On the strength of non-determinism for Büchi VASS

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UNIVERSITY OF WARSAW

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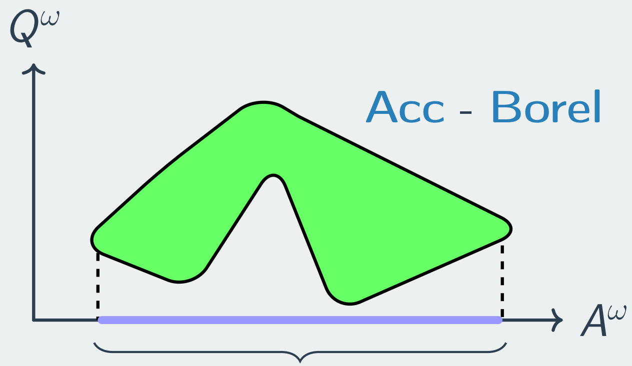

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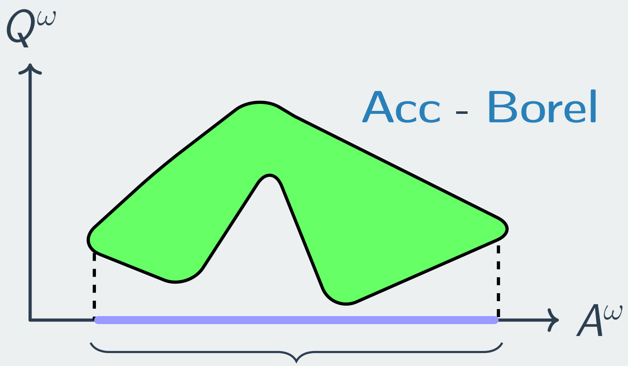

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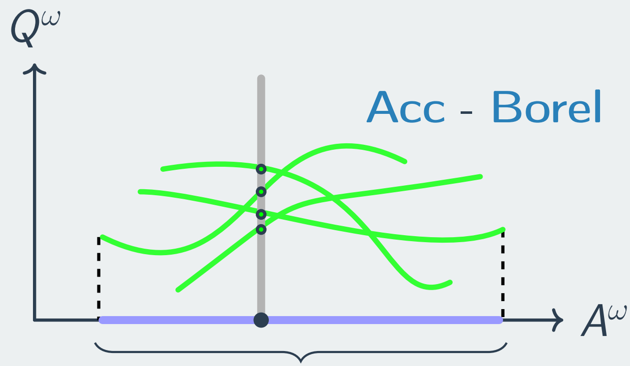

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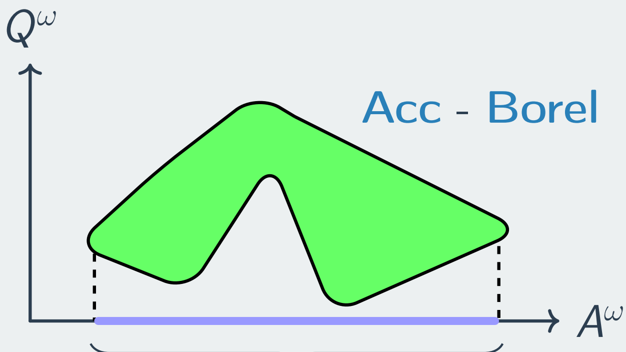

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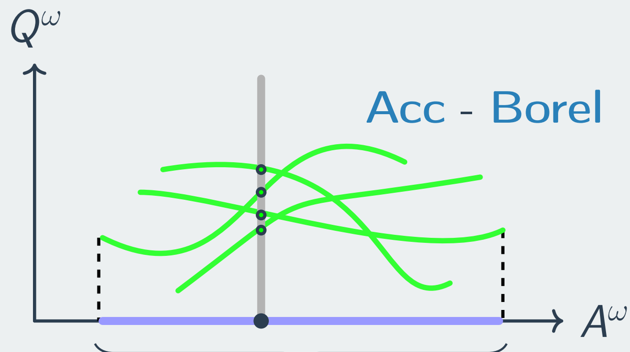

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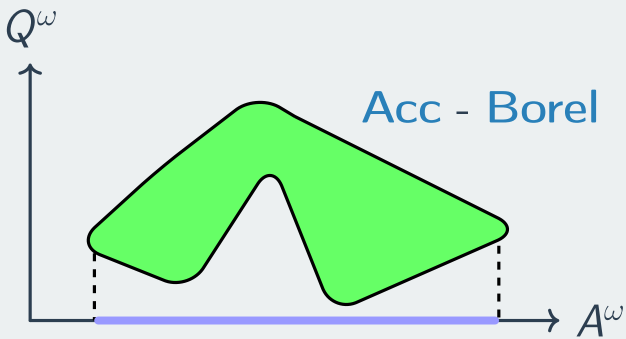

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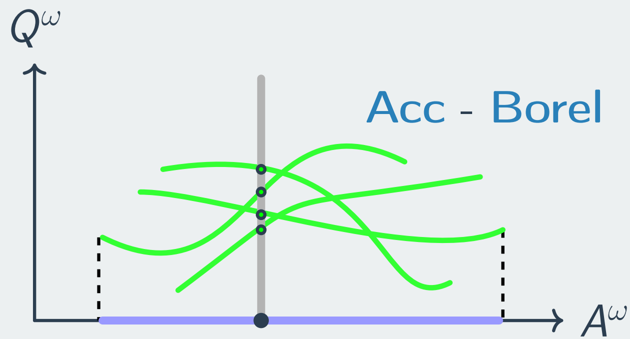

