On guidable index of tree automata

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MFCS 2021, Tallinn

UNIVERSITY
OF WARSAW

Infinite trees t

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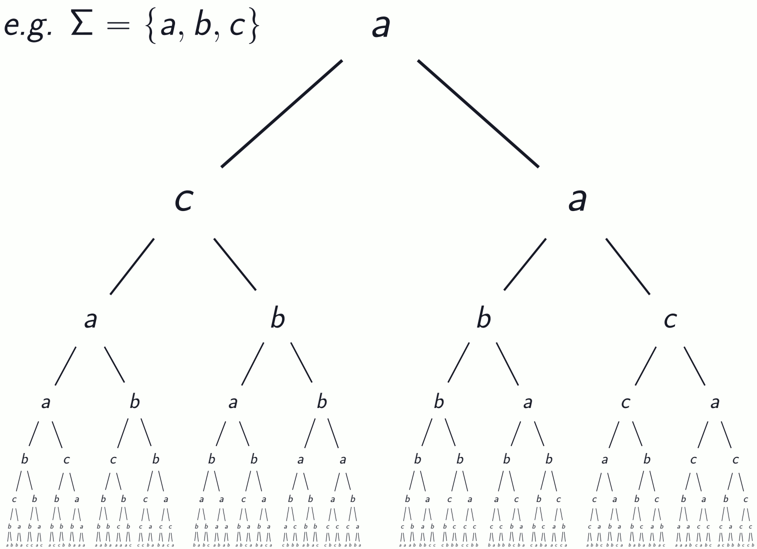

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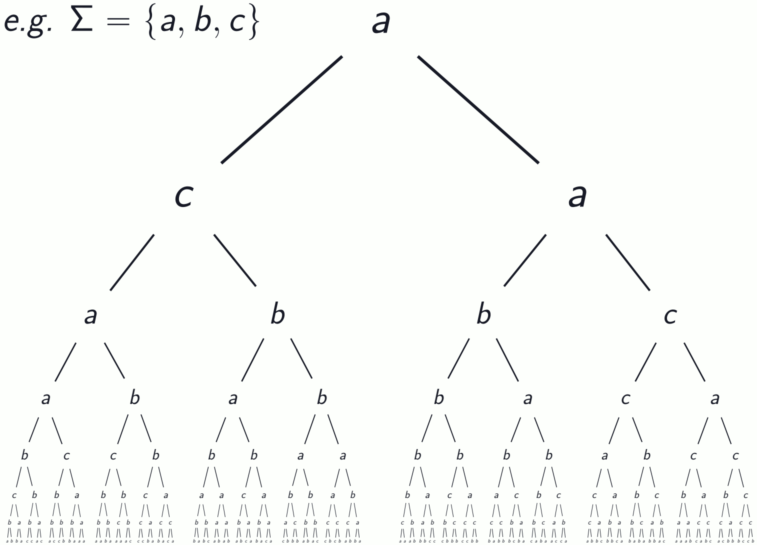

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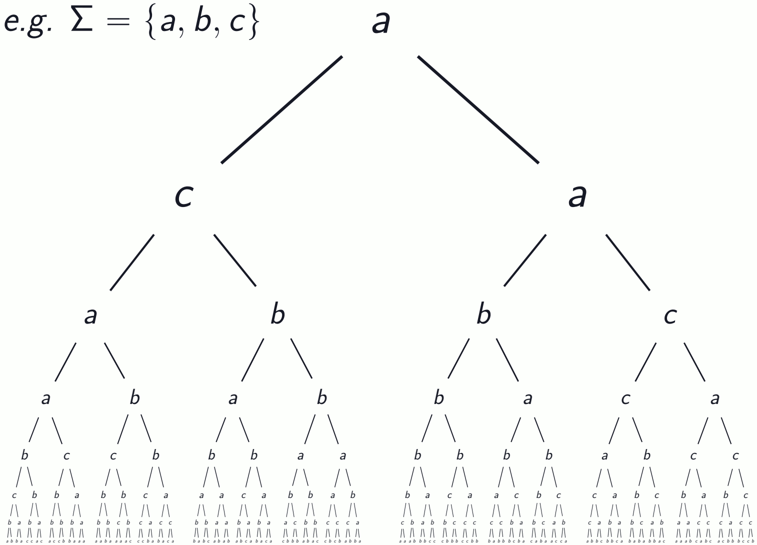

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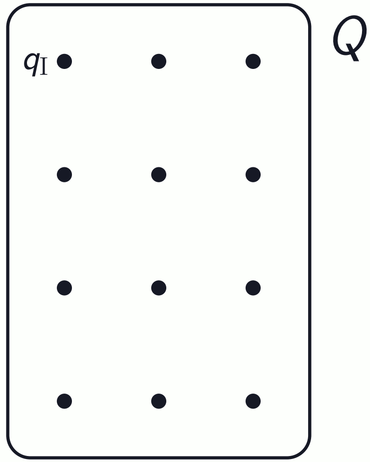

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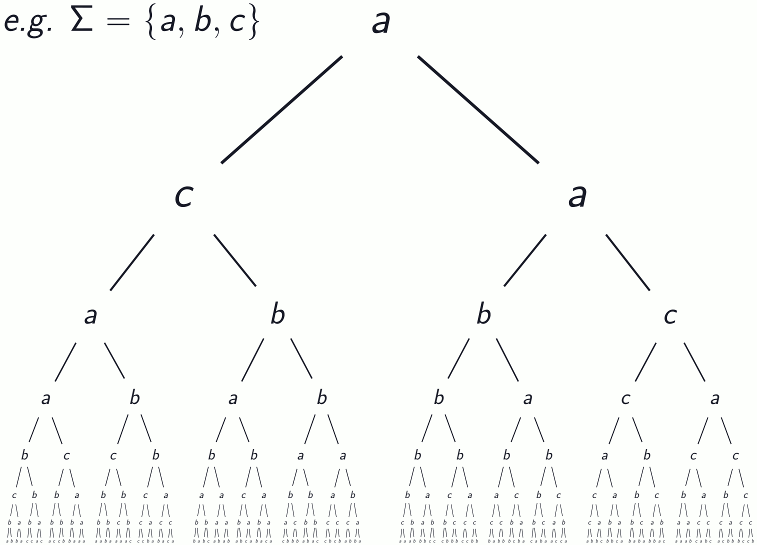

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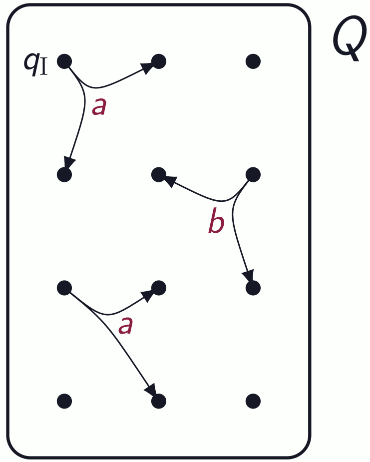

