Deciding the topological complexity of Büchi languages

Michał Skrzypczak **Igor Walukiewicz**

Highlights 2016
Brussels

Infinite trees

Infinite trees

Logic **Automata**

Infinite trees

Logic | **Automata**

Weak Monadic Second-Order (WMSO)

- only $\exists_x$ and $\exists_X^{\mathsf{fin}}$ quantifiers

Infinite trees

Logic

Weak Monadic Second-Order (WMSO)

- only $\exists_x$ and $\exists_X^{\mathsf{fin}}$ quantifiers

Automata

weak alternating

Infinite trees

Logic		Automata
Weak Monadic Second-Order - only $\exists_x$ and $\exists_X^{\text{fin}}$ quantifiers	(WMSO)	weak alternating
Existential Monadic Second-Order - $\exists_{X_1} \ldots \exists_{X_n} \psi$ for $\psi \in$ WMSO	($\exists$MSO)	

Infinite trees

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Weak Monadic Second-Order - only $\exists_x$ and $\exists^{\text{fin}}_X$ quantifiers	(WMSO)	weak alternating
Existential Monadic Second-Order - $\exists_{X_1} \ldots \exists_{X_n}\ \psi$ for $\psi \in$ WMSO	($\exists$MSO)	non-deterministic Büchi alternating Büchi

Infinite trees

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Full Monadic Second-Order - $\exists_x$, $\exists^{\text{fin}}_X$, and $\exists_X$ quantifiers	(MSO)	

Infinite trees

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Infinite trees

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	$\subseteq$	
Existential Monadic Second-Order - $\exists_{X_1} \ldots \exists_{X_n}\ \psi$ for $\psi \in$ **WMSO**	(**∃MSO**)	non-deterministic Büchi alternating Büchi
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Infinite trees

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Theorem (Colcombet, Kuperberg, Löding, Vanden Boom [2013])

It is decidable if a given ∃MSO formula is equivalent to a WMSO formula.

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Proof

Theory of regular cost functions

Infinite trees

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Infinite trees

Logic | **Automata**

Weak Monadic Second-Order (WMSO) — weak alternating

- only $\exists_x$ and $\exists_X^{\text{fin}}$ quantifiers

$\not\supseteq$

Existential Monadic Second-Order ($\exists$MSO) — non-deterministic Büchi, alternating Büchi

- $\exists_{X_1} \ldots \exists_{X_n}\ \psi$ for $\psi \in$ WMSO

$\not\supseteq$

Full Monadic Second-Order (MSO) — non-deterministic parity, alternating parity

- $\exists_x$, $\exists_X^{\text{fin}}$, and $\exists_X$ quantifiers

Theorem (Colcombet, Kuperberg, Löding, Vanden Boom [2013])

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reduction of Colcombet and Löding to

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Proof

Theory of regular cost functions
+
reduction of Colcombet and Löding to
a boundedness problem

Topology of infinite trees

Topology of infinite trees

$$A^{(\{\mathtt{L},\mathtt{R}\}^*)}$$

Topology of infinite trees

$$A^{(\{\mathtt{L},\mathtt{R}\}^*)}$$

$$\cong$$

Topology of infinite trees

$$A^{(\{\mathtt{L},\mathtt{R}\}^*)}$$

$$\cong$$

$$\{0,1\}^\omega$$

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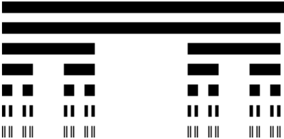
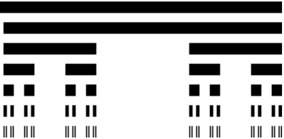


the Cantor set

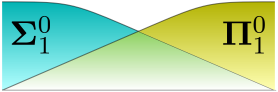


Descriptive set theory

Descriptive set theory

Start from simple sets

$\rightsquigarrow$ open ($\mathbf{\Sigma}^0_1$) and closed ($\mathbf{\Pi}^0_1$)



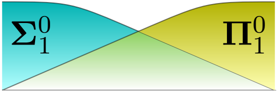
Descriptive set theory

Start from simple sets

$\rightsquigarrow$ open ($\mathbf{\Sigma}^0_1$) and closed ($\mathbf{\Pi}^0_1$)