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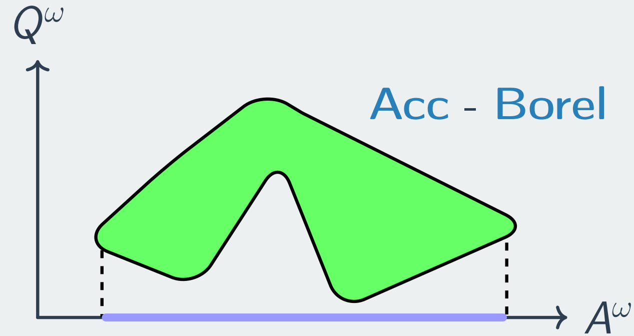

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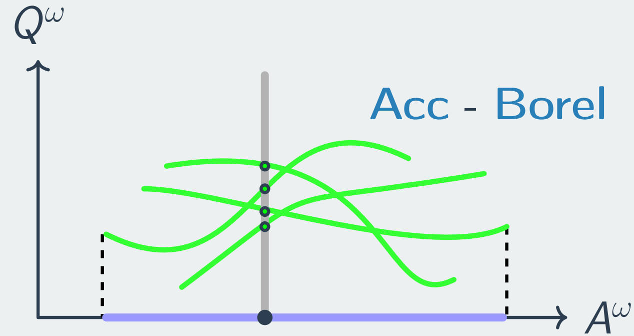

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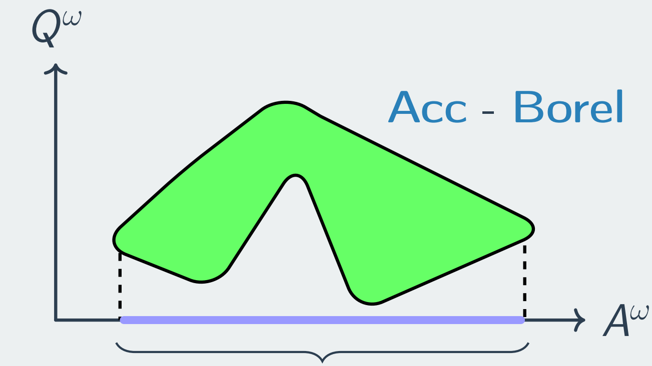

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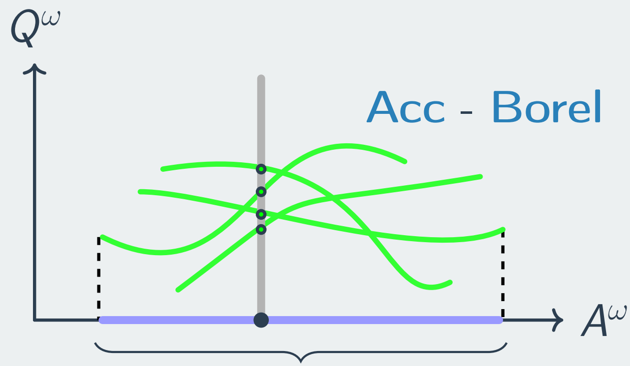

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There exists a **non Borel** ω-language recognisable by a one-counter Büchi VASS.

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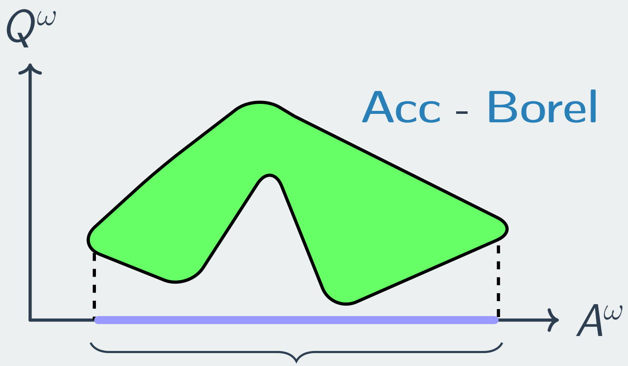

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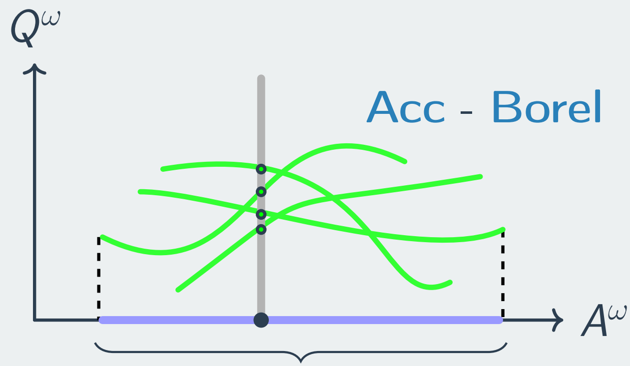

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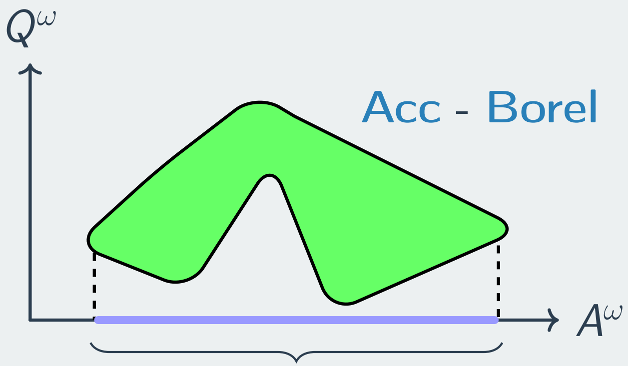