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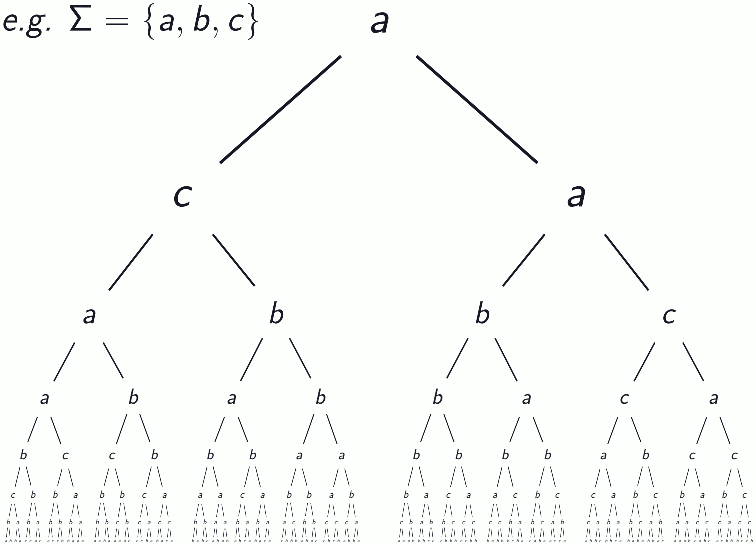

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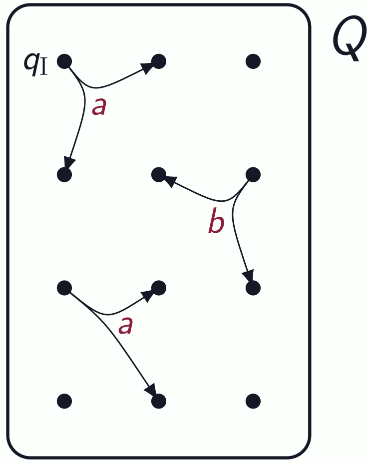

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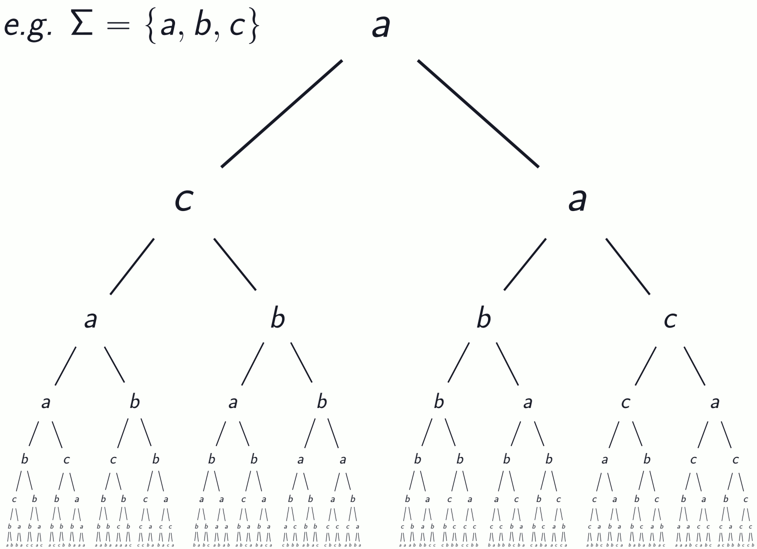

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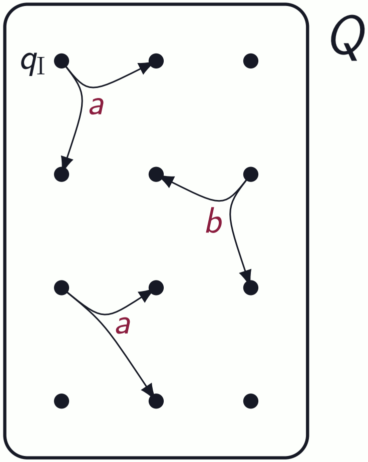

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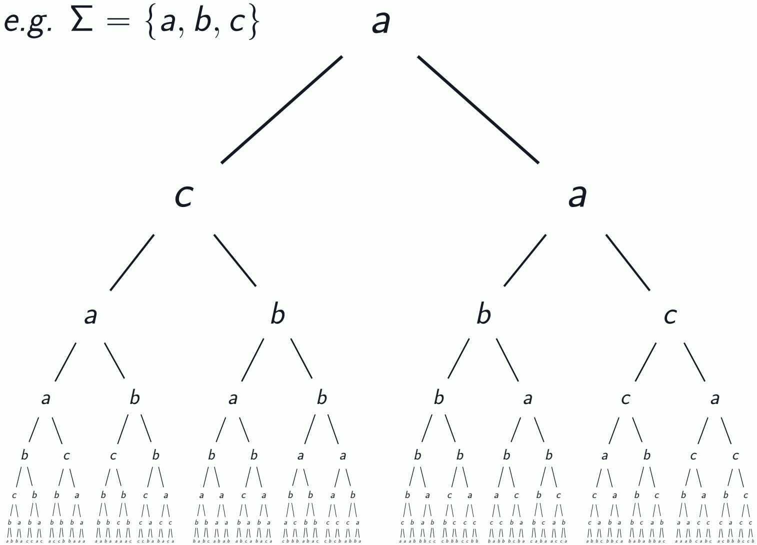

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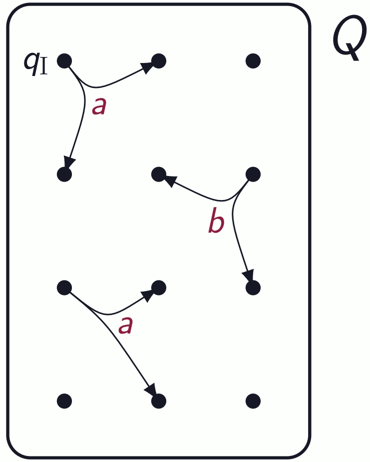

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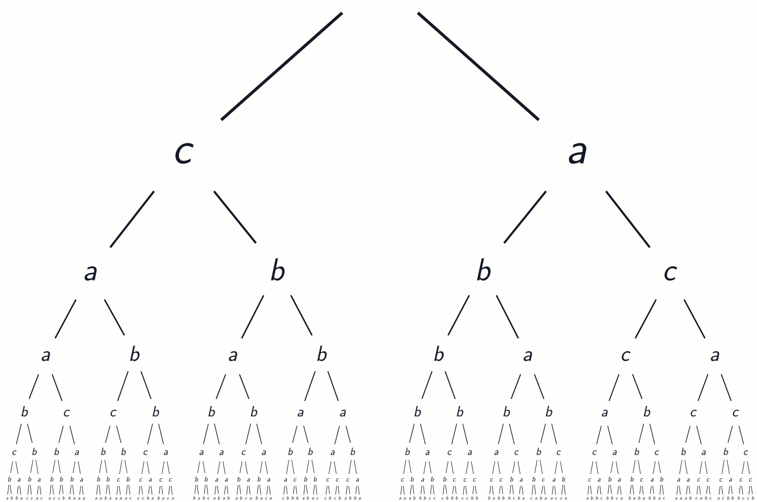