Apply countable unions ($\bigcup$)

and countable intersections ($\bigcap$)



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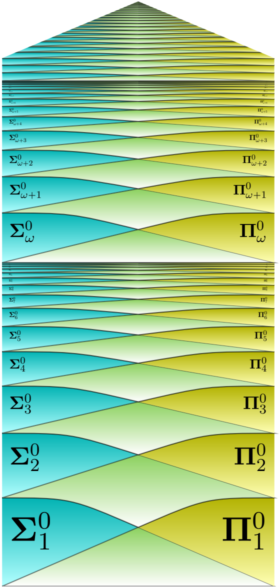

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$\rightsquigarrow$ open ($\mathbf{\Sigma}^0_1$) and closed ($\mathbf{\Pi}^0_1$)

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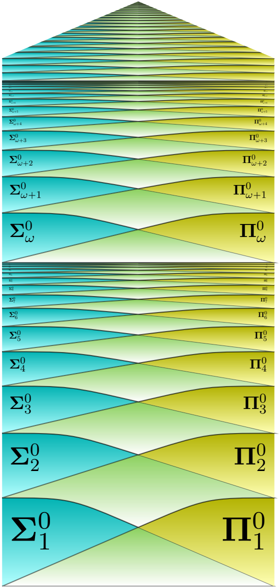
$\rightsquigarrow$ Borel sets: $\mathbf{\Sigma}^0_\eta$, $\mathbf{\Pi}^0_\eta$ for $\eta < \omega_1$



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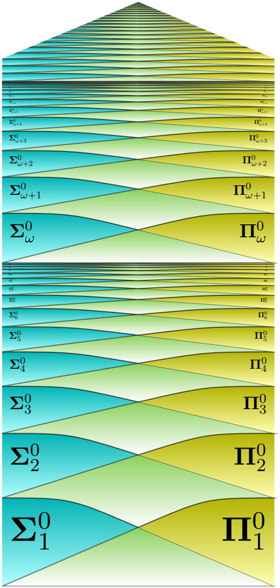
Apply projection and co-projection



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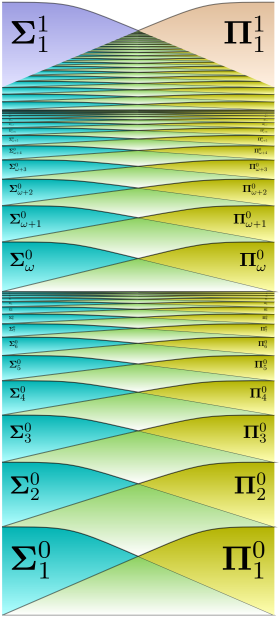
$\rightsquigarrow$ analytic ($\mathbf{\Sigma}^1_1$) and co-analytic ($\mathbf{\Pi}^1_1$)



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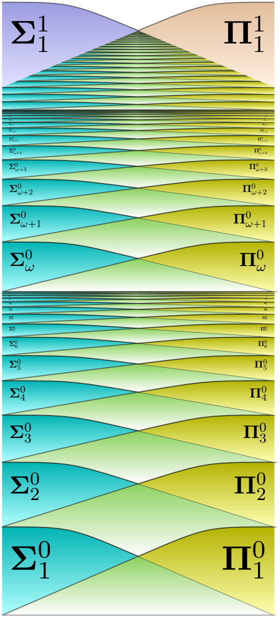
⇝ analytic ($\mathbf{\Sigma}^1_1$) and co-analytic ($\mathbf{\Pi}^1_1$)

By induction



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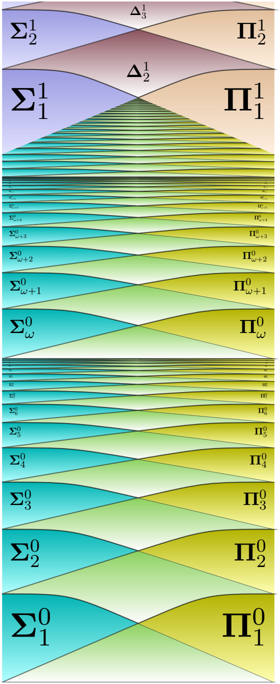
⇝ Borel sets: $\mathbf{\Sigma}^0_\eta$, $\mathbf{\Pi}^0_\eta$ for $\eta < \omega_1$

