Deterministic and game separability of tree languages via games

Lorenzo Clemente, Michał Skrzypczak

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UW MIM

Parity index for ω-regular languages

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Theorem (Büchi, Landweber '69)

ω-regular games are **effectively finite-memory** determined.

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Wadge game for $L \leqslant_{\mathrm{W}} P_C$

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$\Updownarrow$

mathematical counterexample

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Theorem (Colcombet '13)

Domination games for **cost automata**.

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Star-height via **games**.

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+ Rabin Separation Theorem

Disjointness Game

$\mathrm{Tr}_A \stackrel{\text{def}}{=} \{\mathtt{L}, \mathtt{R}\}^* \to A$

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Non-deterministic tree automata:

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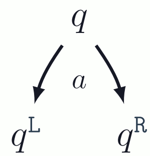

Non-deterministic tree automata: branching transitions

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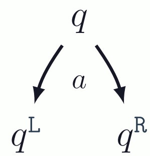

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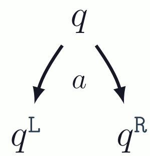

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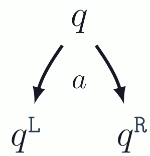

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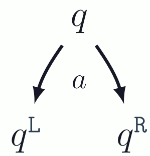

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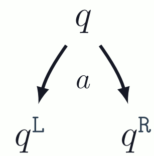

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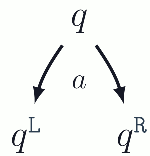

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$$\left[\ (p_n, q_n) \to \qquad\qquad\qquad\qquad\qquad \right]$$

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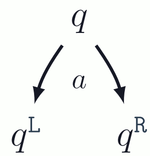

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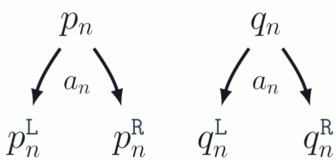

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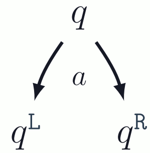

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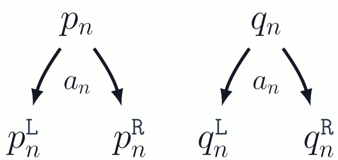

PATHFINDER: $d_n \in \{\mathtt{L}, \mathtt{R}\}$

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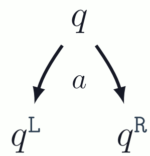

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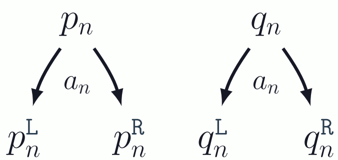

PATHFINDER: $d_n \in \{\mathtt{L}, \mathtt{R}\}$

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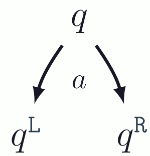

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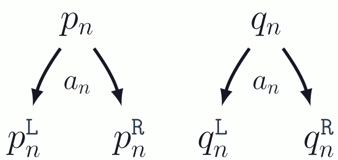

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$\to (p_n^{d_n}, q_n^{d_n})$

AUTOMATON wins

iff

$(p_n)_{n\in\omega}$ and $(q_n)_{n\in\omega}$ are accepting

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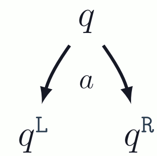

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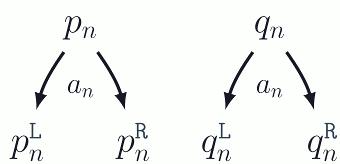

PATHFINDER: $d_n \in \{\mathtt{L},\mathtt{R}\}$

$\to (p_n^{d_n}, q_n^{d_n})$

AUTOMATON wins
iff
$(p_n)_{n\in\omega}$ and $(q_n)_{n\in\omega}$ are accepting

PATHFINDER wins
iff
one of $(p_n)_{n\in\omega}$, $(q_n)_{n\in\omega}$ is rejecting

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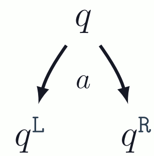

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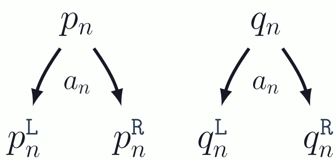
Problem Given two automata $\mathcal{A}$ and $\mathcal{B}$, decide whether $\mathrm{L}(\mathcal{A}) \cap \mathrm{L}(\mathcal{B}) \neq \emptyset$?