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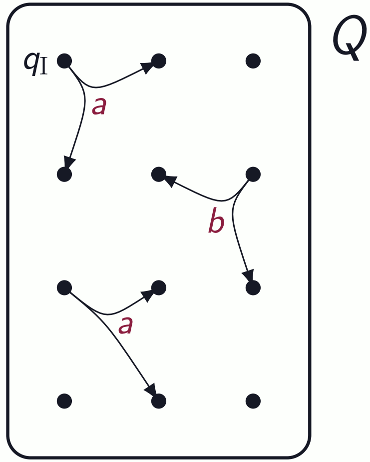

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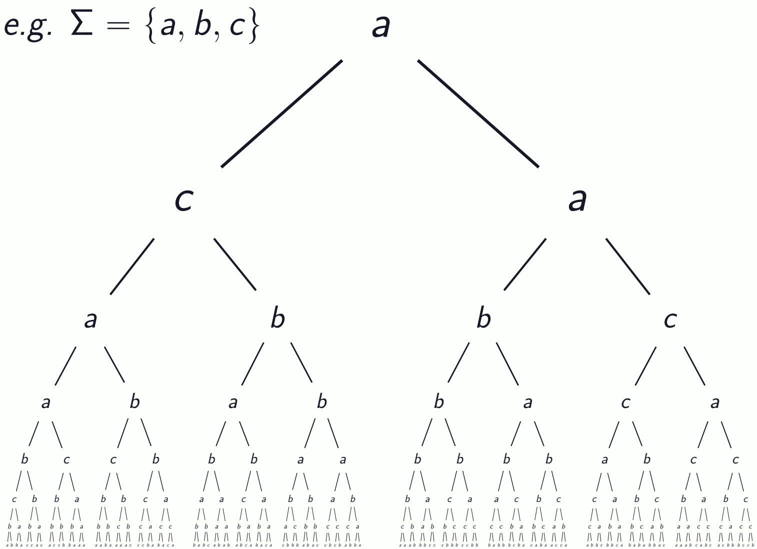

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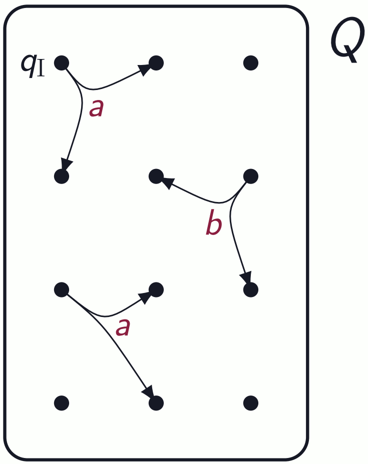

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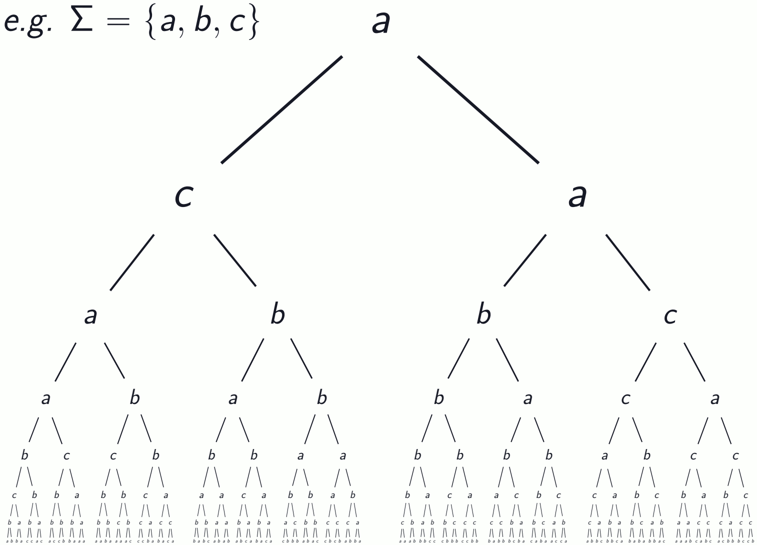

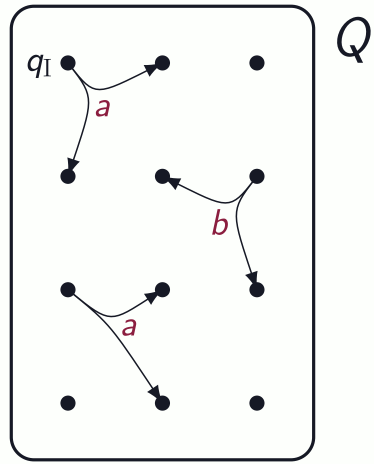

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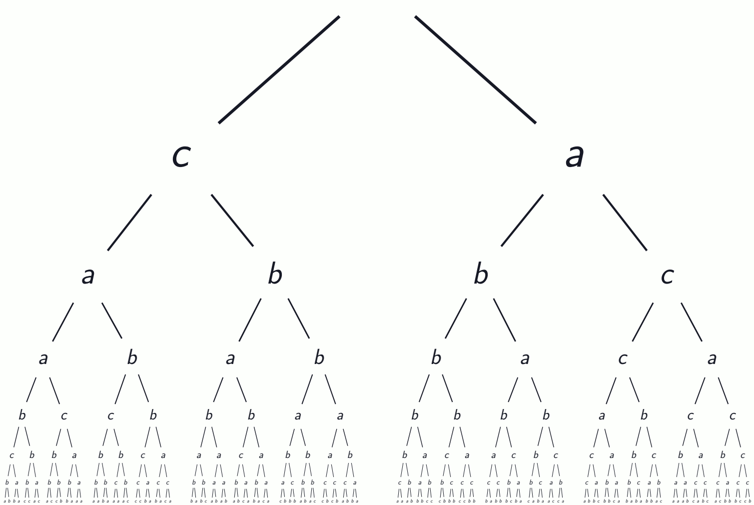