Apply projection and co-projection

⇝ analytic ($\mathbf{\Sigma}^1_1$) and co-analytic ($\mathbf{\Pi}^1_1$)

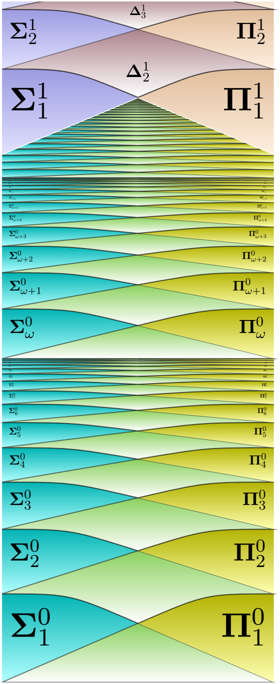
By induction

⇝ projective sets: $\mathbf{\Sigma}^1_n$, $\mathbf{\Pi}^1_n$ for $n < \omega$



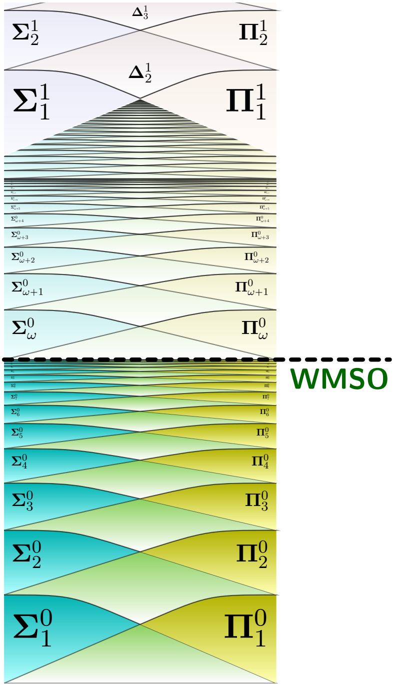
Upper bounds

Upper bounds



Upper bounds

$$L \in \textbf{WMSO} \implies L \in \mathbf{\Sigma}^0_n \quad \text{(for some } n\text{)}$$



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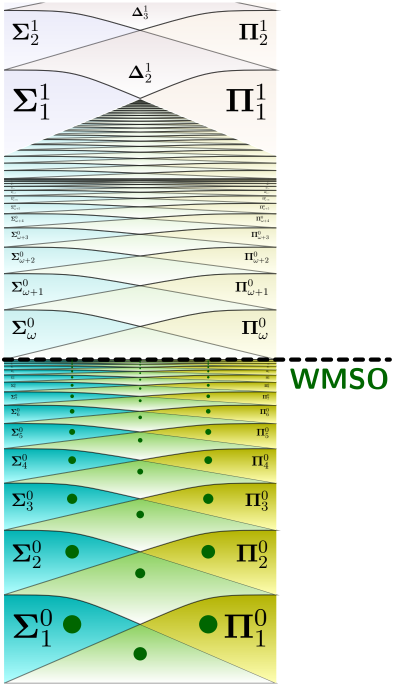

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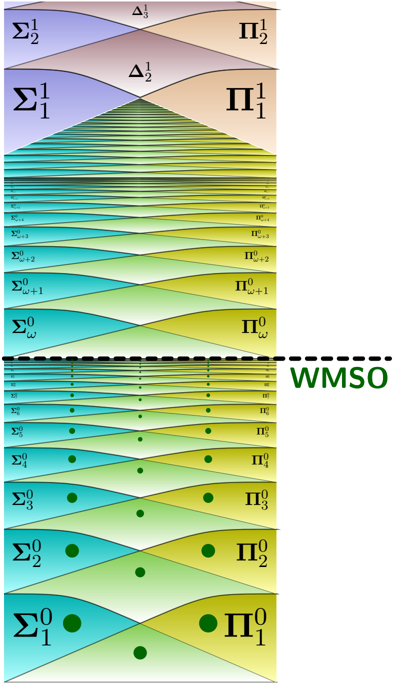