$(p_n, q_n) \to$

AUTOMATON:



PATHFINDER: $d_n \in \{\mathtt{L}, \mathtt{R}\}$

$\to (p_n^{d_n}, q_n^{d_n})$

AUTOMATON wins
iff
$(p_n)_{n\in\omega}$ and $(q_n)_{n\in\omega}$ are accepting

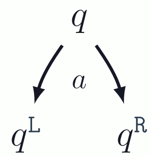
PATHFINDER wins
iff
one of $(p_n)_{n\in\omega}$, $(q_n)_{n\in\omega}$ is rejecting

(win. strategy of AUTOMATON)

$\mathrm{Tr}_A \stackrel{\text{def}}{=} \{\mathtt{L}, \mathtt{R}\}^* \to A$

Disjointness Game

Non-deterministic tree automata: branching transitions



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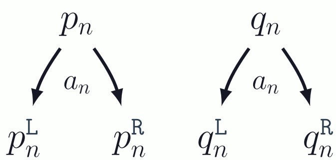

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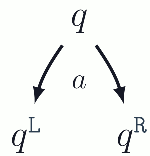
Disjointness Game



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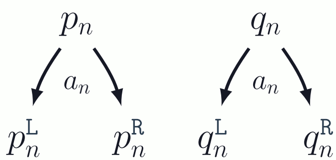

Pathfinder: $d_n \in \{\mathtt{L}, \mathtt{R}\}$

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Disjointness Game

Non-deterministic tree automata: branching transitions $q \xrightarrow{a} (q^{\mathtt{L}}, q^{\mathtt{R}})$

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Problem Given two automata $\mathcal{A}$ and $\mathcal{B}$, decide whether $\mathrm{L}(\mathcal{A}) \cap \mathrm{L}(\mathcal{B}) \neq \emptyset$?

$(p_n, q_n) \to$

Automaton: $p_n \xrightarrow{a_n} (p_n^{\mathtt{L}}, p_n^{\mathtt{R}})$, $q_n \xrightarrow{a_n} (q_n^{\mathtt{L}}, q_n^{\mathtt{R}})$

Pathfinder: $d_n \in \{\mathtt{L}, \mathtt{R}\}$

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Automaton wins
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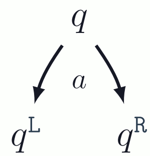
Non-deterministic tree automata: branching transitions



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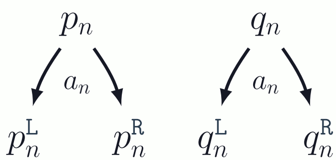

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\+ **positional** determinacy for PATHFINDER

Rabin Separation Theorem

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Theorem (Rabin '70)

If $\mathcal{A}$, $\mathcal{B}$ are Büchi tree automata and $\mathrm{L}(\mathcal{A}) \cap \mathrm{L}(\mathcal{B}) = \emptyset$

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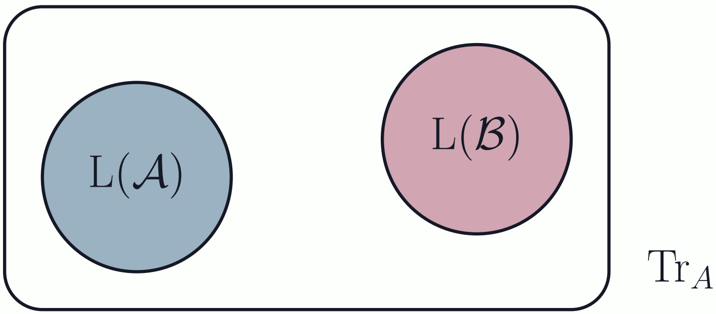

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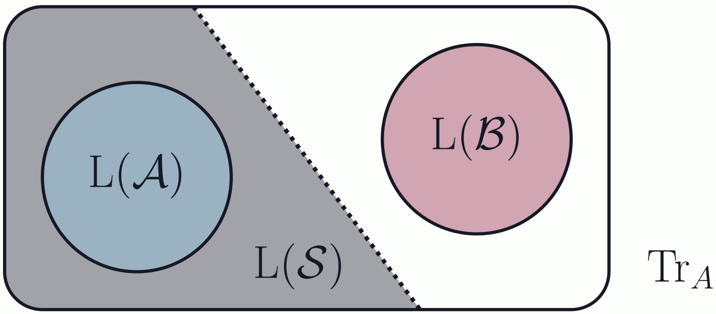

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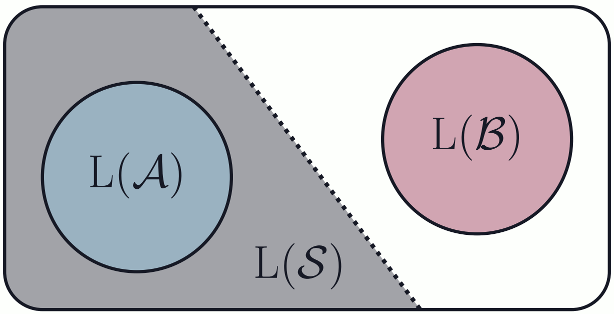

Tr_A

If $L(\mathcal{B}) = L(\mathcal{A})^c$ then $L(\mathcal{S}) = L(\mathcal{A})$.

