On determinisation of Good-For-Games automata

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joint work with Denis Kuperberg

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LIAFA
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Synthesis

Synthesis

φ — specification (i.e. MSO)

Synthesis

φ — specification (i.e. MSO)

$\rightsquigarrow$

— machine

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Synthesis

φ — specification (i.e. MSO)

⇝

 — machine

Environment:

Synthesis

Environment: I_0,

φ — specification (i.e. MSO)

 — machine

Synthesis

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$\rightsquigarrow$

 — machine

Environment: $I_0,$

$\downarrow$

$\downarrow$

$O_0,$

Synthesis

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⇝

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Environment: $I_0, \ I_1,$

↓

↓

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Synthesis

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Environment: I_0, I_1,

$\downarrow$

$\downarrow$

O_0, O_1,

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Environment: $I_0, \;\; I_1, \;\; I_2,$

$\downarrow$

$\downarrow$

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Synthesis

Environment: $I_0, \; I_1, \; I_2,$

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Environment: $I_0, \ I_1, \ I_2, \ \ldots$

$\downarrow$

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$O_0, \ O_1, \ O_2, \ \ldots$

$$\big(I_0, O_0, \ I_1, O_1, \ \ldots\big) \models \varphi$$

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⇝

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Game semantics

Two players, perfect information

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Game semantics

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$\forall$: I_i

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$\exists$ wins if $\underbrace{\big(I_0, O_0, \; \ldots\big) \models \varphi}_{\omega\text{-regular winning condition}}$

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(Büchi, Landweber ['69])

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ω-regular winning condition

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$\mathbf{det}$: transition of $\mathcal{A}_{\mathbf{det}}$

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$\forall$: I_i

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$\mathbf{det}$: transition of $\mathcal{A}_{\mathbf{det}}$

$\exists$ wins if

the run of $\mathcal{A}_{\mathbf{det}}$ is accepting

Complexity

Complexity

$\varphi \rightsquigarrow \mathcal{A}_{\mathbf{det}}$ — det. automaton

Complexity

$$\underbrace{\varphi \rightsquigarrow \mathcal{A}_{\mathbf{det}} \text{ — det. automaton}}_{\textbf{expensive}!}$$

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[e.g. 2-EXP for LTL]

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$$\underbrace{\varphi \rightsquigarrow \mathcal{A}_{\mathbf{non-det}}}_{\textbf{exponentially} \text{ more succinct!}}$$

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$\forall$: I_i

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$\exists$: transition of $\mathcal{A}_{\mathbf{non-det}}$

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$\forall$ may cheat!

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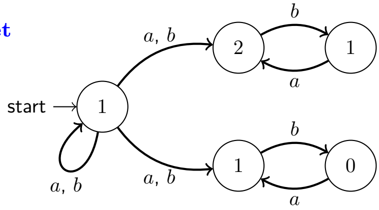

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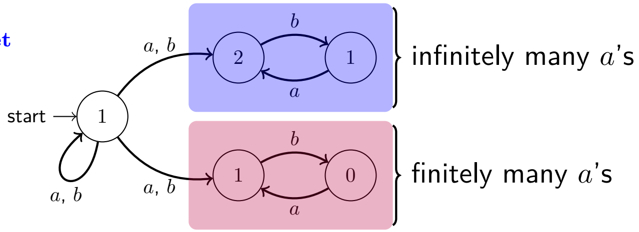

Good-For-Games automata

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$$\exists_{\sigma\colon \Sigma^* \to Q} \qquad \forall_{w \in \mathrm{L}(\mathcal{A}_{\mathbf{non-det}})} \qquad \sigma(w) \text{ is accepting}$$

Good-For-Games automata

$\mathcal{A}_{\mathbf{non-det}}$ is Good-For-Games (GFG) if

$\Big(\sigma(), \sigma(w_0), \sigma(w_0 w_1), \ldots\Big)$

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Synthesis (Henzinger, Piterman ['06])

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$\sigma(w)$ ← $\big(\sigma(), \sigma(w_0), \sigma(w_0 w_1), \ldots\big)$

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$\varphi \rightsquigarrow \mathcal{A}_{\mathbf{GFG}}$ — GFG automaton

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$\forall$: I_i

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History determinism (Colcombet, Löding ['08])

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where $\sigma(w)$ denotes $\big(\sigma(), \sigma(w_0), \sigma(w_0 w_1), \ldots\big)$

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[replaces determinism for counter automata]

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GFG $\equiv$ Good-For-Trees (derived languages $\forall\textsc{path}(\mathrm{L})$)

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What

is the blow-up of

$\mathcal{A}_{\mathbf{GFG}} \rightsquigarrow \mathcal{A}_{\mathbf{det}}$

?