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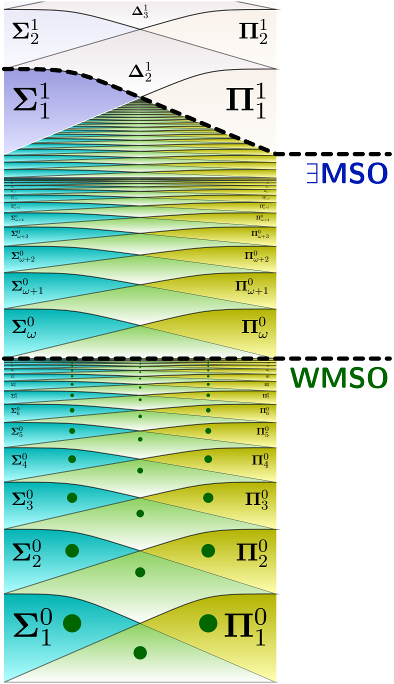
$$L \in \exists\textbf{MSO} \implies L \in \mathbf{\Sigma}^1_1$$



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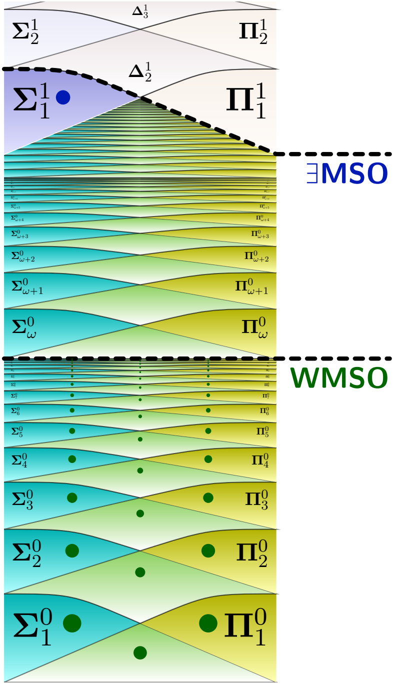
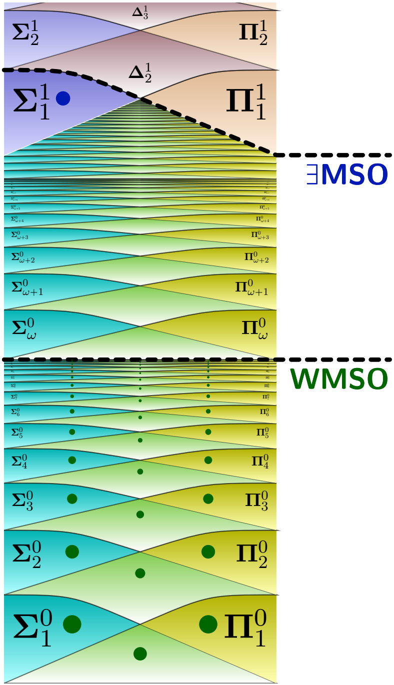


Upper bounds

$L \in$ WMSO $\implies L \in \Sigma^0_n$ (for some n)

$L \in \exists$MSO $\implies L \in \Sigma^1_1$



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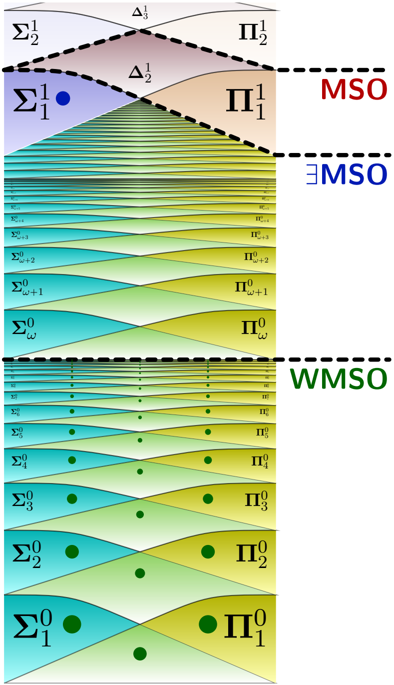
$$L \in \exists\textbf{MSO} \implies L \in \mathbf{\Sigma}^1_1$$