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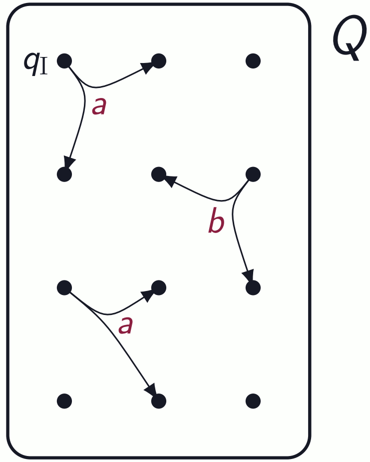

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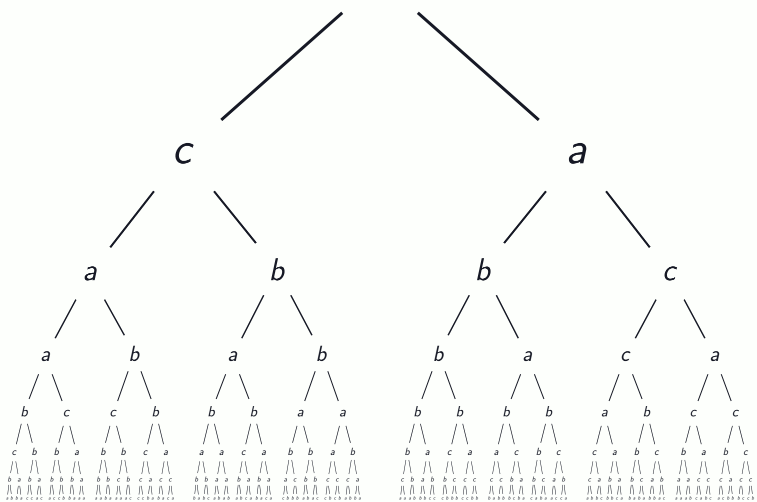

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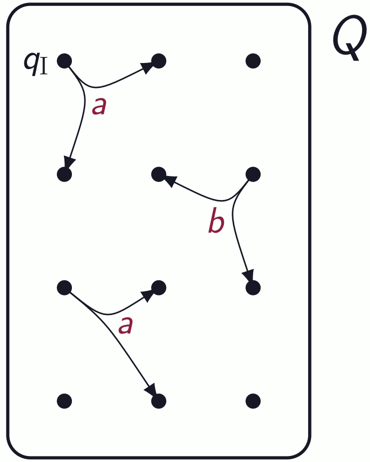

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⇝ regular tree languages ⇝ model checking of **Monadic Second-Order logic**

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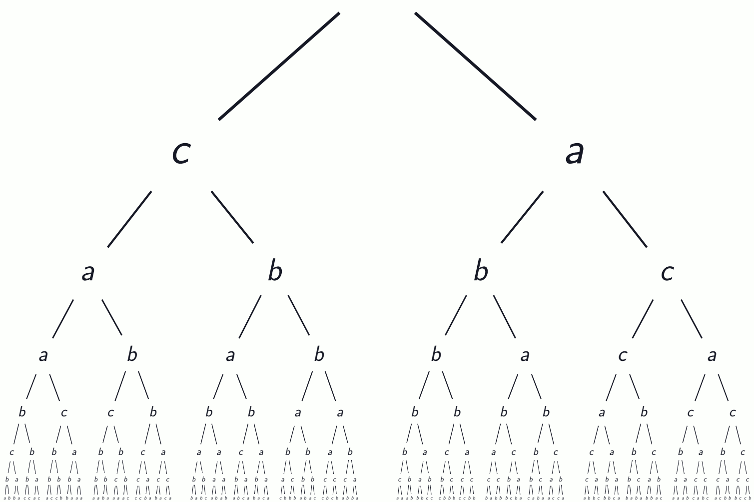

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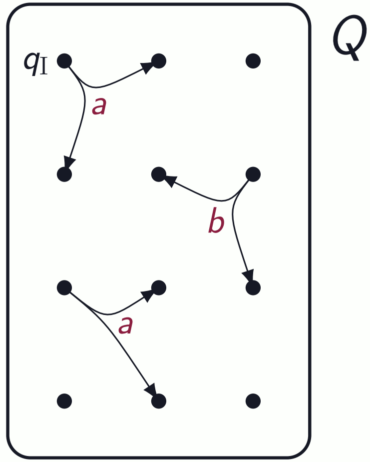

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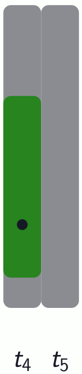

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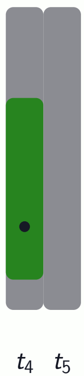

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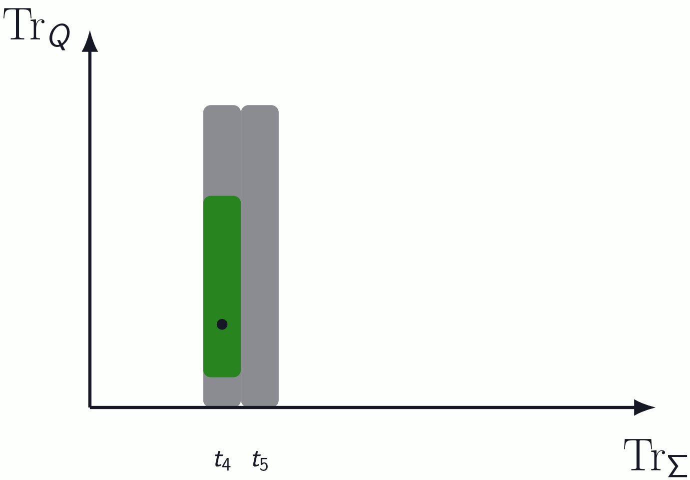

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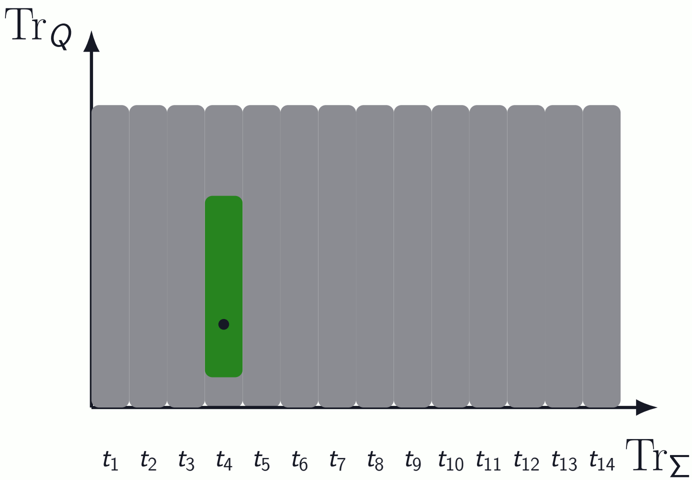

