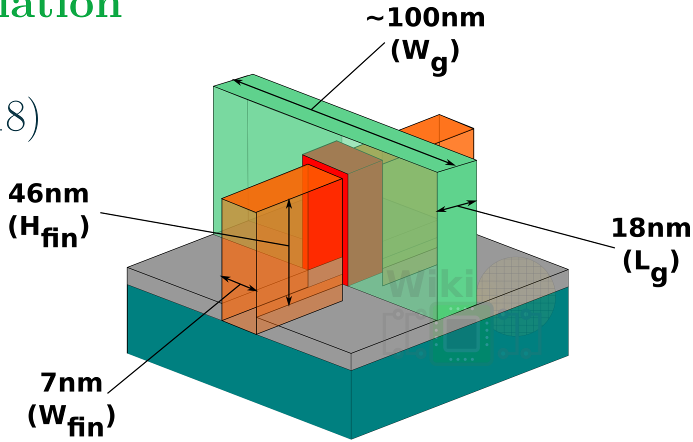

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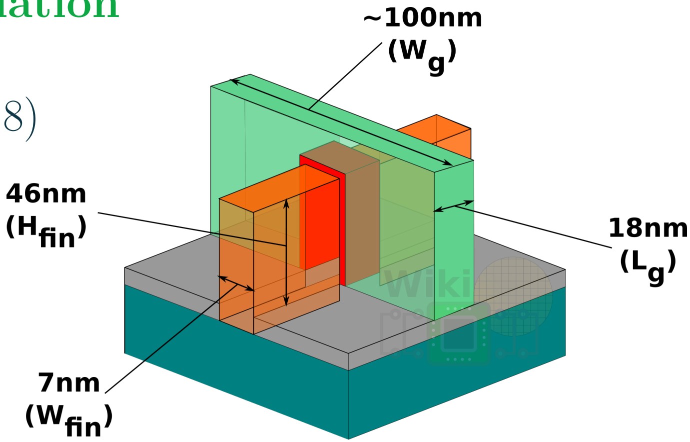

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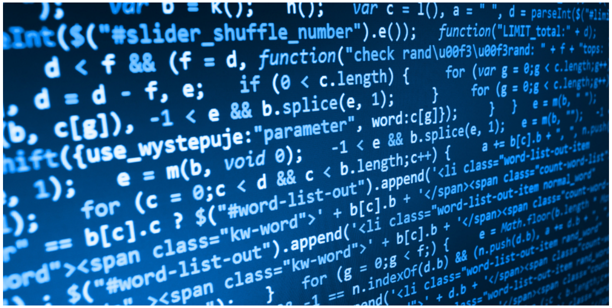

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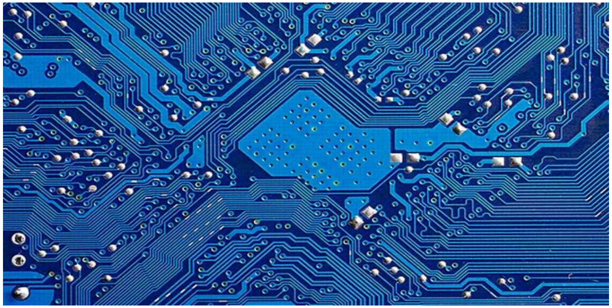

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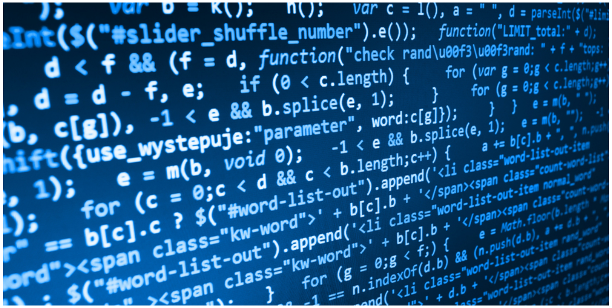

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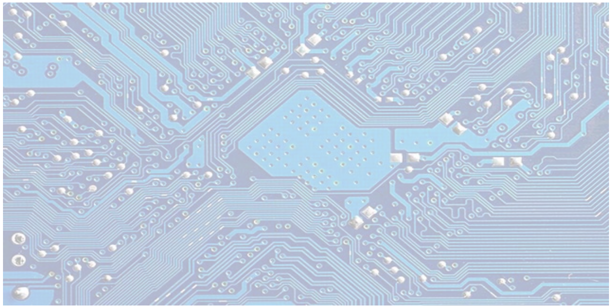

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⇝ 6 seriously overdosed patients, at least 3 fatalities

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$\rightsquigarrow$ explosion in 30th second of flight, estimated loss of 442 milion €

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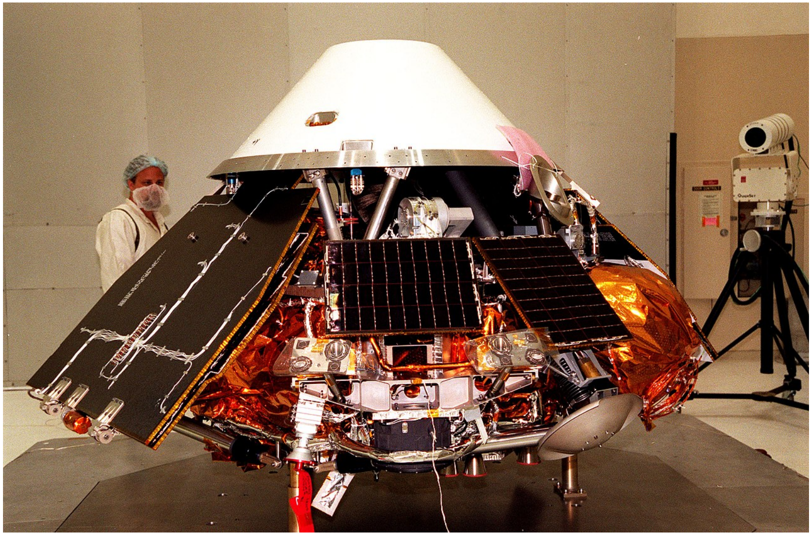

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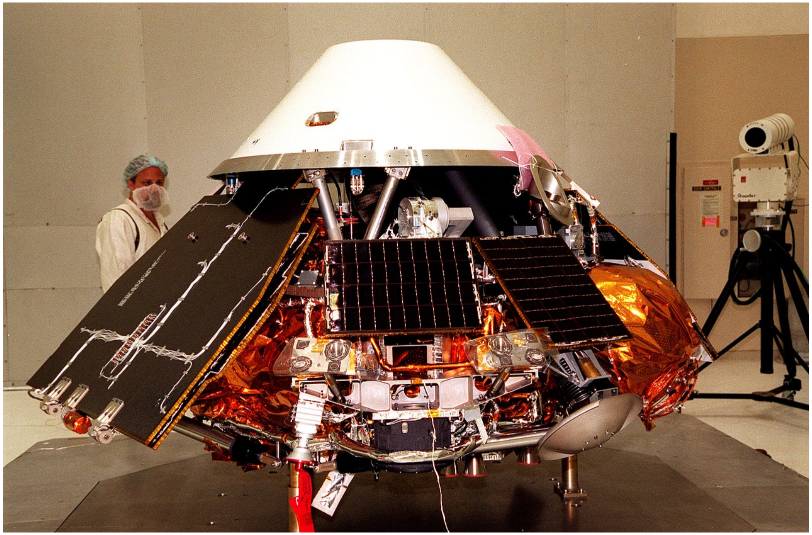