$$L \in \textbf{MSO} \implies L \in \mathbf{\Delta}^1_2$$



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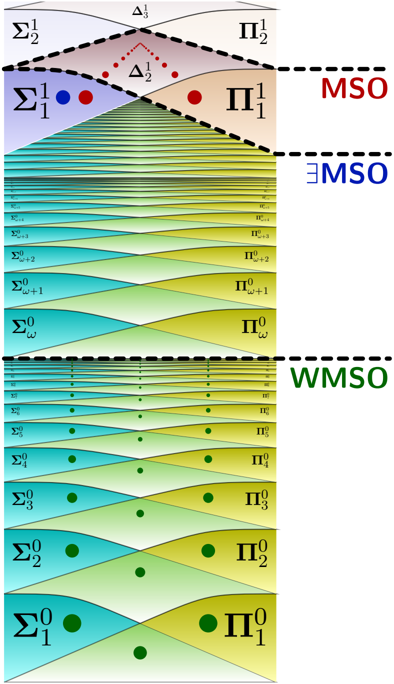

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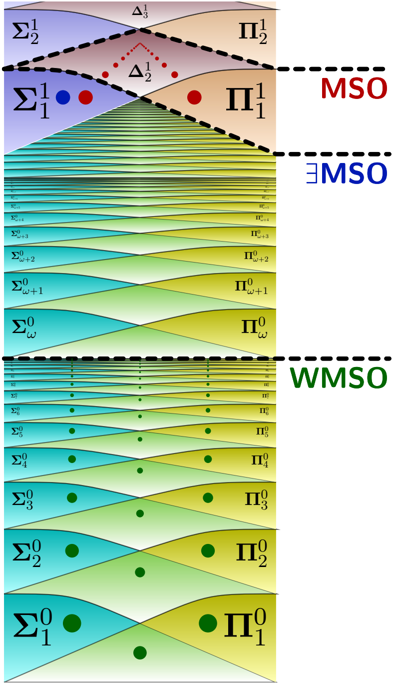

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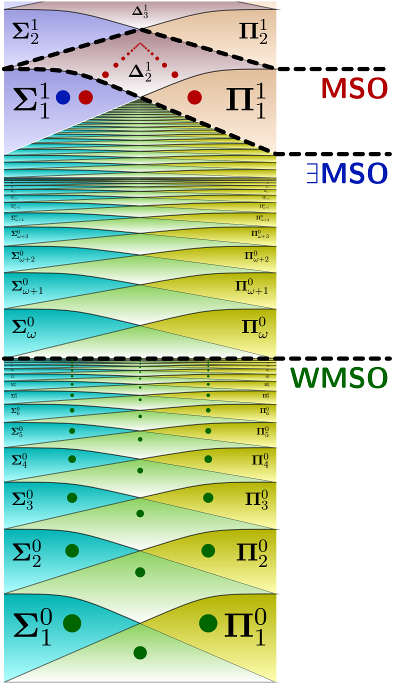
WMSO $\subsetneq$ ∃MSO $\subsetneq$ MSO



Gap property

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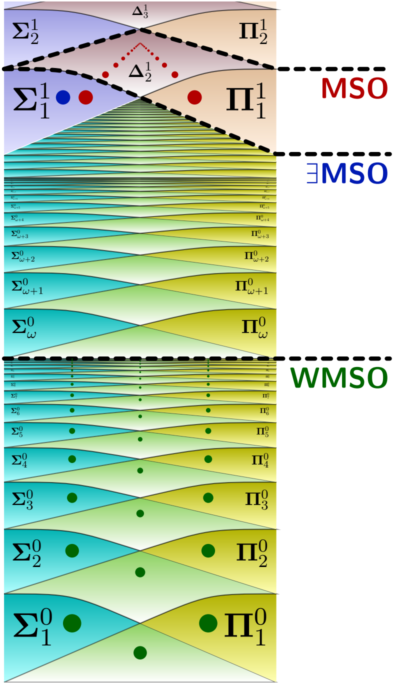