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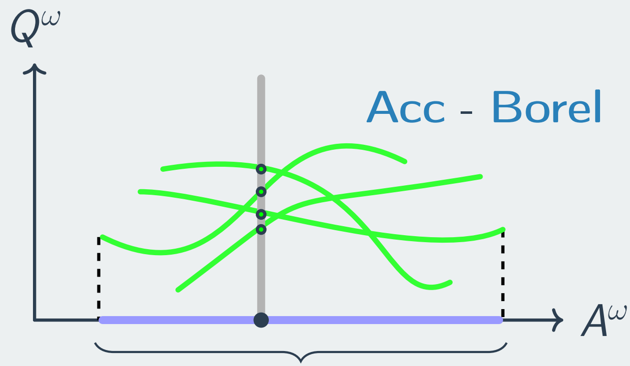

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$\longrightarrow$ Similar result (+Wadge) with four counters in (Finkel ['18]) $\longleftarrow$

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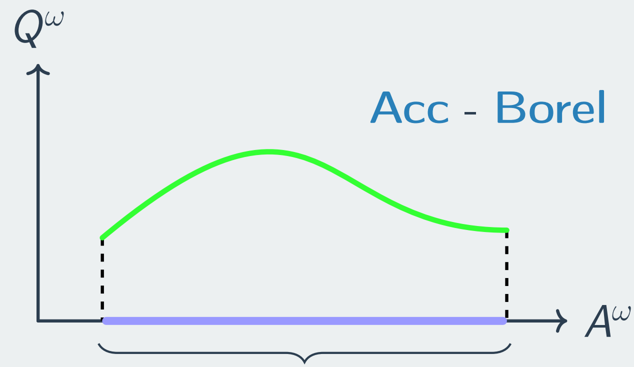

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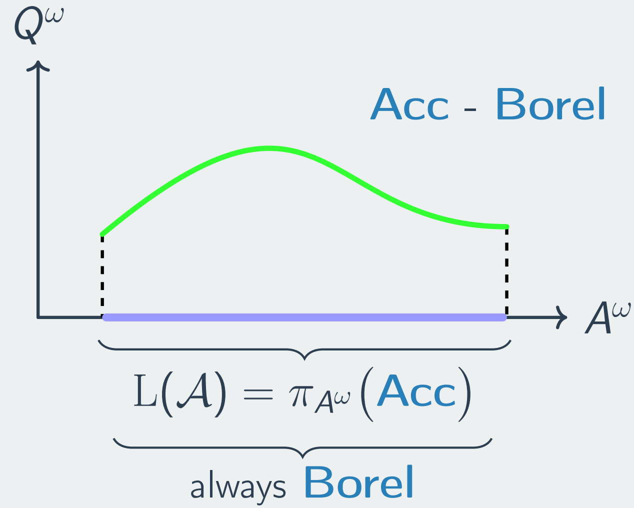

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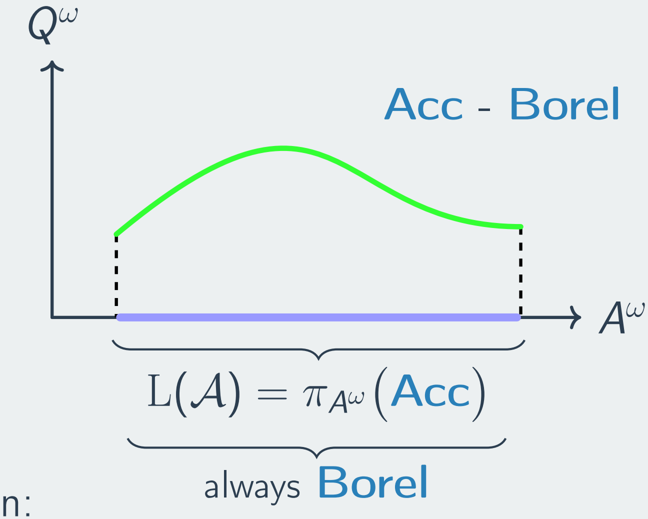

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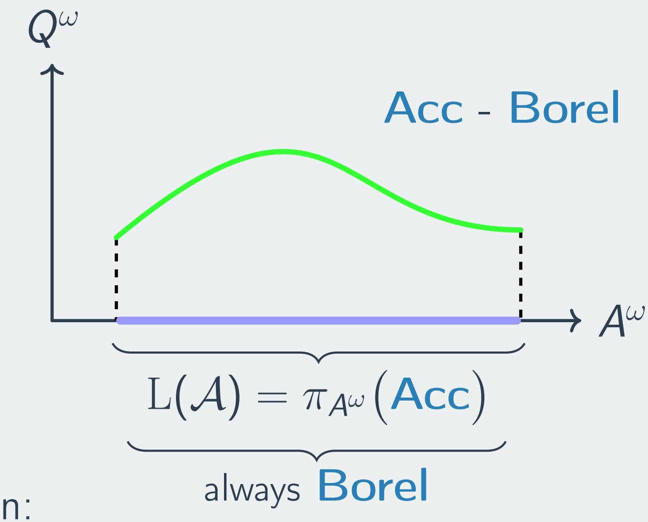

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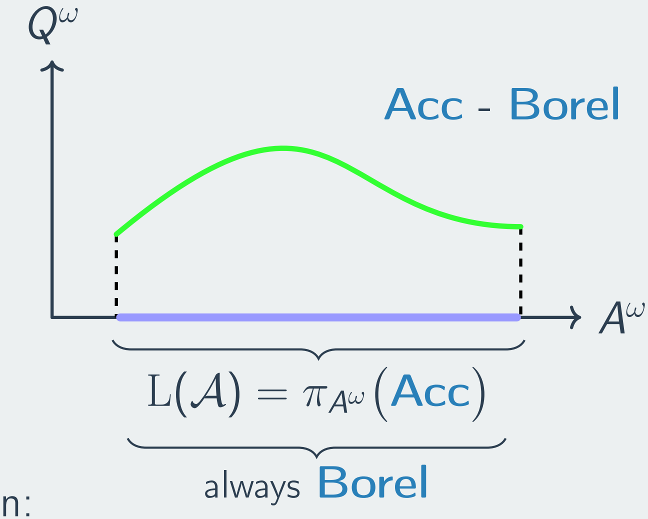