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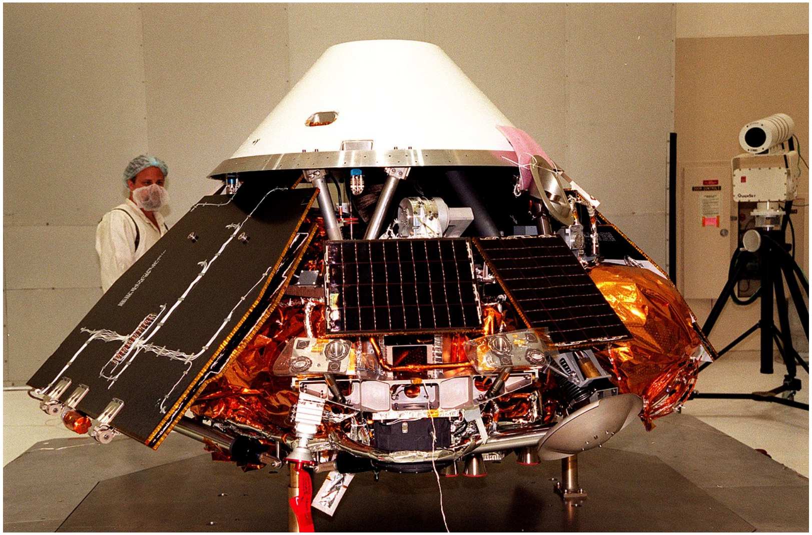

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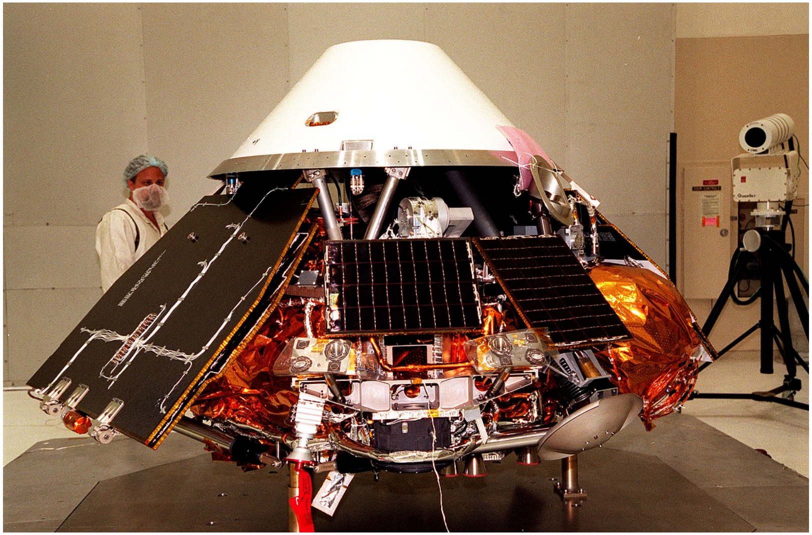

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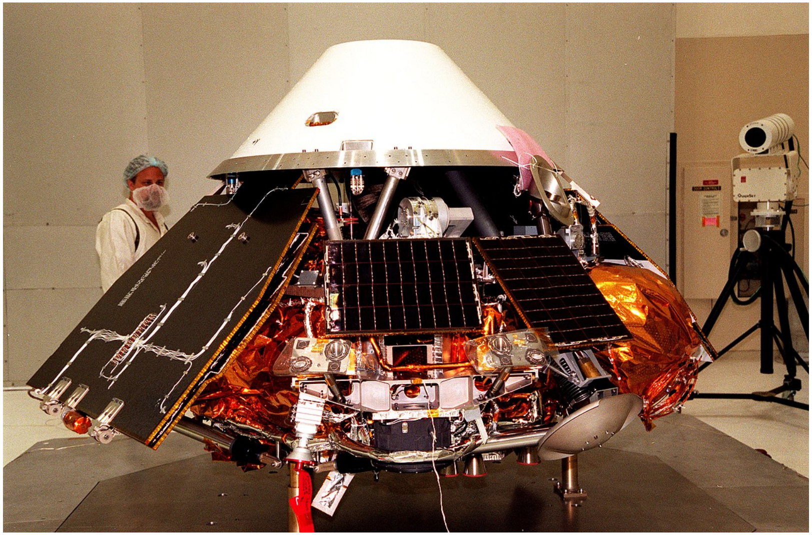
⇝ impact at 22 m/s instead of 2.4 m/s, estimated loss of 100 milion $



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MATHEMATICS!

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$\mathcal{P}$

program

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$\rightsquigarrow$ **formal verification** of programs

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...the program used for **4.** is short and simple and everyone trusts it...

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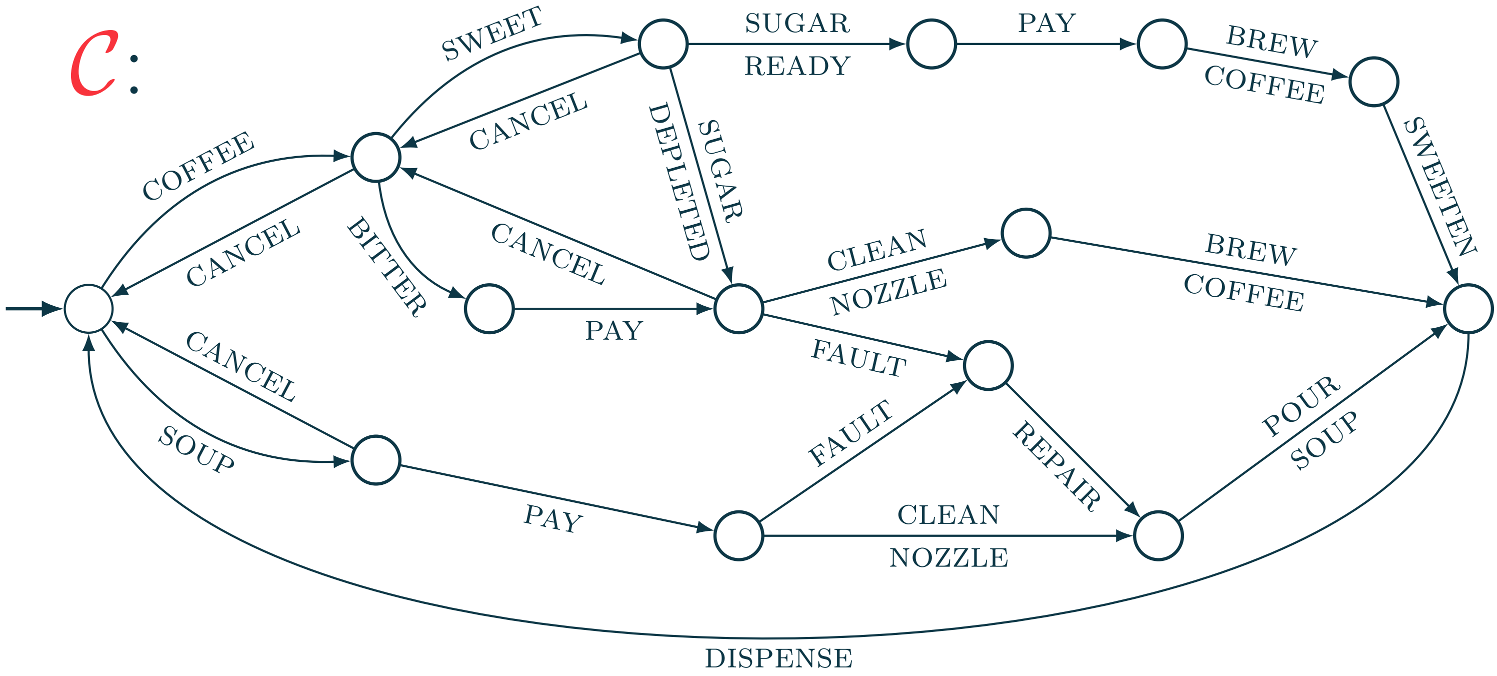
[even worse, as a program can enumerate proofs]

„It is all because of numbers”