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Every GFG automaton over finite words contains an equivalent deterministic subautomaton.

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Conjecture (Colcombet ['12])

The same holds for ω-words (parity automata).

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There exists a Büchi GFG automaton with **no** equivalent deterministic subautomaton.

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Does every GFG automaton admit polynomial determinisation?

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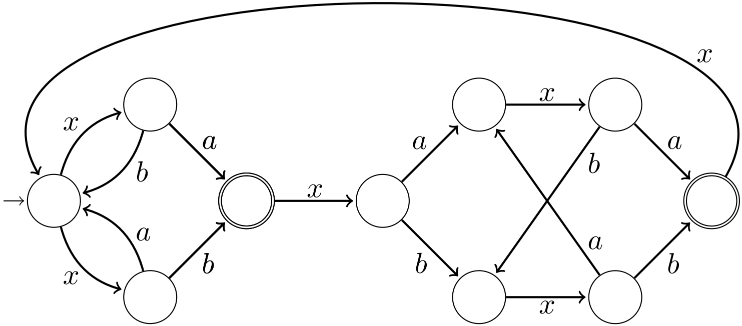

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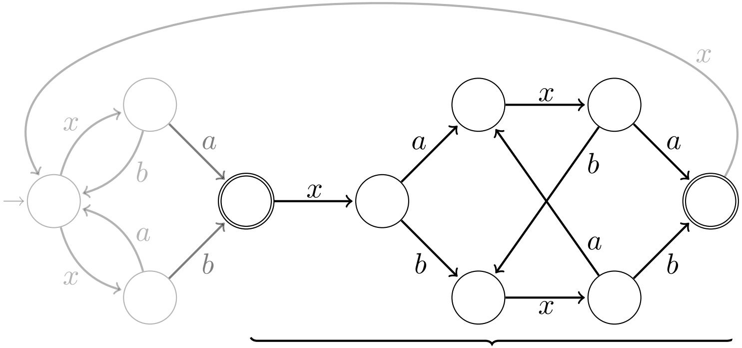
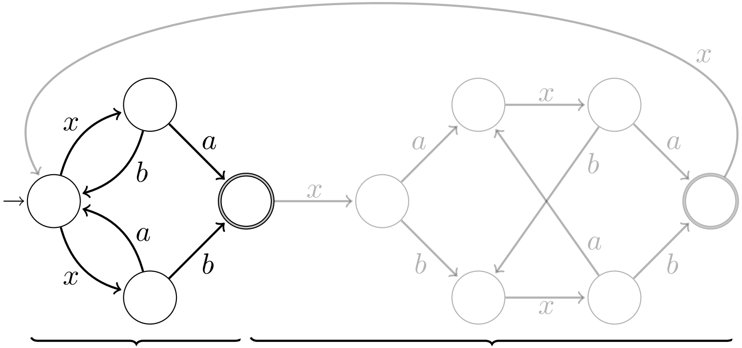


$$\mathrm{L} = \big(x(a,b)\big)^* \cdot \big(xaxa, xbxb\big)$$

deterministic

Boker's example — Büchi GFG automaton w.o. det. subautomaton



$\mathrm{L} = \big(x(a,b)\big)^*$

non-det.

$\mathrm{L} = \big(x(a,b)\big)^* \cdot \big(xaxa, xbxb\big)$

deterministic

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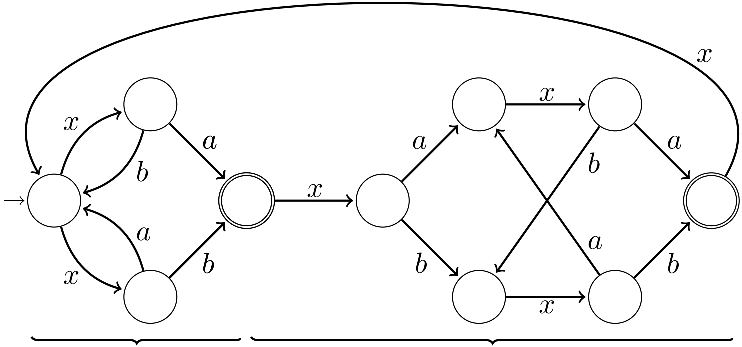