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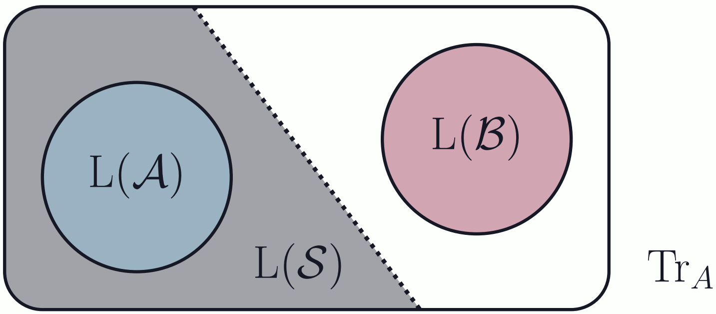

If $L(\mathcal{B}) = L(\mathcal{A})^c$ then $L(\mathcal{S}) = L(\mathcal{A})$.

$\rightsquigarrow$ **separation** $\Longrightarrow$ **membership**

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$$\underbrace{\hphantom{xxxxxxxxxx}}$$

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1. Fix a **positional** (or **f-m**) win. strategy $\boldsymbol{\sigma}$ of Pathfinder

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3. Stratify $\mathcal{A} \times \mathcal{B} \times \boldsymbol{\sigma}$ into **layers**.

Rabin Separation Theorem

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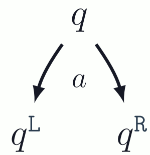
+ extension to higher indices (Arnold, Santocanale '05)

Deterministic and game automata

Deterministic and game automata

Deterministic automata

For each $q \in Q$, $a \in A$ there is a **unique** transition



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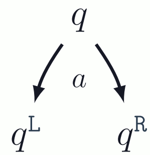

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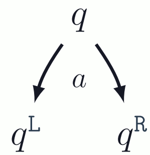

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Acceptance game for $t \overset{?}{\in} \mathrm{L}(\mathcal{A})$

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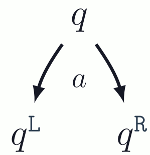

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$$\left[\ (q_n, u_n) \rightarrow \qquad\qquad \right]$$

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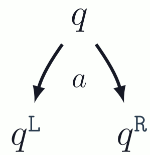

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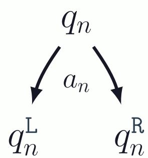

Deterministic and game automata

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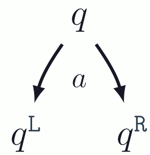
For each $q \in Q$, $a \in A$ there is a **unique** transition 

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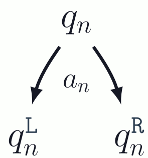
TREE: $a_n = t(u_n)$, 

$\forall$: $d_n \in \{\mathtt{L}, \mathtt{R}\}$

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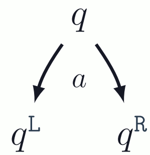

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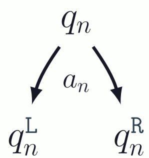
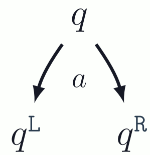


$\forall$: $d_n \in \{\mathtt{L}, \mathtt{R}\}$

$\rightarrow (q_n^{d_n}, u_n d_n)$

Deterministic and game automata

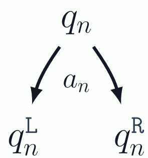
Deterministic automata

For each $q \in Q$, $a \in A$ there is a **unique** transition 

$\rightsquigarrow$ **unique** run on every tree

Acceptance game for $t \overset{?}{\in} \mathrm{L}(\mathcal{A})$

$(q_n, u_n) \to$

TREE: $a_n = t(u_n)$, 

$\forall$: $d_n \in \{\mathtt{L}, \mathtt{R}\}$

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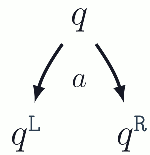

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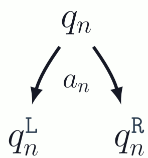

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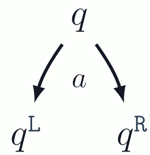

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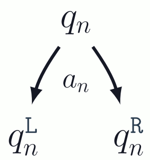

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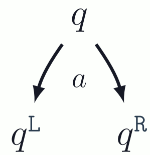

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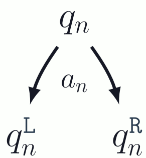

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