„It is all because of numbers” $\rightsquigarrow$ consider *numberless* machines = **automata**

$\mathcal{C}$:



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COFFEE
CANCEL
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BITTER
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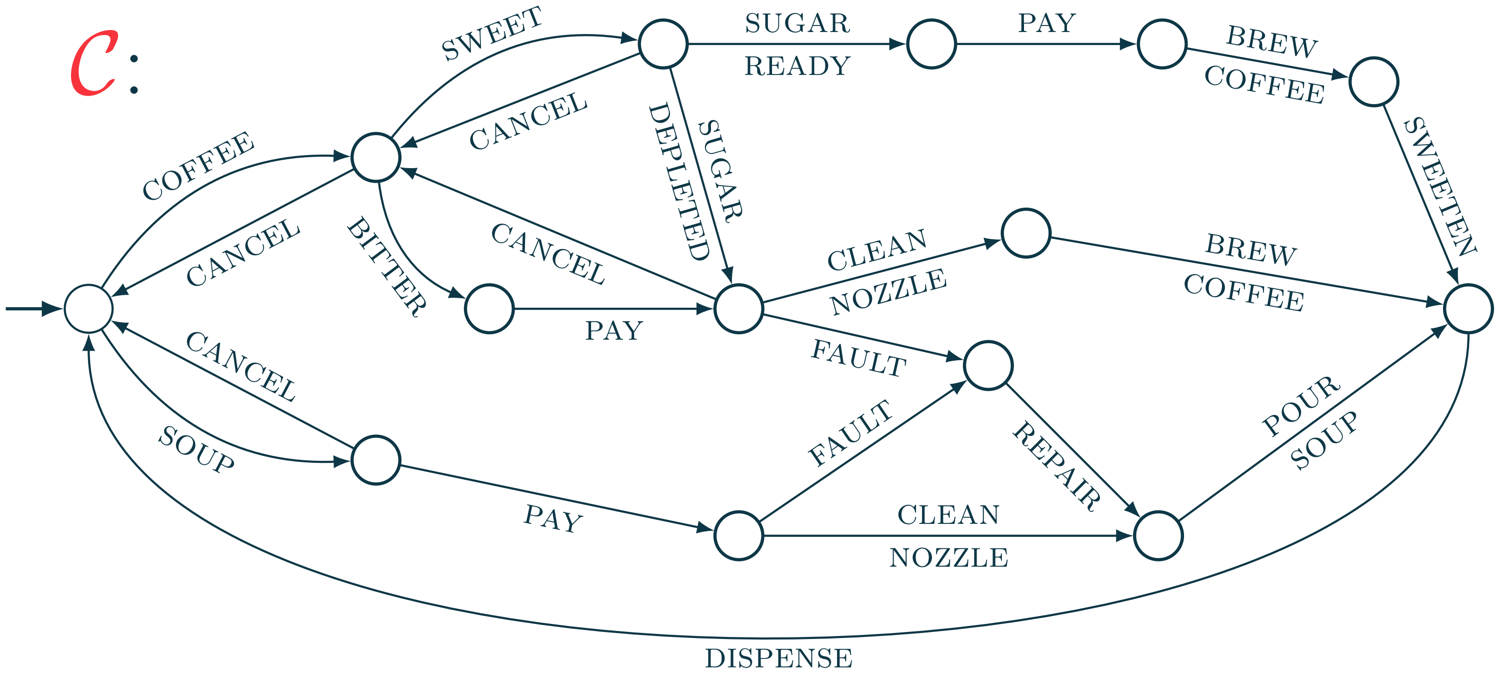

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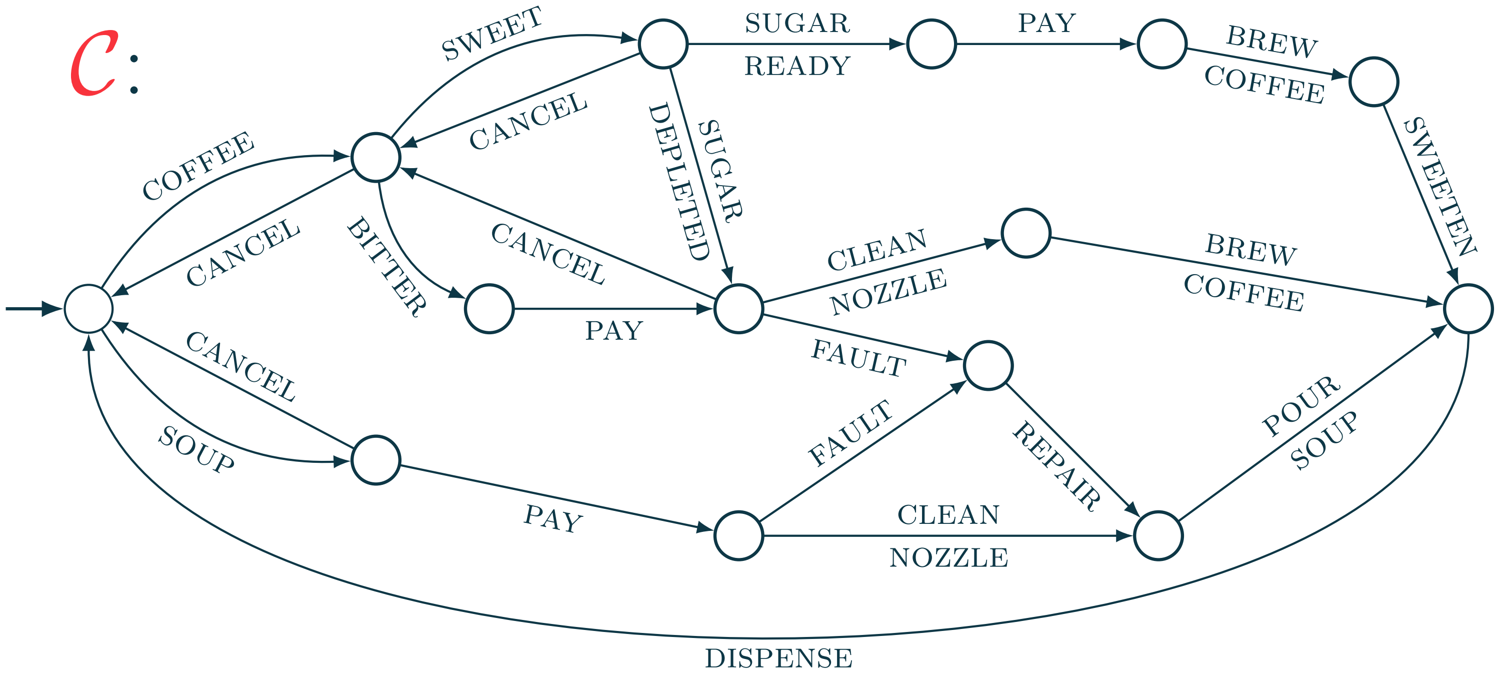