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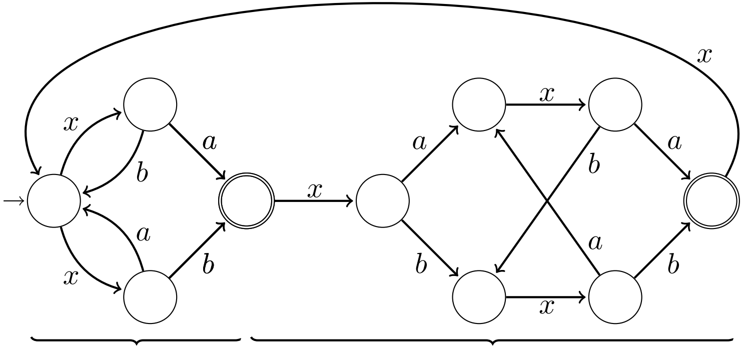

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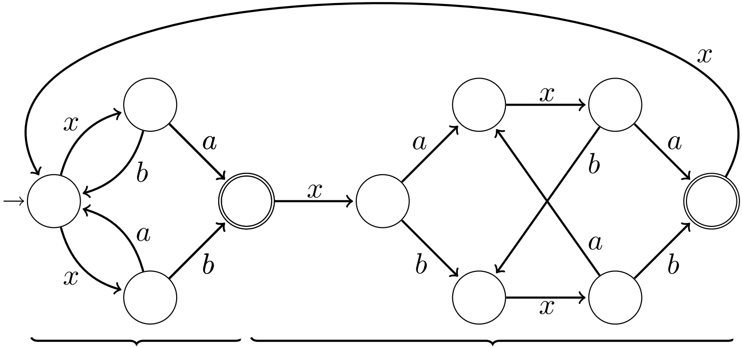

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[two memory states]

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i.e. $|\mathcal{A}_{\mathbf{det}}| \sim |M|$

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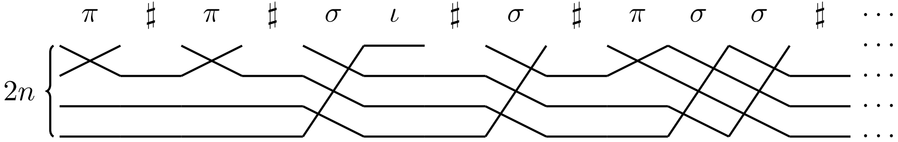

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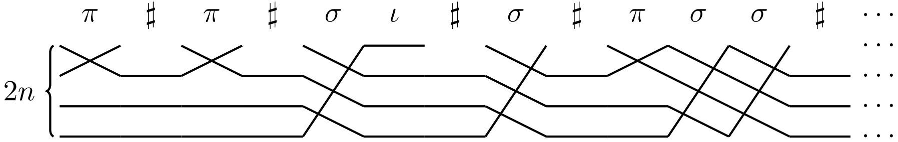

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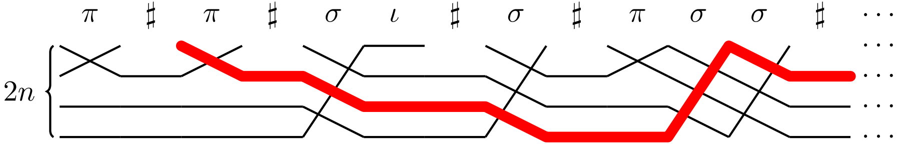

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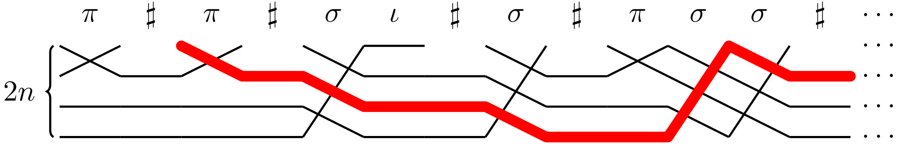

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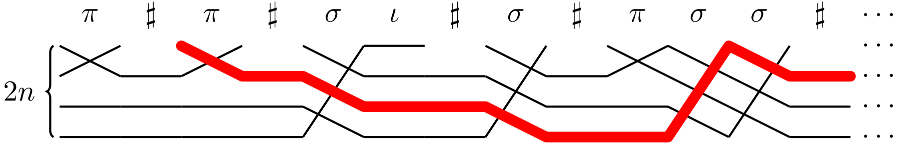