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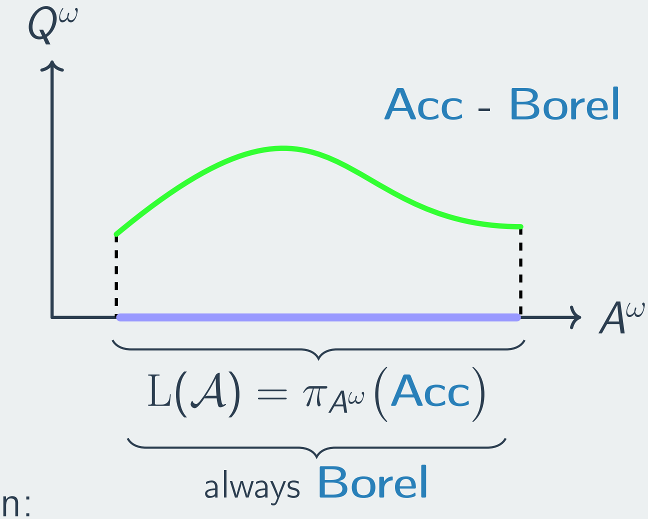

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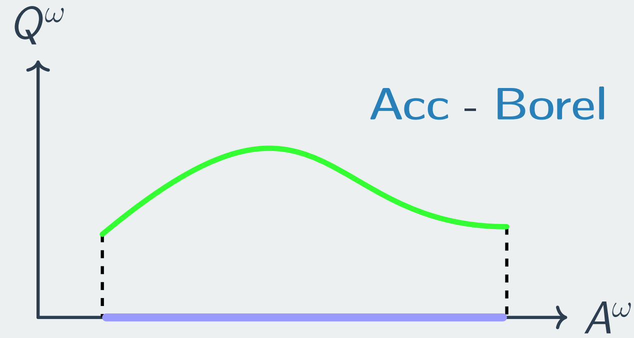

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Unambiguous VASS, when reading $w \in A^*$,

can reach **at most one** configuration per sector in $Q \times \mathbb{N}^C$.

Sectors / clubs of configurations

$$Q \times \mathbb{N}^C$$

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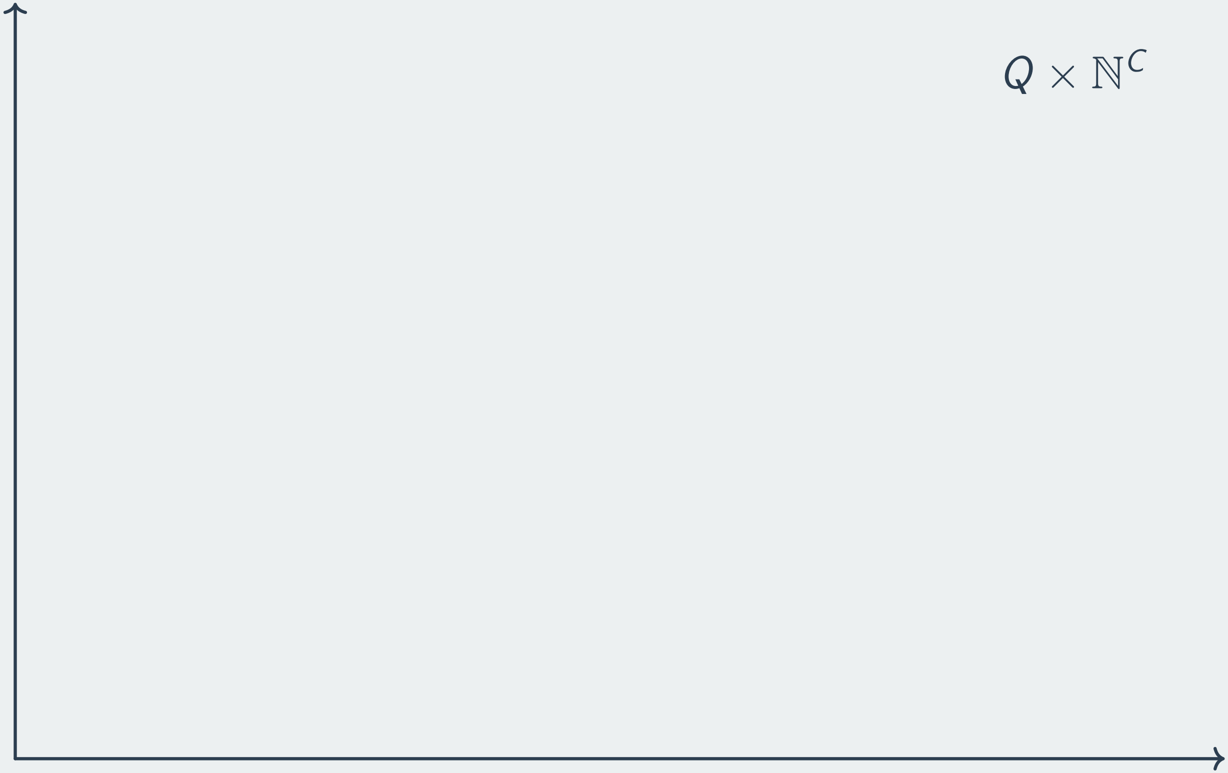