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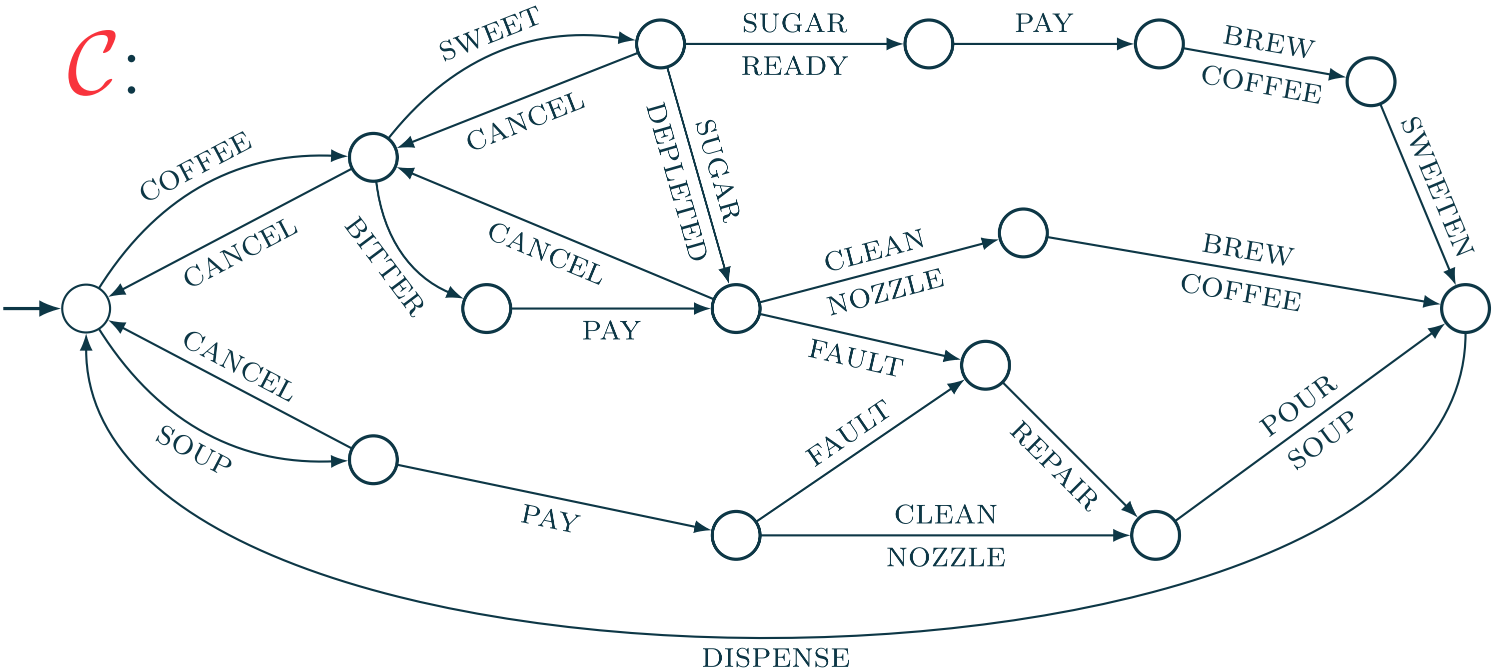

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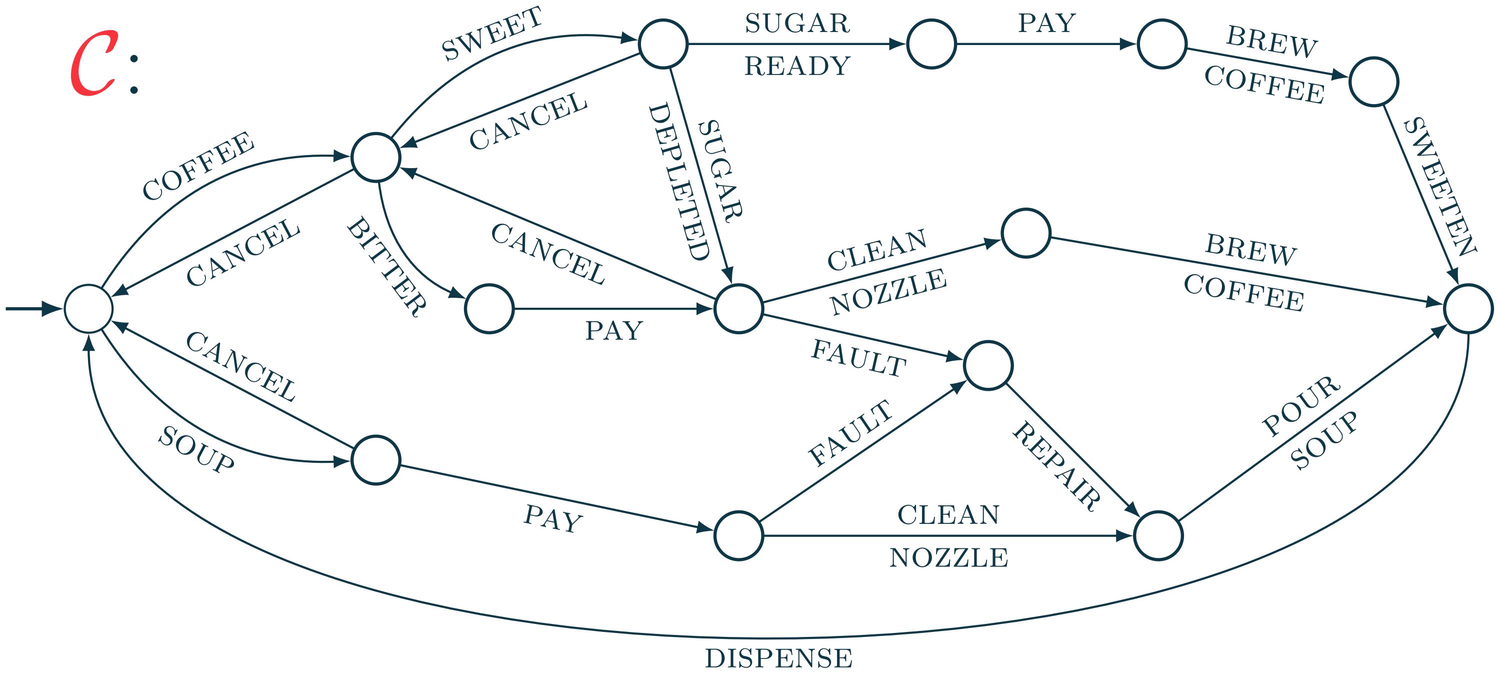

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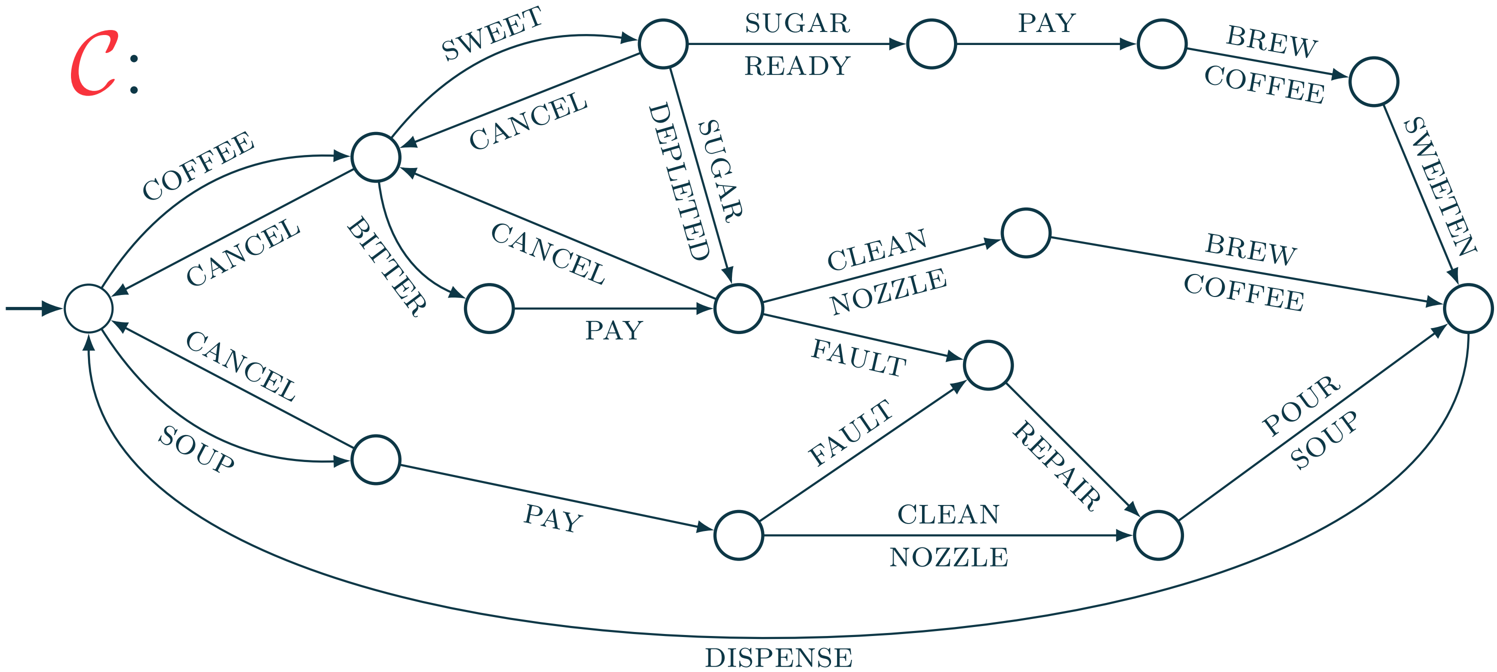