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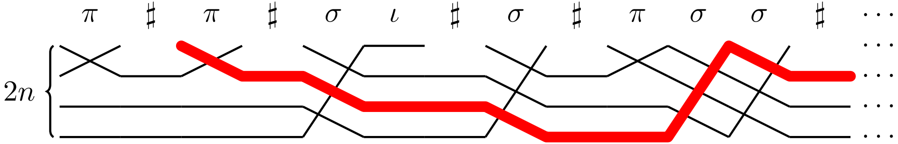

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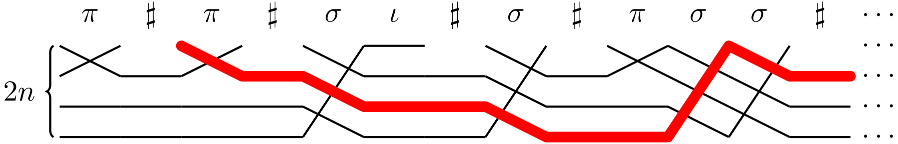

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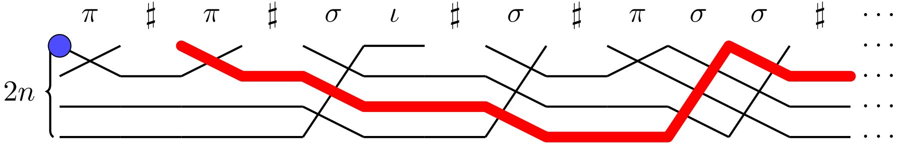

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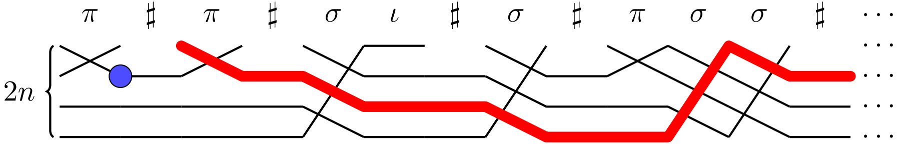

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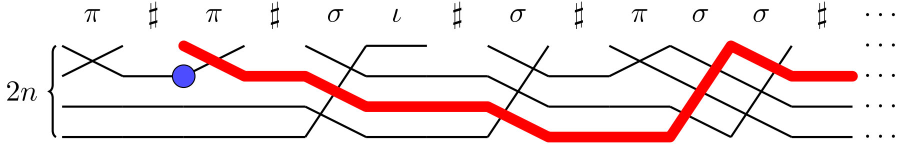

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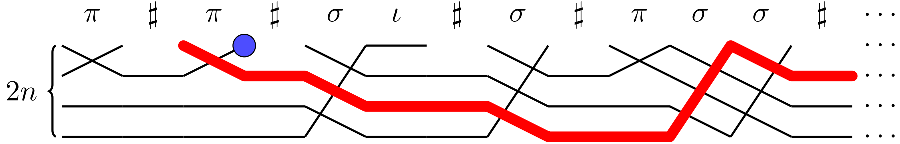

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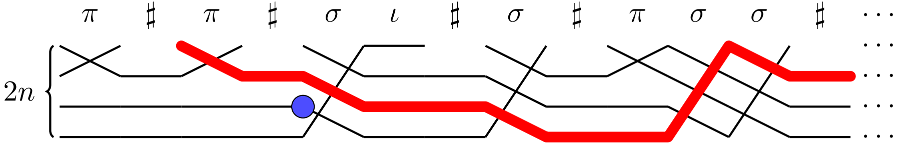

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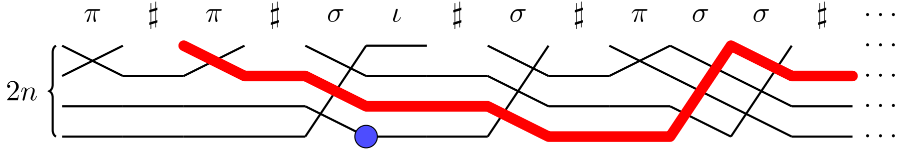