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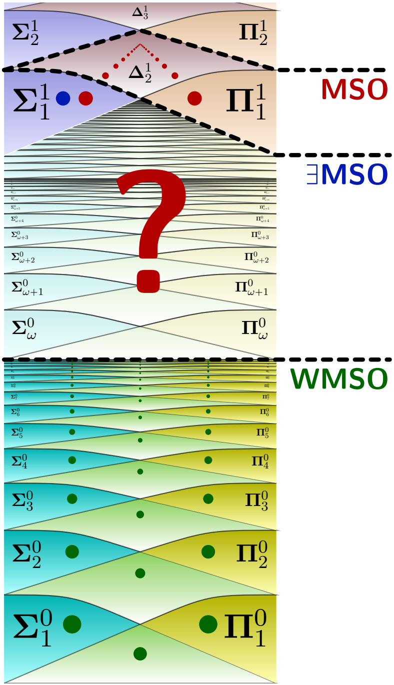
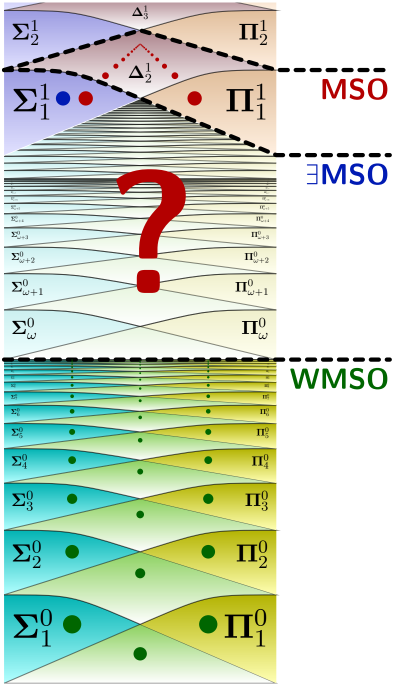


Gap property

Conjecture (Skurczyński [1993])

A Borel MSO-definable language is WMSO-definable.



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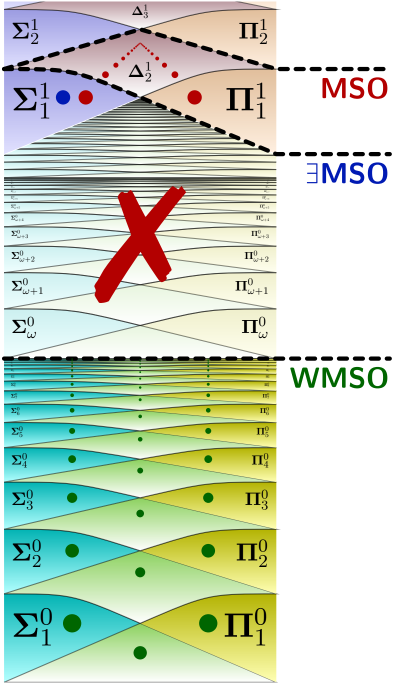

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An ∃MSO-definable language is either:

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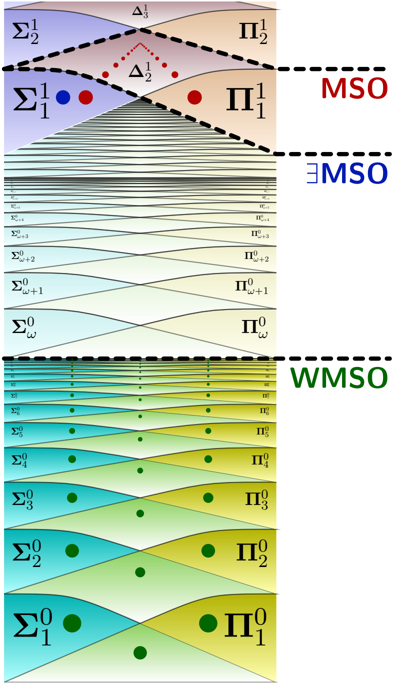
— **not** WMSO-definable and Σ^1_1-comp.