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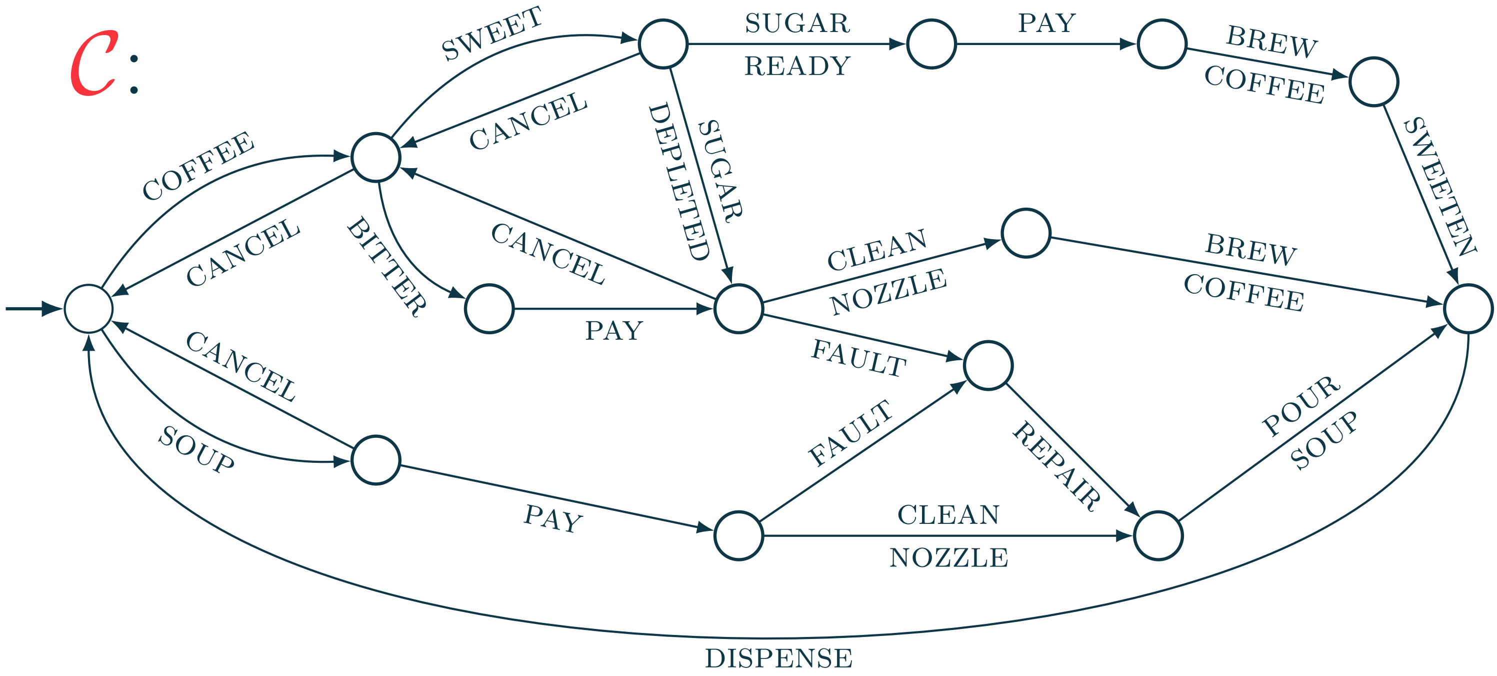

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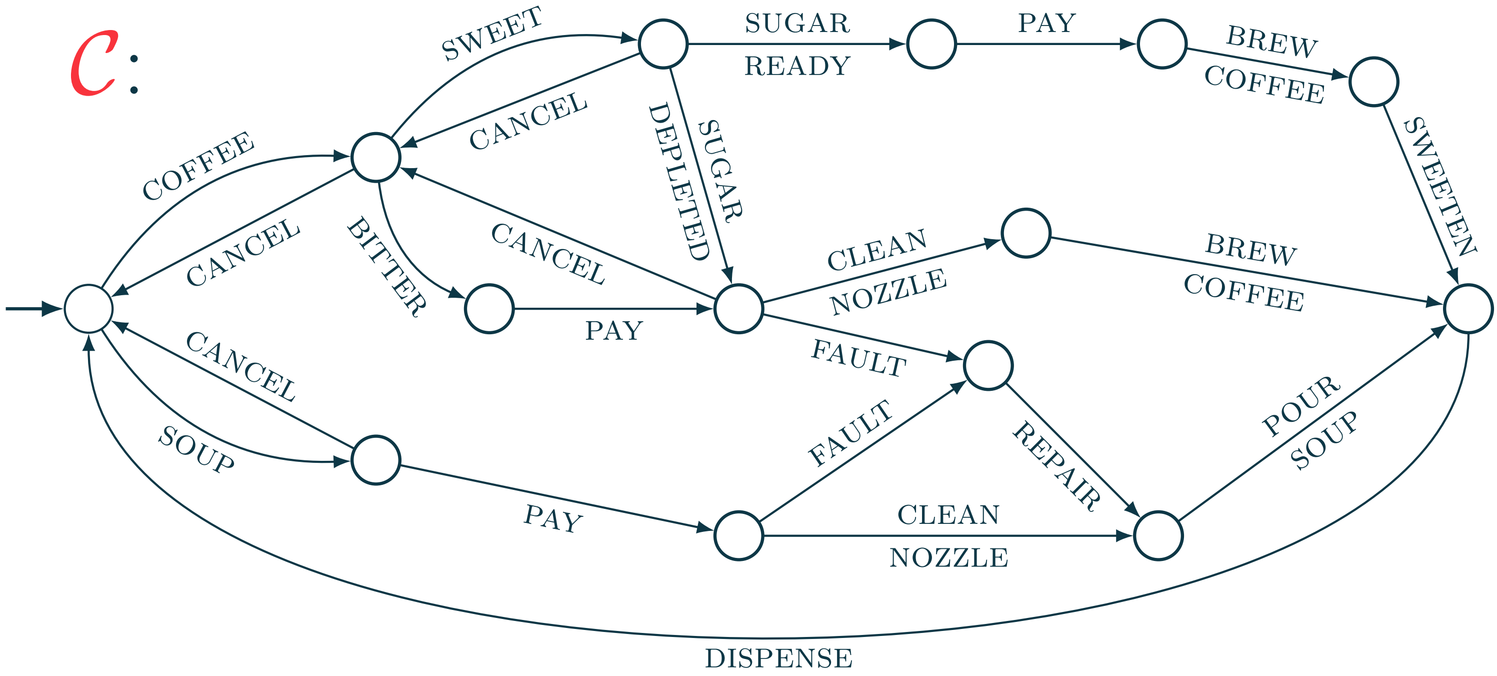