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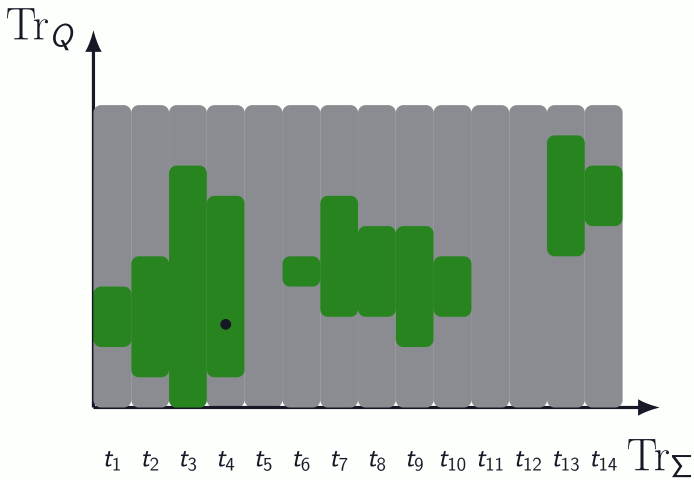

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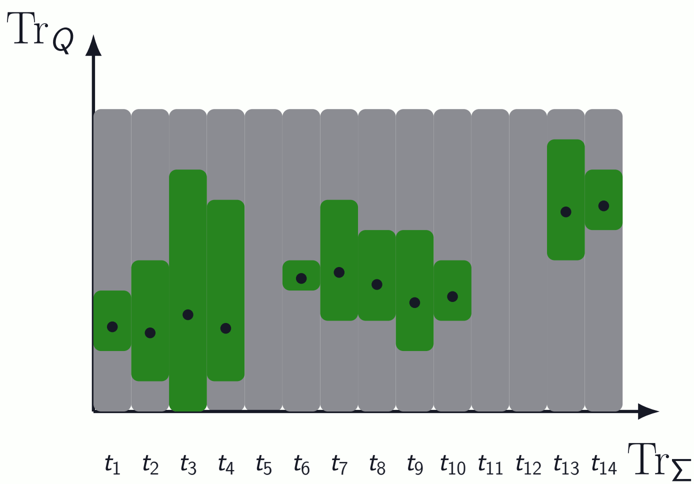

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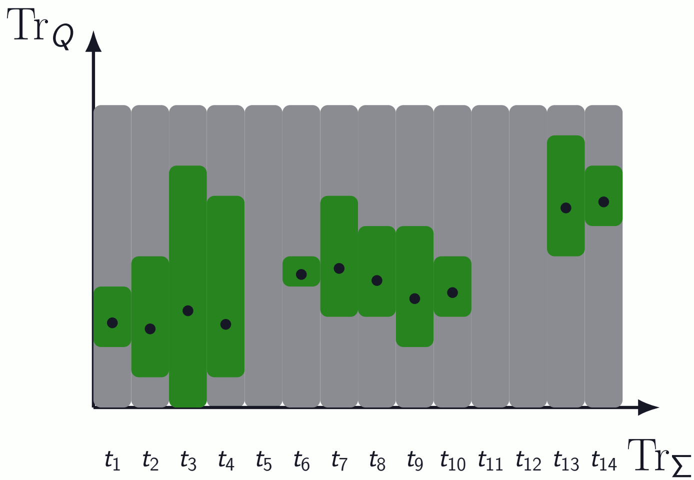

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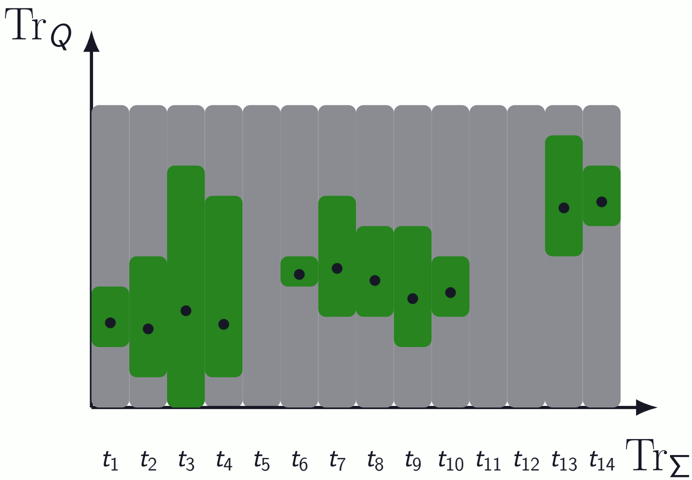

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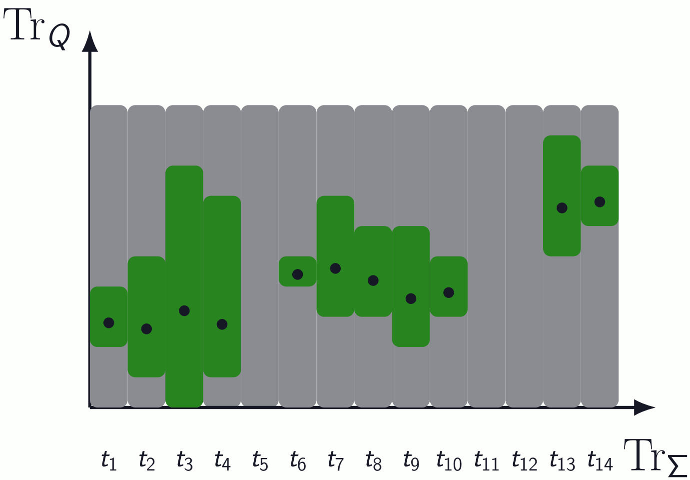
$$\mathrm{L}(\mathcal{A}) \stackrel{\text{def}}{=} \{t \in \mathrm{Tr}_\Sigma \mid \underbrace{\exists \rho \in \mathrm{Tr}_Q}_{\text{run}}.\ \underbrace{\forall \pi \in \{\mathtt{L}, \mathtt{R}\}^\omega.\ \text{states of } \rho \text{ on } \pi \text{ are } \Omega\text{-accepting}}_{\text{accepting}}\}$$

$$\mathrm{L}(\mathcal{A}) = \{\ldots, t_4, \not{t_5}, \ldots\}$$



⇝ **no** unambiguity / uniformisation

⇝ **no** minimality / canonical form

⇝ several **decidability questions** still open for automata on infinite trees...