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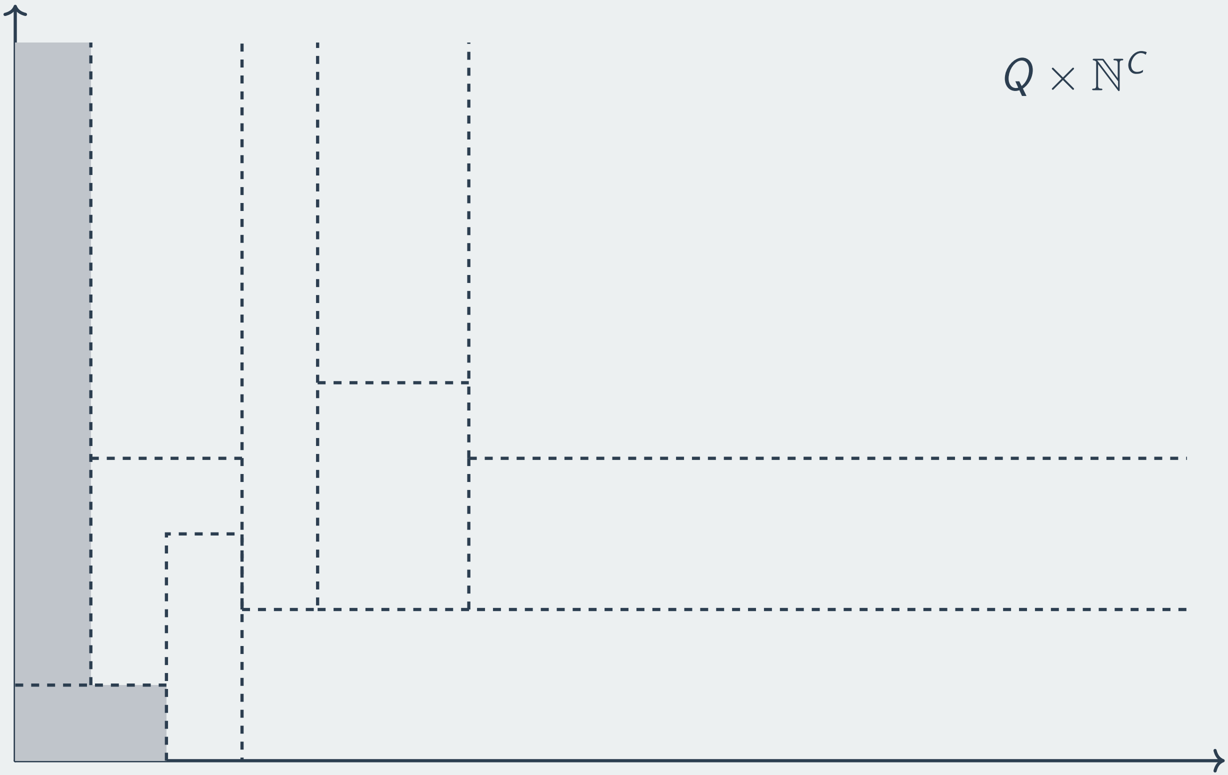

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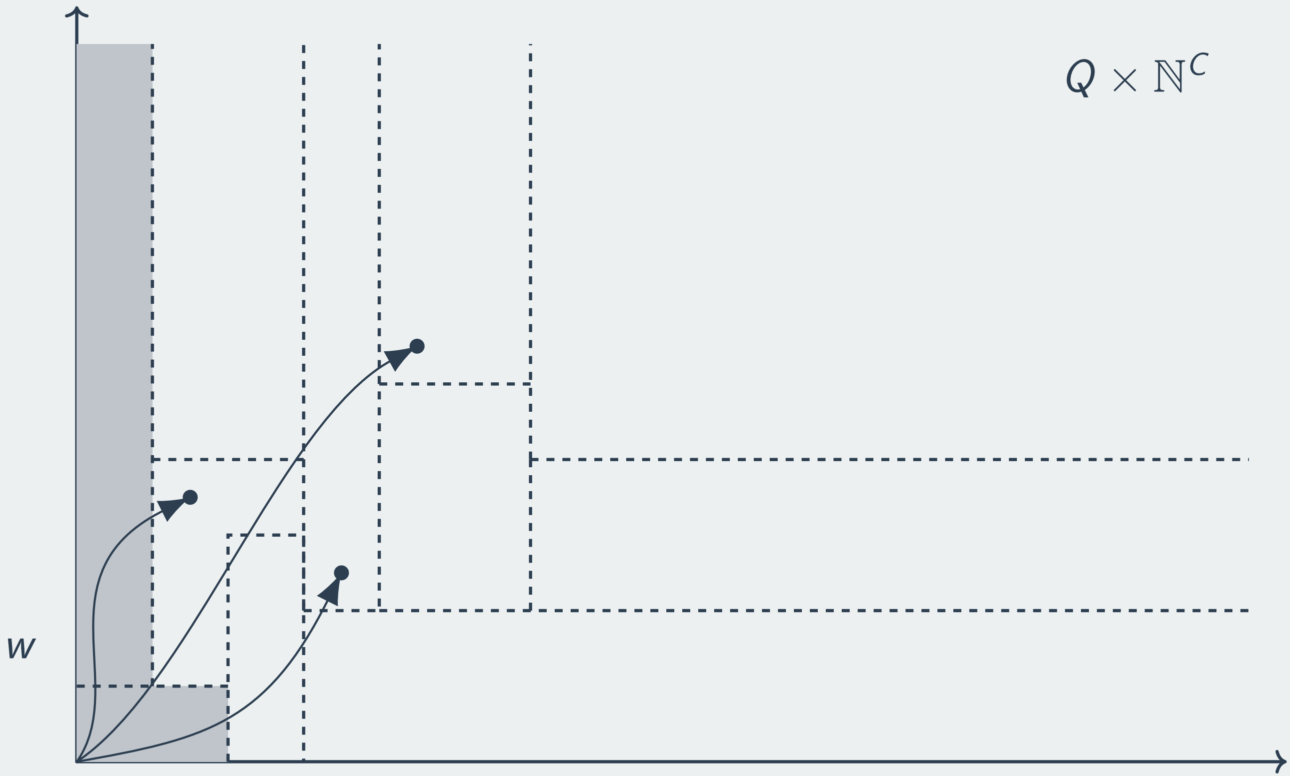