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Model-checking problem:

$$[\![\mathcal{C}]\!] \overset{???}{\subseteq} [\![\mathcal{S}]\!]$$

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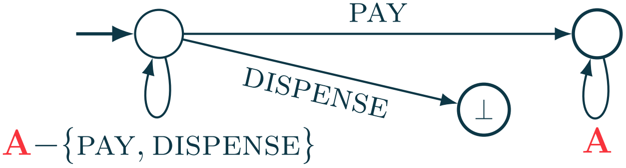

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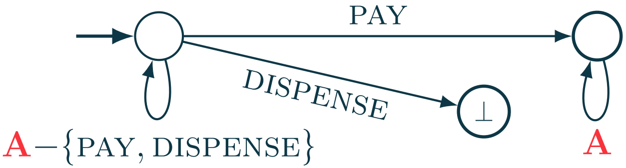

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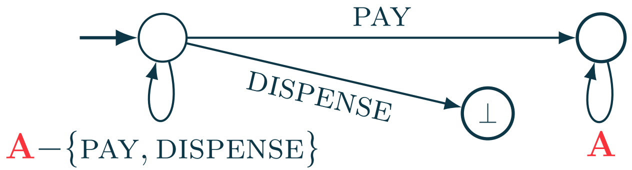

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Theorem (Büchi, Elgot, Trachtenbrot [~1960])

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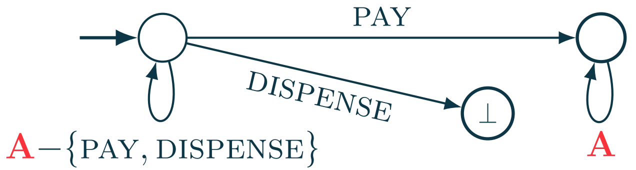

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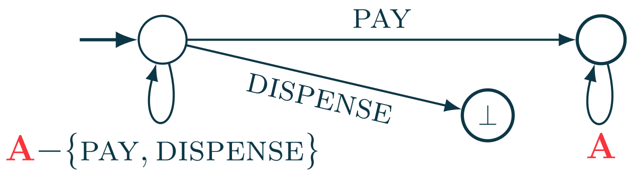

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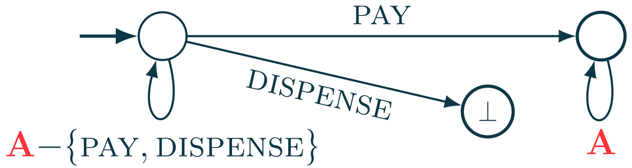

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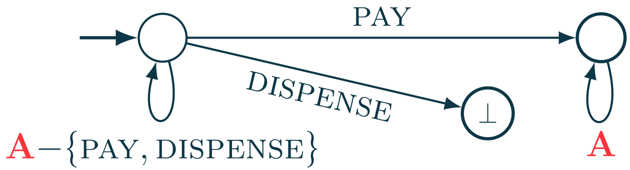

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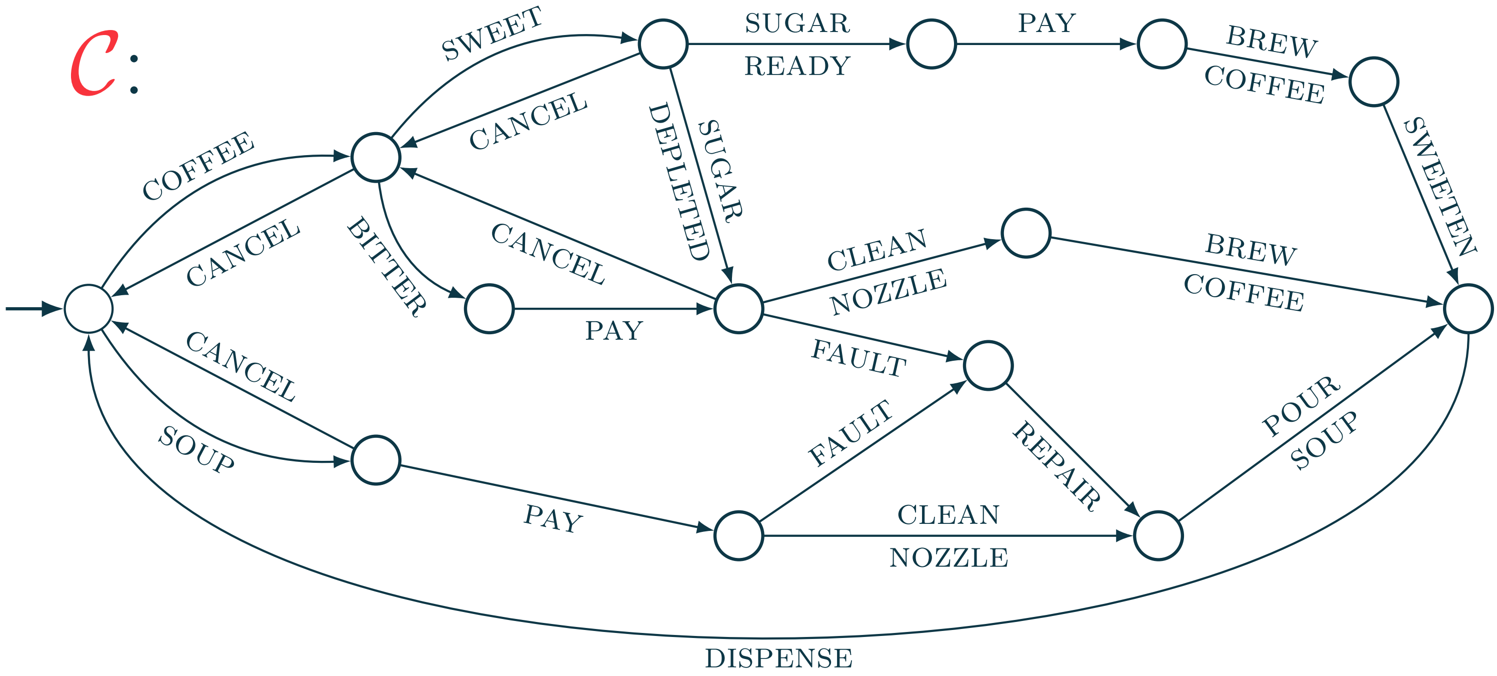
$$\mathcal{C} \times \mathcal{V} \longrightarrow^* (\_, \bot)$$

If that's not enough. . .

$\mathcal{C}$:



SWEET
SUGAR
READY
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BREW
COFFEE
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SWEETEN
COFFEE
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BITTER
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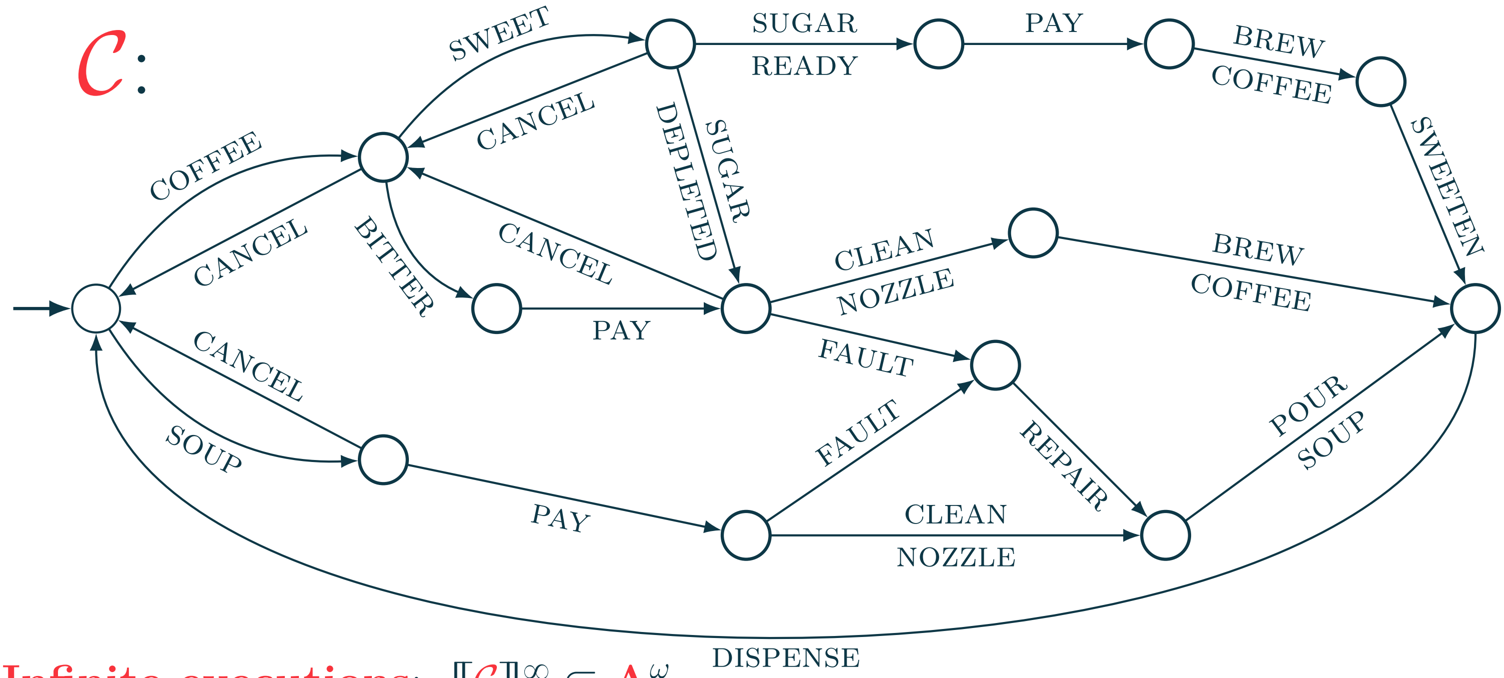

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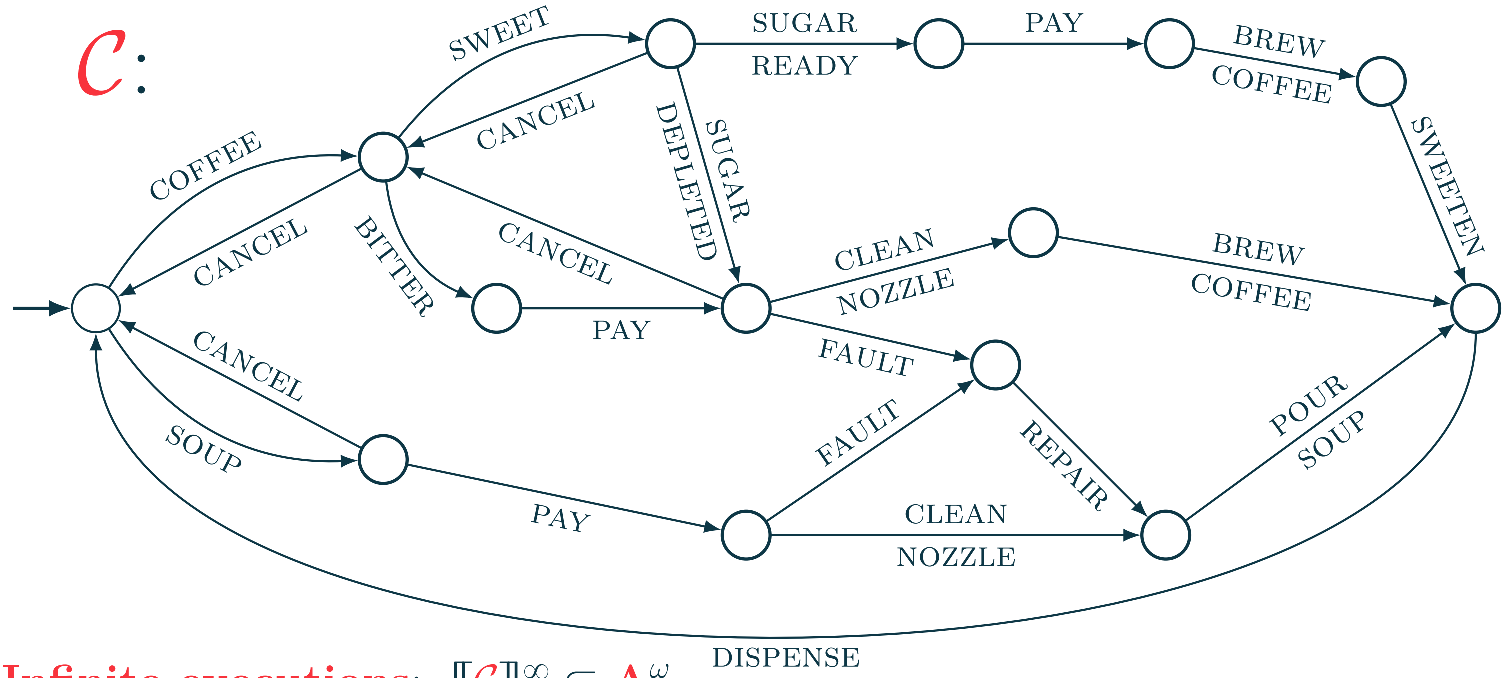

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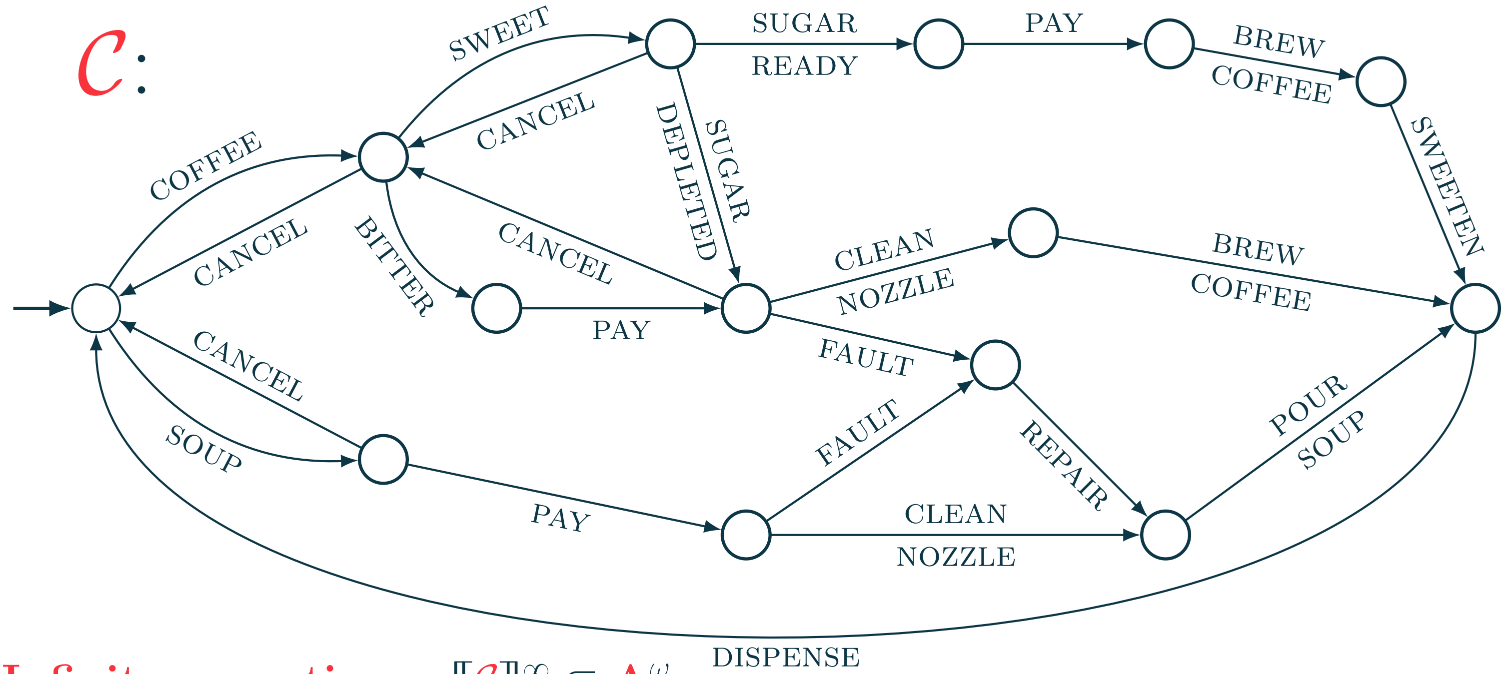