Guidability relation

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(Colcombet, Löding [2008])

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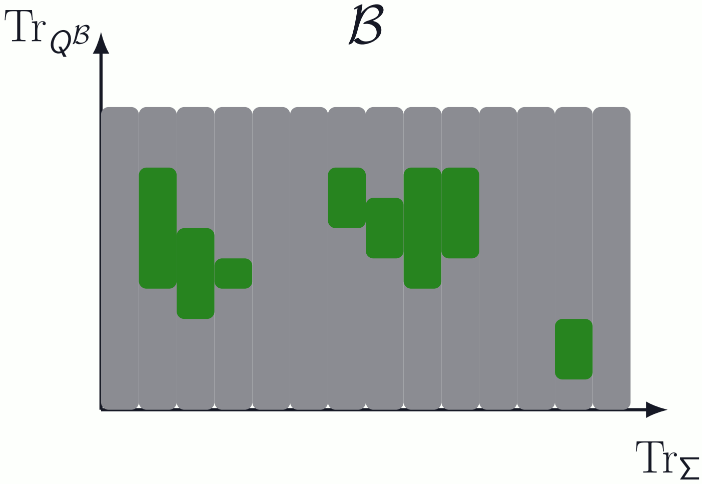

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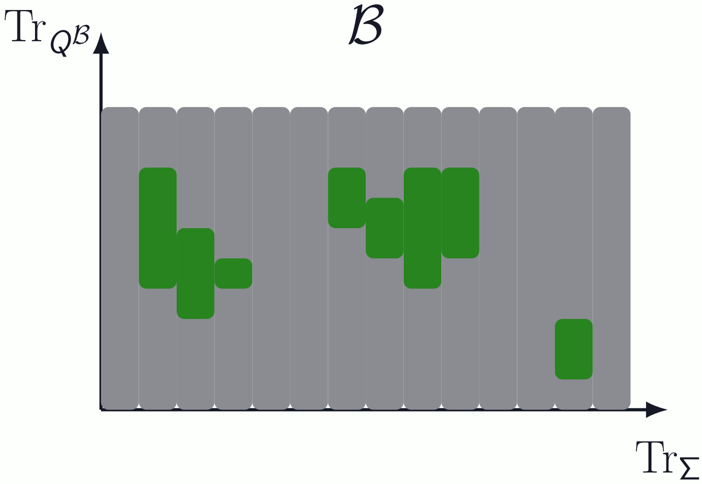

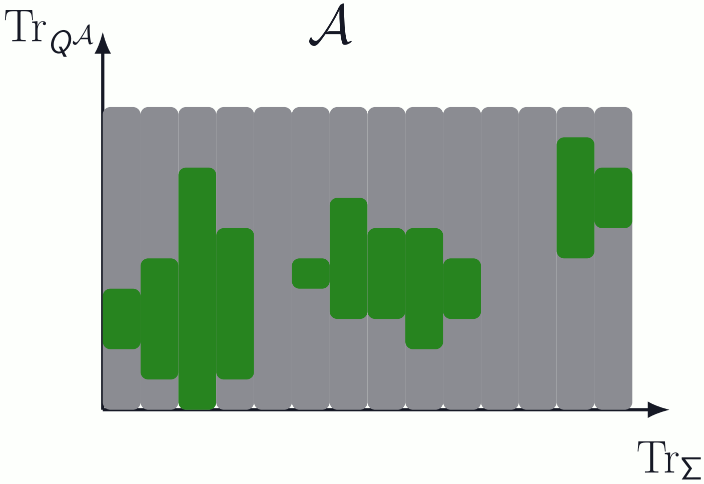

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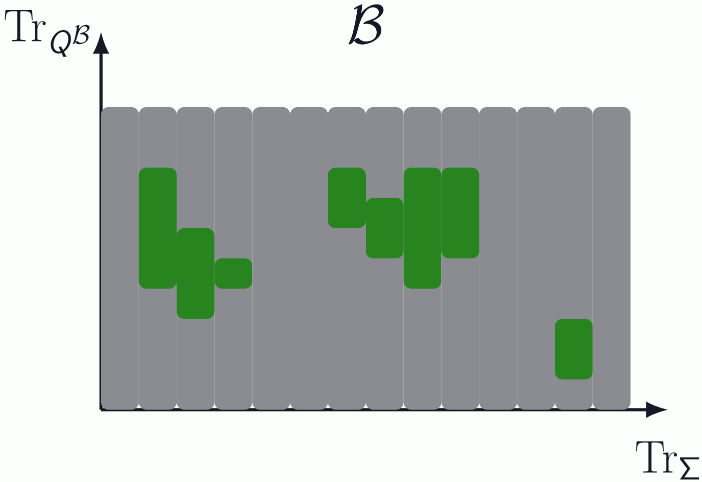

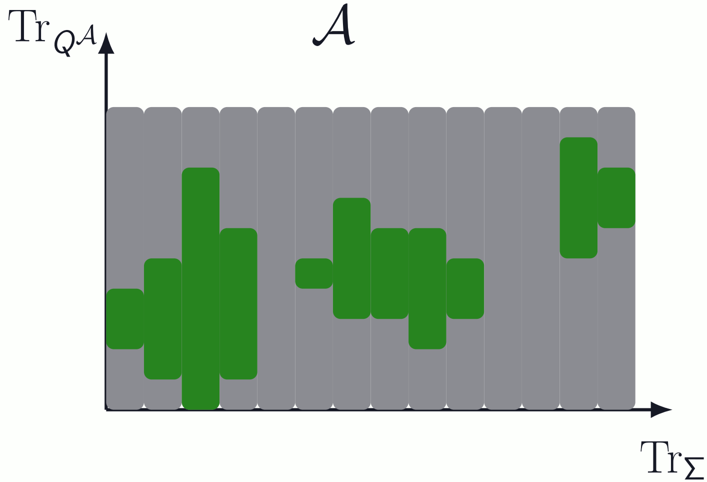

$\mathcal{B}$ **guides** $\mathcal{A}$

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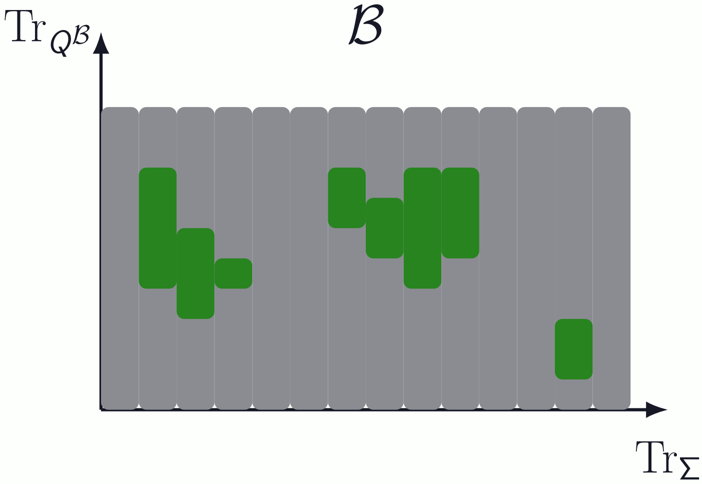