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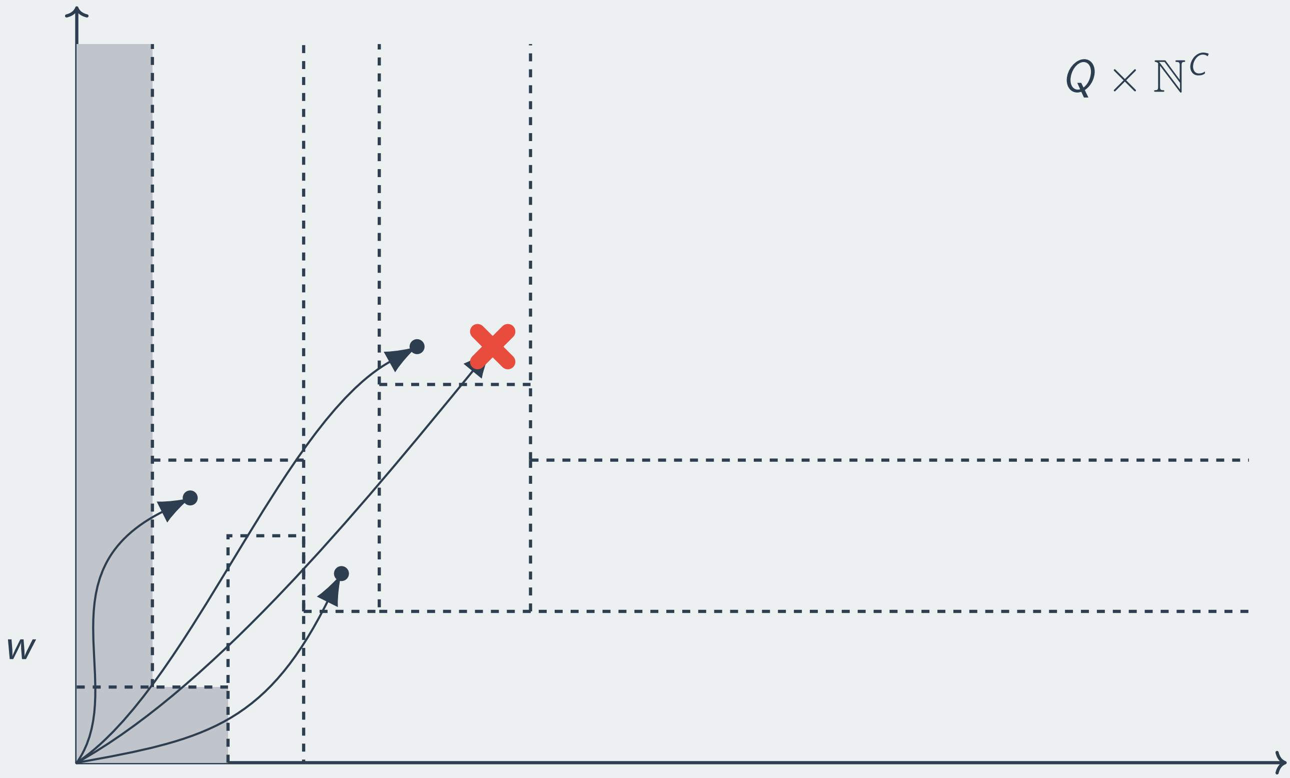

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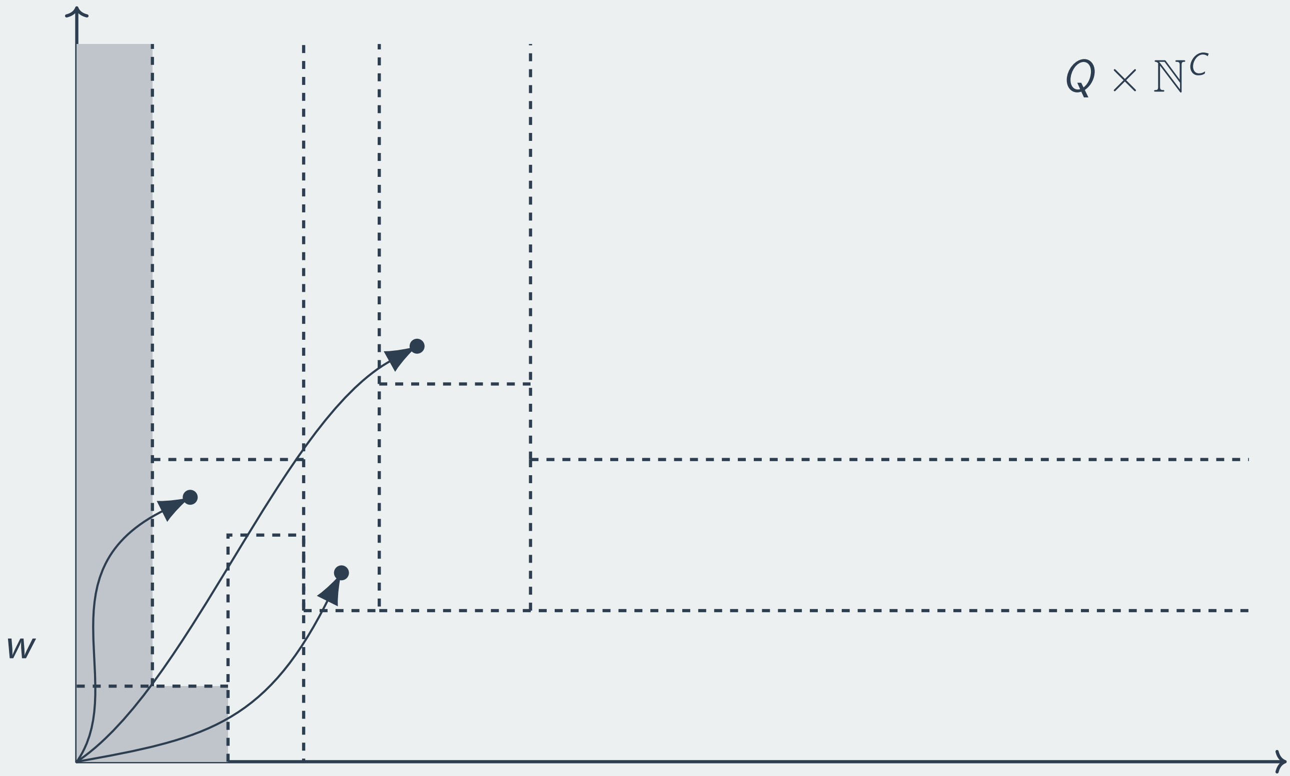