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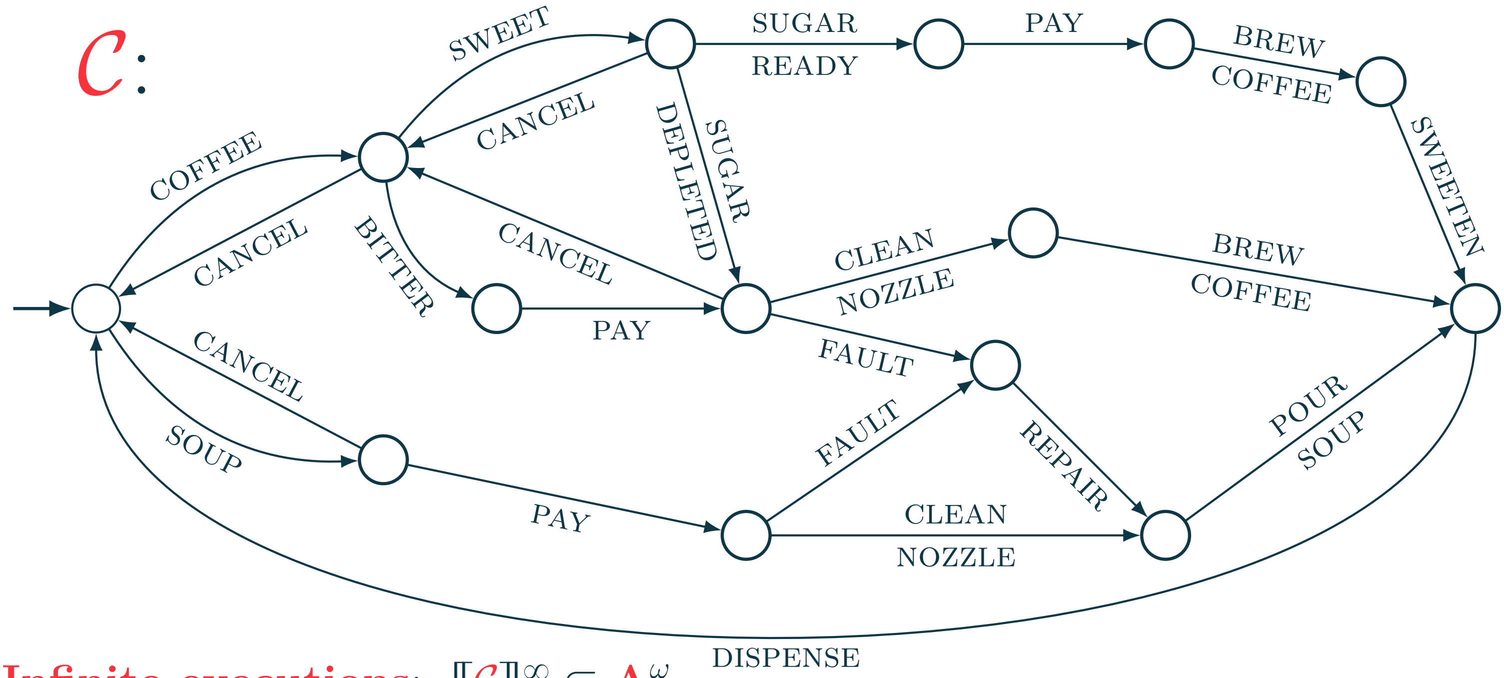

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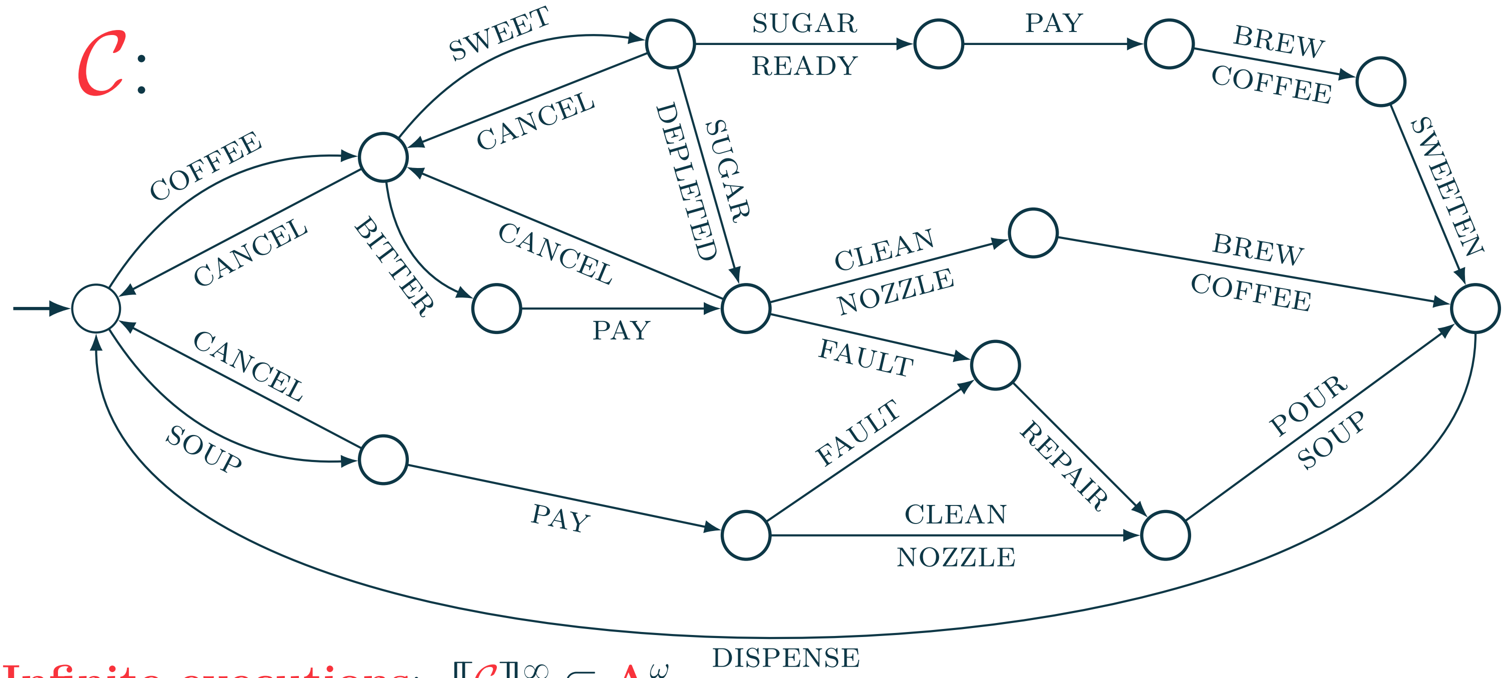

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