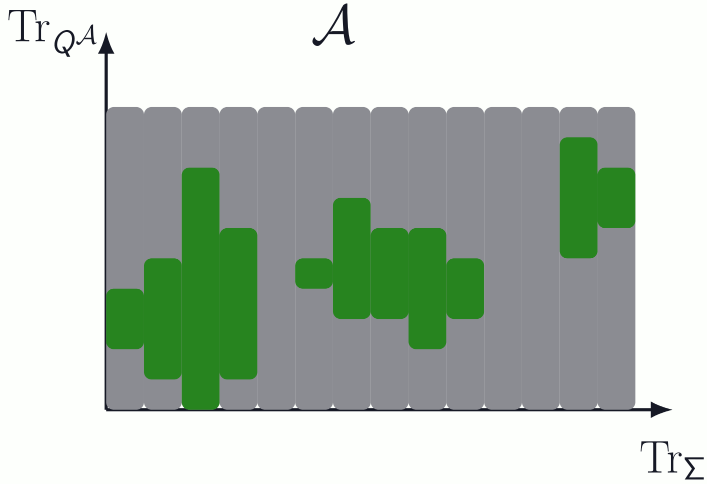

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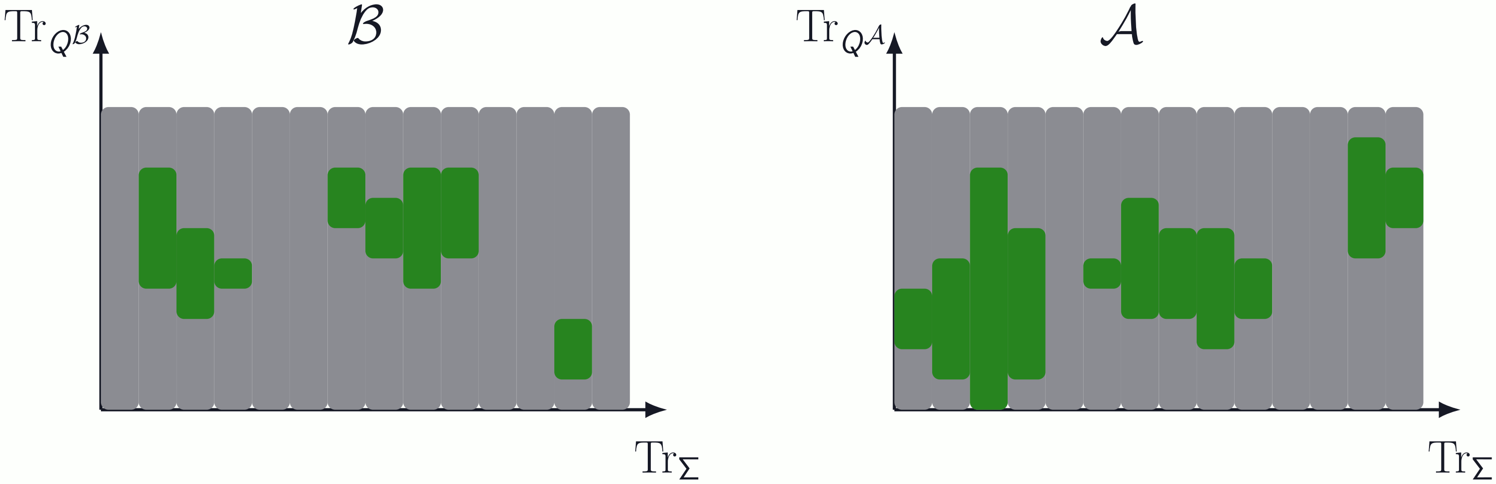

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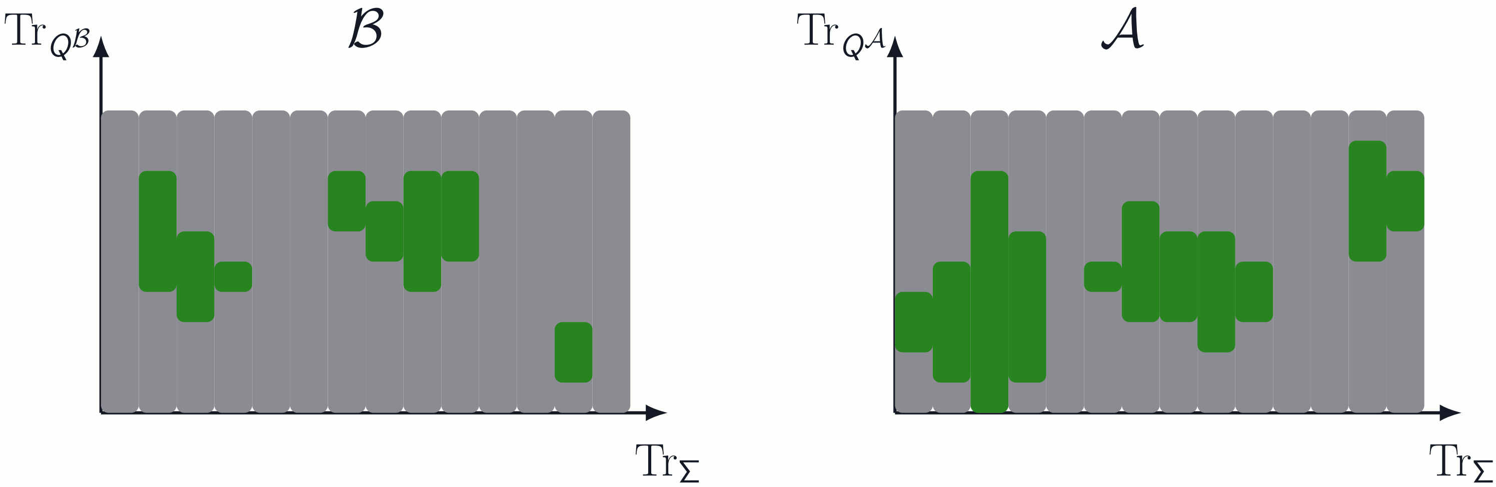