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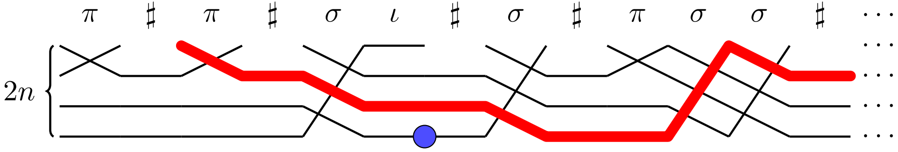

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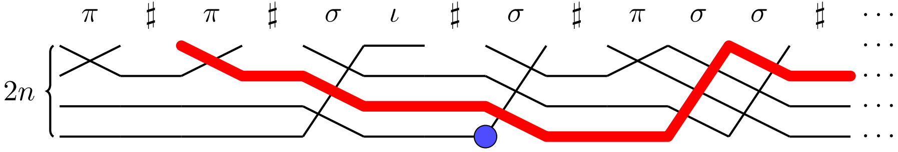

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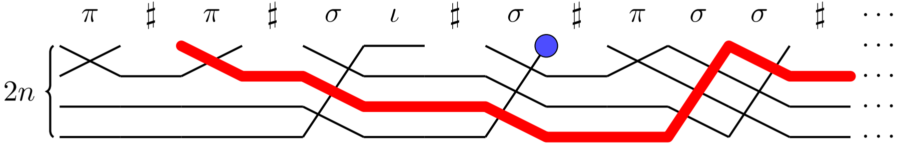

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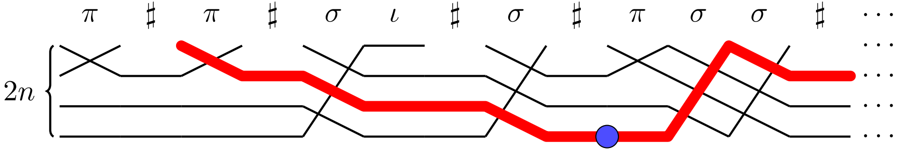

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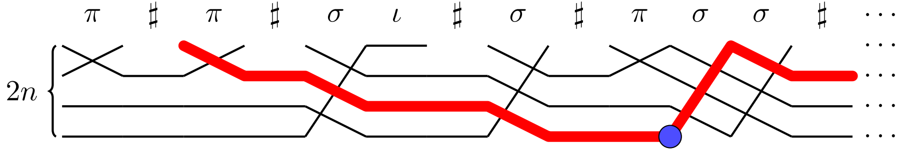

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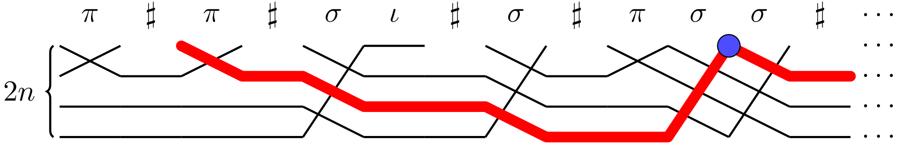

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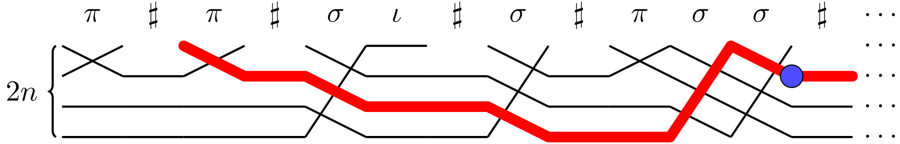

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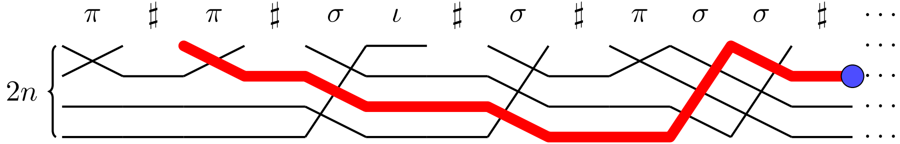

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Assume M with $|M| < 2^n/2n$

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Assume M with $|M| < 2^n/2n$ $\mathcal{A}_{\mathbf{GFG}} \times M$ — det. aut. for L

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Partial runs of $\mathcal{A}_{\mathbf{GFG}} \times M$