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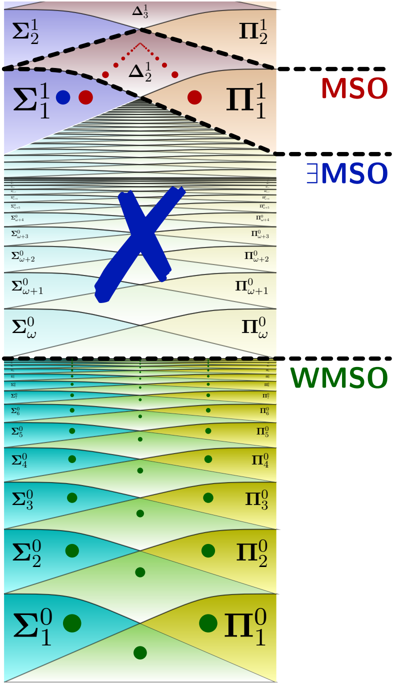

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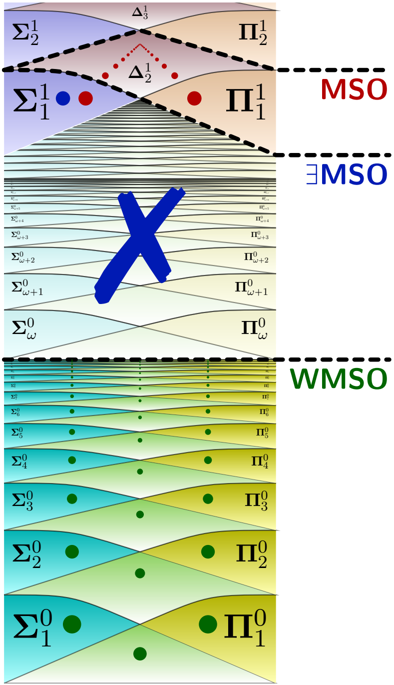
And the dychotomy is decidable.



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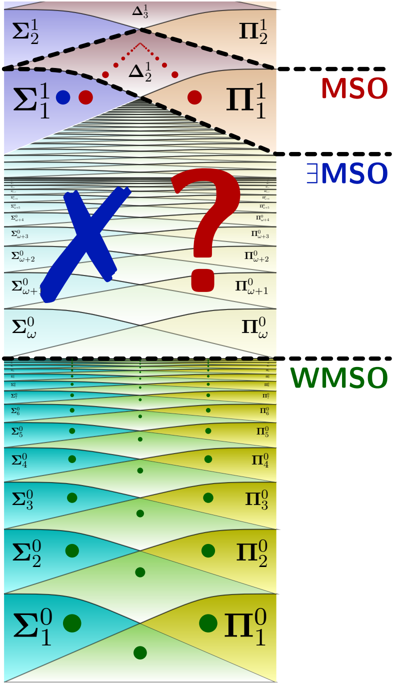

Theorem (Walukiewicz, S. [2016])

$L \in$ ∃MSO is effectively either:

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Theorem (Walukiewicz, S. [2016])

$L \in \exists$MSO is effectively either:

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— Σ_1^1-complete

Proof

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Proof

Input $\varphi \in$ **∃MSO** (or a Büchi automaton) for L

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Construct a finite game $\mathcal{F}$ with an ω-regular winning condition

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$\rightsquigarrow$ If $\forall$ wins $\mathcal{F}$:

Theorem (Walukiewicz, S. [2016])

$L \in \exists$MSO is effectively either:

— WMSO-definable

— Σ^1_1-complete

Proof

Input $\varphi \in \exists$MSO (or a Büchi automaton) for L

Construct a finite game $\mathcal{F}$ with an ω-regular winning condition

$\rightsquigarrow$ If $\forall$ wins $\mathcal{F}$:

$\sigma_\forall$ — finite memory winning strategy

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$\rightsquigarrow$ If ∀ wins $\mathcal{F}$:

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$\rightsquigarrow$ If ∃ wins $\mathcal{F}$:

$\sigma_\exists$ — a winning strategy $\rightsquigarrow$ L is Σ^1_1-comp.

$\sigma_\exists$ gives a continuous reduction of IF to L

Summary

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Theorem (CKLV [2013])

It is decidable for $L \in$ **∃MSO**

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Theorem (CKLV [2013])

It is decidable for $L \in \exists\text{MSO}$

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It is decidable for $L \in \exists\textsc{mso}$ if $L \in \textsc{wmso}$.

Proof

Regular cost functions

Summary

Theorem (CKLV [2013])

It is decidable for $L \in \exists$MSO
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Proof

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+

Boundedness problem

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(direct constructions)

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