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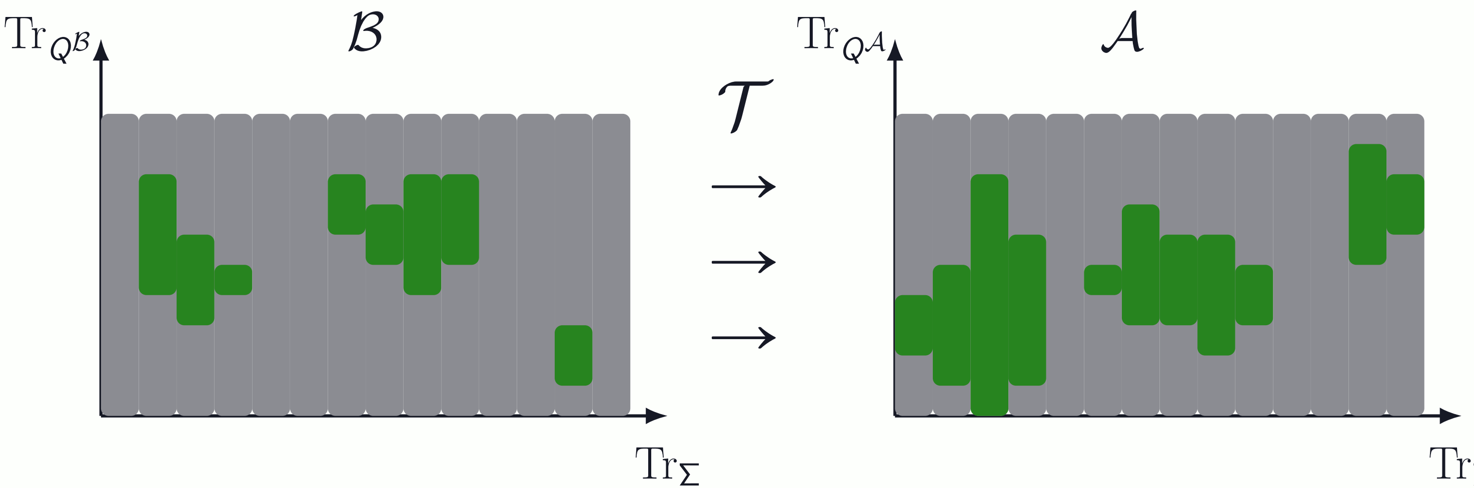

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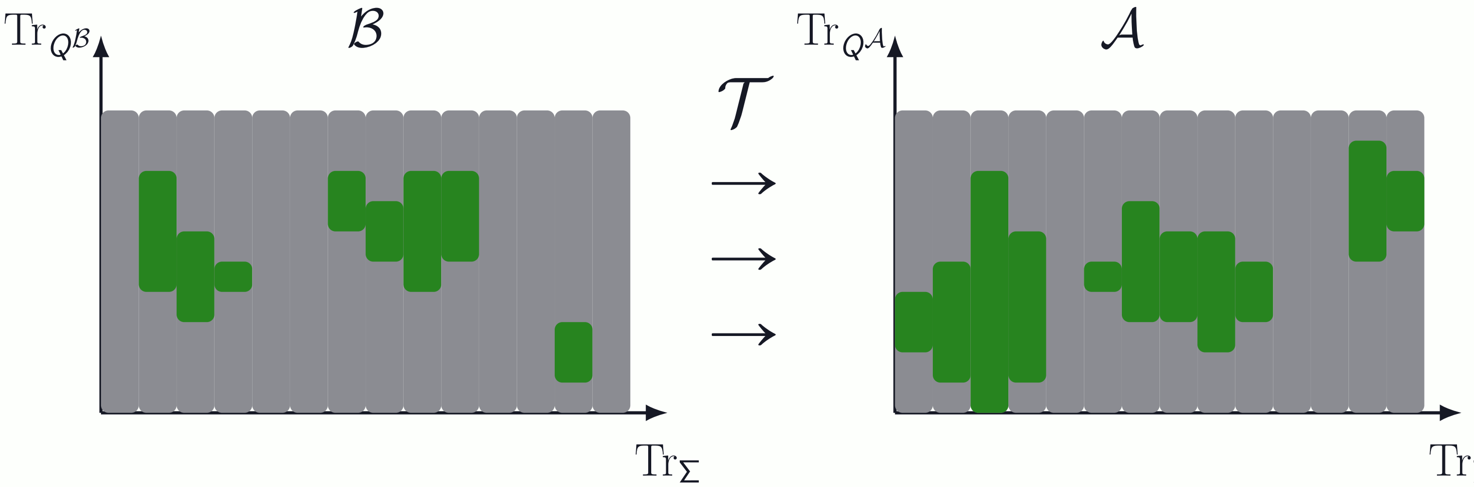

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In particular: $\mathcal{B} \hookrightarrow \mathcal{A} \implies \mathrm{L}(\mathcal{B}) \subseteq \mathrm{L}(\mathcal{A})$

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$\rightsquigarrow$ every regular tree language is recognised by some guidable automaton

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Lemma (Colcombet, Löding [2008]; Löding [2009])

If $\mathcal{B} \hookrightarrow \mathcal{A}$ then there exists a **memoryless** transducer

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Top-down deterministic automata are guidable.

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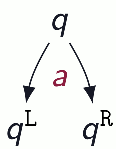
Fact

Top-down deterministic automata are guidable.

(Top-down deterministic:) For each $q \in Q$, $a \in \Sigma$
exactly one transition



Deterministic and game automata

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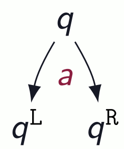
Deterministic automata

For each $q \in Q$, $a \in \Sigma$ there is a **unique** transition



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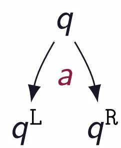

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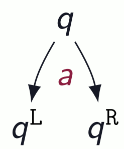

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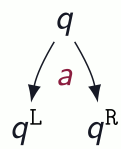

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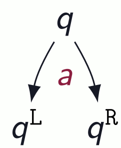

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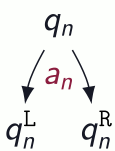

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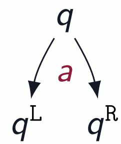

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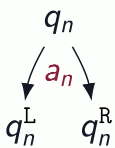

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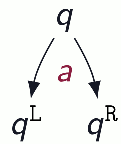

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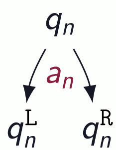

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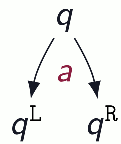

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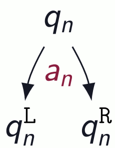

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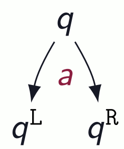

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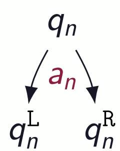

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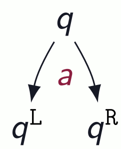

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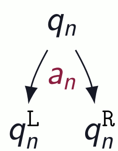

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As above, but the player $\exists$ / $\forall$ choosing d_n depends on (q_n, a_n).

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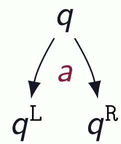

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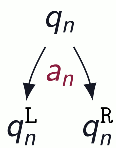

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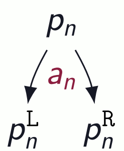

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Claim $\exists$ has a **winning strategy** **iff** there exists such $\mathcal{B}$

Index transfer

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Theorem

Assume that $\mathbf{\Delta} \subseteq \mathbf{\Gamma}$ are some of the above classes.

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Remark

But **no** such transfer for $\mathbf{\Gamma} =$ **(Non-det.)**!

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+ New game-based characterisation!