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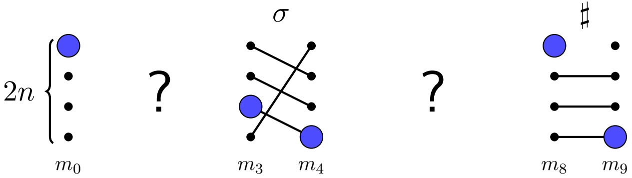

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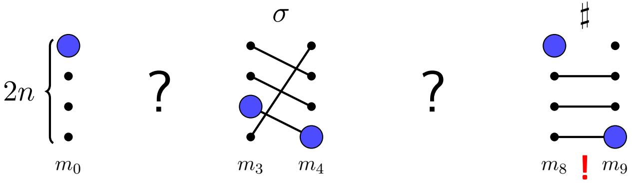

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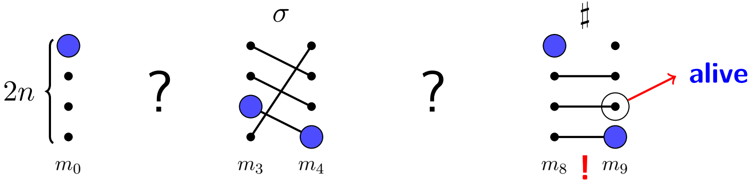

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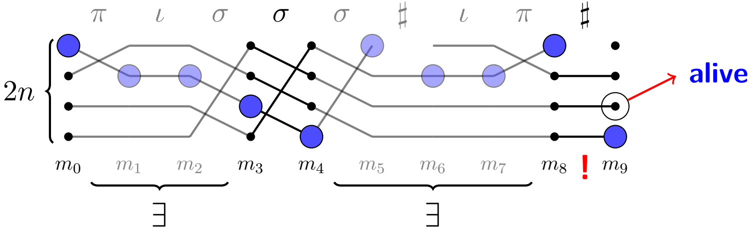
$\left[|\mathcal{A}_{\mathbf{GFG}} \times M| < 2^n\right]$

Assume M with $|M| < 2^n/2n$ $\qquad$ $\mathcal{A}_{\mathbf{GFG}} \times M$ — det. aut. for L

Partial runs of $\mathcal{A}_{\mathbf{GFG}} \times M$



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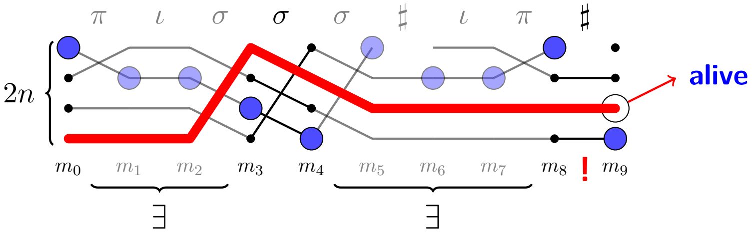

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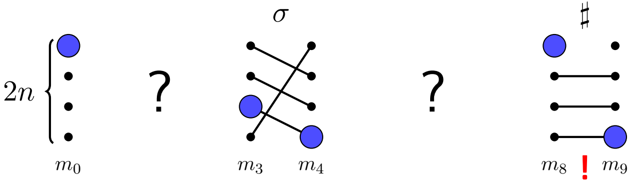

Aim: construct rejecting partial runs

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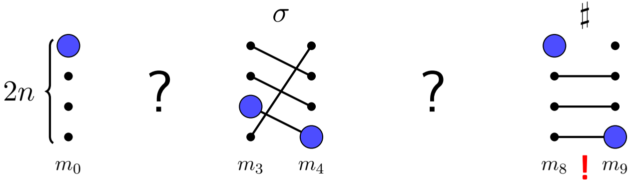

Aim: construct rejecting partial runs with $>n$ alive values

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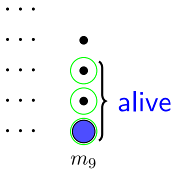

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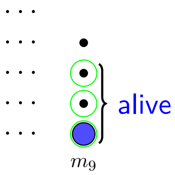

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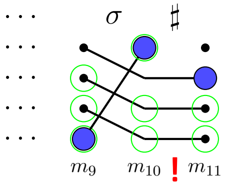

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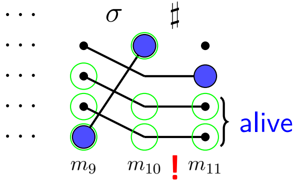