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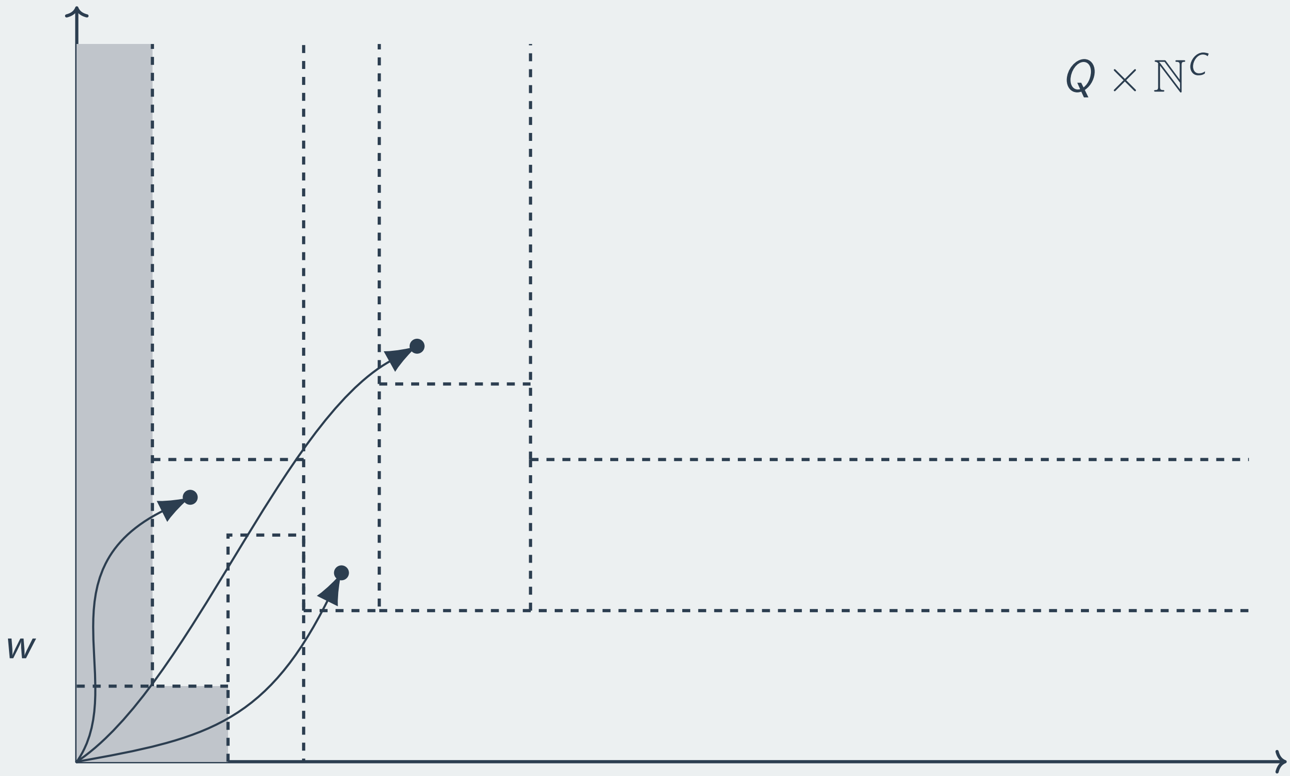

$\rightsquigarrow$ **finitary** representation of all runs

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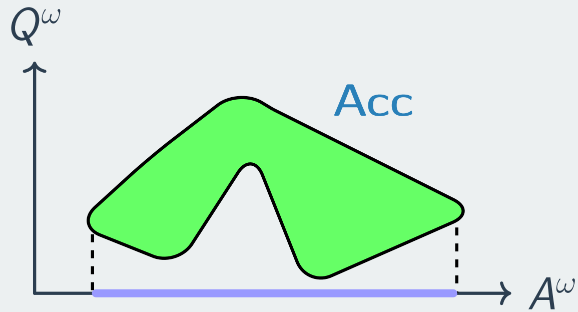

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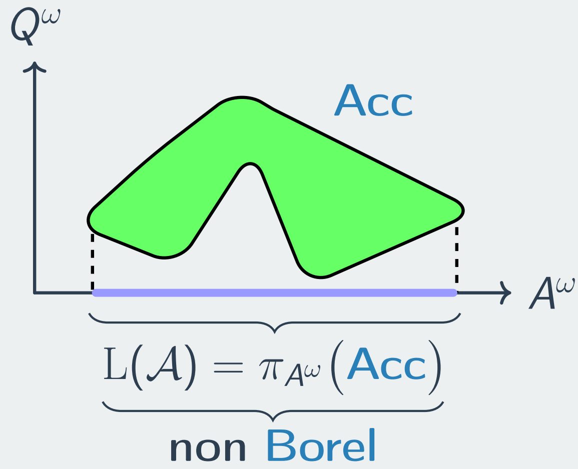