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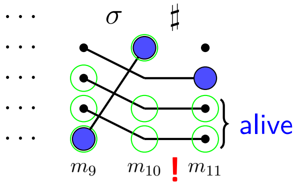

Aim: construct rejecting partial runs with $>n$ alive values

1. force a rejecting transition (using $\sharp$)
2. increase the number of alive values $n \mapsto n{+}1$

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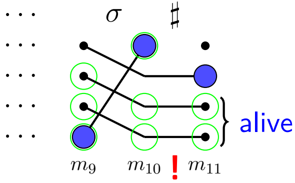

Aim: construct rejecting partial runs with $>n$ alive values

1. force a rejecting transition (using $\sharp$)

2. increase the number of alive values $n \mapsto n+1$

- consider 2^n permutations distinct on alive values

Exponential blow-up

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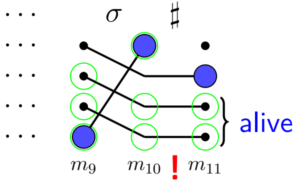

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1. force a rejecting transition (using ♯)

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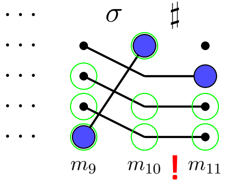

π/π'

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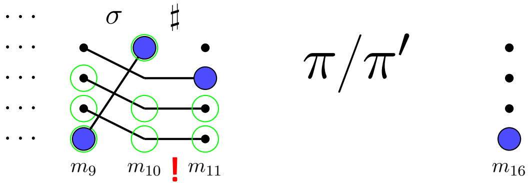

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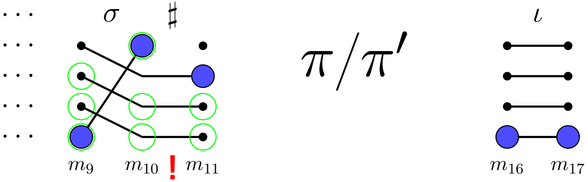

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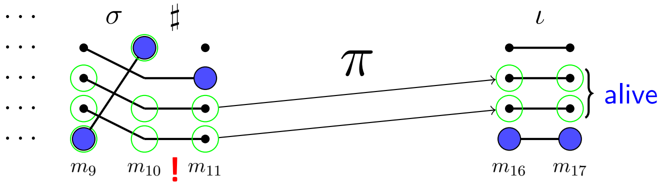

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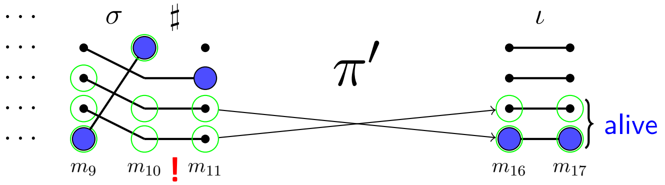

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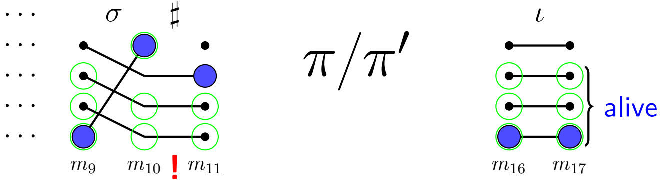

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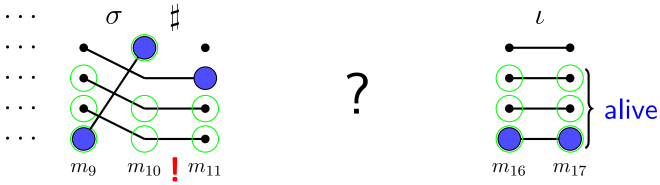

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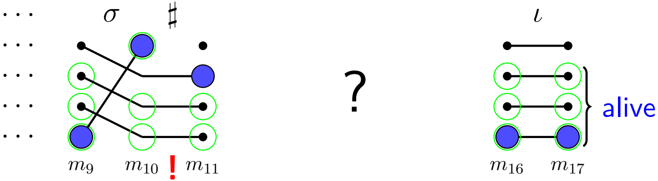

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$\rightsquigarrow w \in L$ s.t. $\mathcal{A}_{\mathbf{GFG}} \times M$ rejects w

Büchi case

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Theorem (Kuperberg, S. ['14])

For Büchi automata $|\mathcal{A}_{\mathbf{det}}| \leqslant |\mathcal{A}_{\mathbf{GFG}}|^2$