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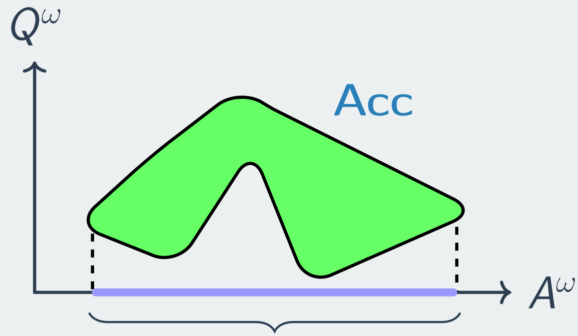

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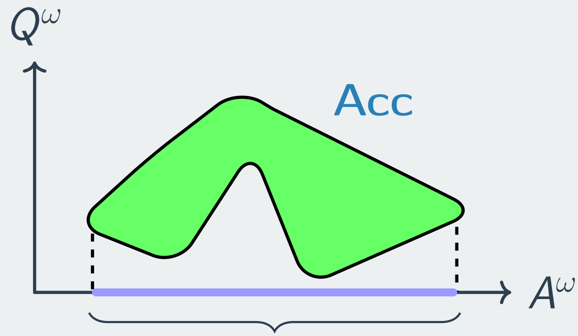

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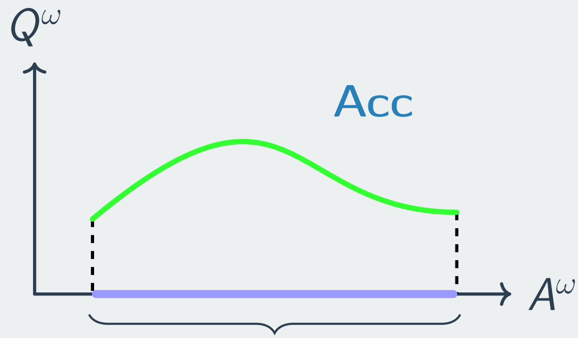

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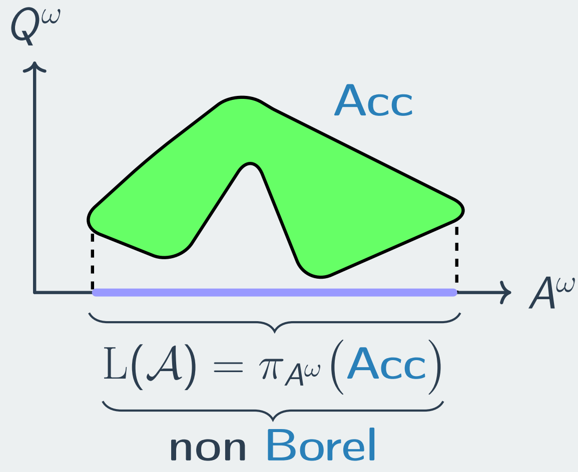

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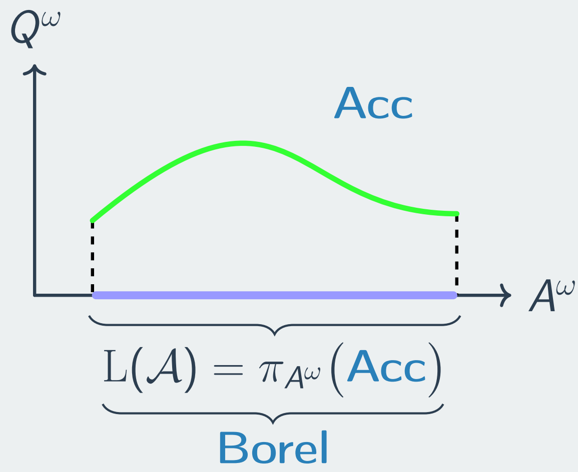

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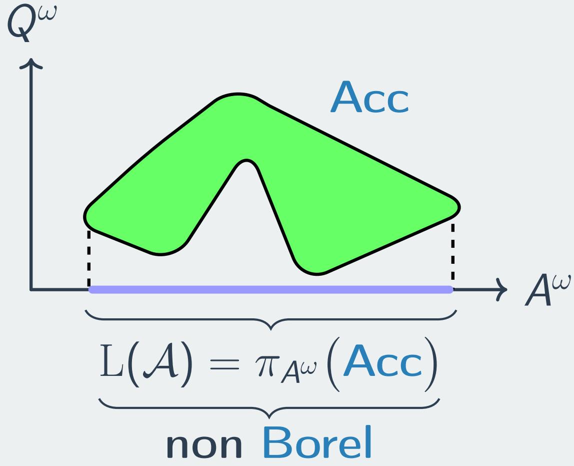

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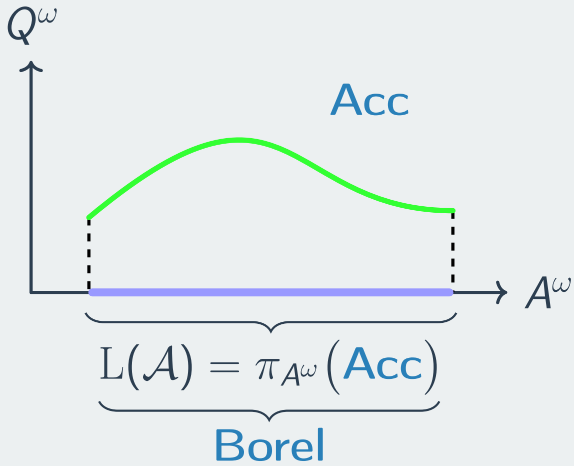

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[combinatorics of **sectors** of configurations]