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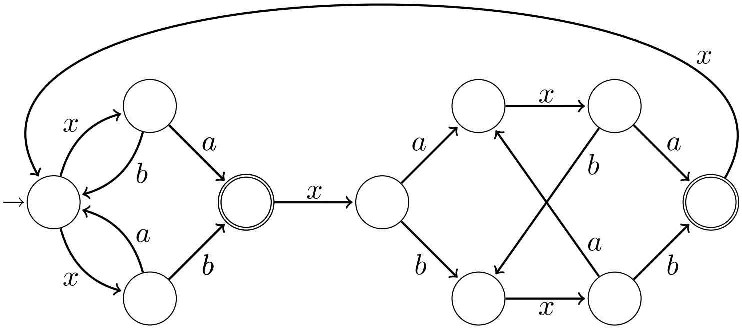

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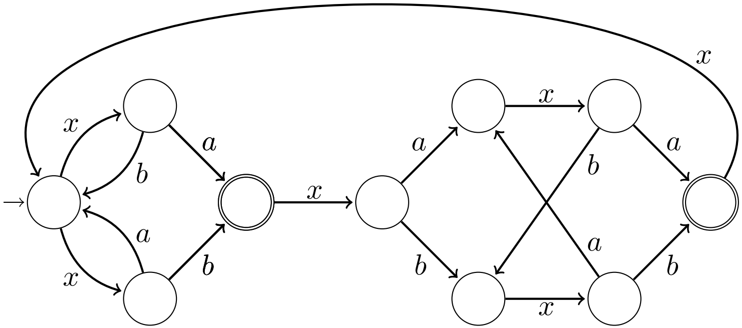

Where the strategy comes from?

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Problem

Input: an automaton $\mathcal{A}_{\mathbf{non-det}}$
Output: is $\mathcal{A}_{\mathbf{non-det}}$ GFG?

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For parity automata at least as hard as solving parity games

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Proof

Joker game. . .

Joker game

Joker game

Arena: $\mathcal{A}_{\mathbf{non-det}} \times \mathcal{A}_{\mathbf{non-det}}$

Joker game

Arena: $\mathcal{A}_{\mathbf{non-det}} \times \mathcal{A}_{\mathbf{non-det}}$

q_i

$\bar{q}_i$

Joker game

Arena: $\mathcal{A}_{\mathbf{non-det}} \times \mathcal{A}_{\mathbf{non-det}}$

$\forall$: a_i

q_i

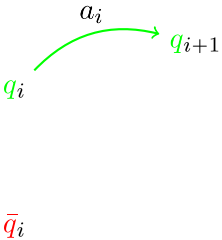
$\bar{q}_i$

Joker game

Arena: $\mathcal{A}_{\mathbf{non-det}} \times \mathcal{A}_{\mathbf{non-det}}$

$\forall$: a_i

$\exists$: $q_i \xrightarrow{a_i} q_{i+1}$



Joker game

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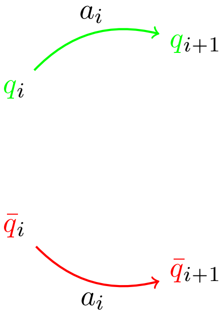

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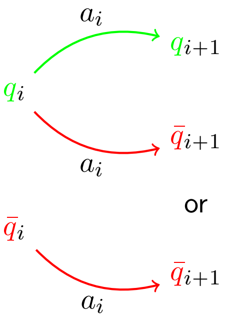

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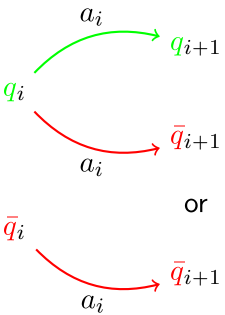

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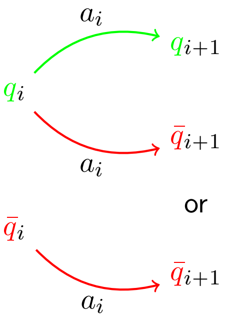

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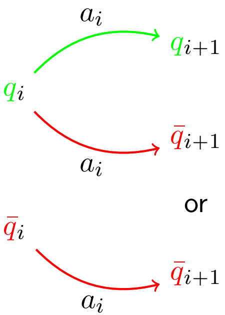

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On the way:

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...