# Determinisation of history-deterministic automata

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## History-deterministic automata

- non-deterministic
- parity $\omega$-word automata
- there exists $\sigma\colon \mathbf{A}^* \to \mathbf{Q}$ satisfying:

  for every word $w \in \mathrm{L}(\mathcal{A}) \subseteq A^\omega$

  the sequence of states $\sigma(w) \in Q^\omega$

  $(\sigma(w)(i) = \sigma(w\restriction_i))$

  is an accepting run of $\mathcal{A}$ on $w$

## Open problem

**What is the state blow-up when determinising a history-deterministic automaton?**

**Solution:** exponential for co-Büchi automata; polynomial for Büchi automata

## Applications

- synthesis (Büchi, Landweber '69)
- branching time verification (Emerson, Sistla '84)
- derived languages (Niwiński, Walukiewicz '98)
- symbolic representation (Henzinger, Piterman '06)
- quantitative models (Colcombet, Löding '10)
- streaming functions (Bojańczyk, K. '14)
- ...

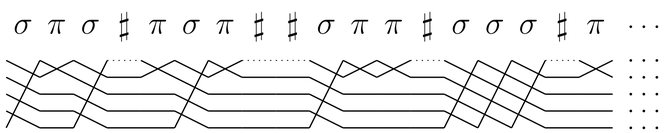
## Co-Büchi automata

**Theorem 1.** *There are* **H-D** *automata* $\mathcal{A}_n$*:*

- $|\mathcal{A}_n| = \Theta(n)$ *and* $\mathcal{A}_n$ *is co-Büchi*
- *if* $\mathcal{B}$ *is deterministic and* $\mathrm{L}(\mathcal{B}) = \mathrm{L}(\mathcal{A}_n)$ *then* $|\mathcal{B}| \geq 2^n$

History-deterministic automata are **succinct**!

*Proof scheme:* Language of permutations:




+ compactness for pumping the limitary behaviour.

## Is $\mathcal{A}$ history-deterministic?

New complexity bounds:

- for co-Büchi: in **P**
- for Büchi: in **NP**
- for parity: at least as hard as parity games

  (**NP** ∩ co-**NP**)

Technical details at **arXiv.org** soon!

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## Büchi automata

**Theorem 2.** *Every* **H-D** *Büchi automaton* $\mathcal{A}$ *admits* $\mathcal{B}$*:*

- $\mathcal{B}$ *is deterministic and Büchi*
- $\mathrm{L}(\mathcal{A}) = \mathrm{L}(\mathcal{B})$ *and* $|\mathcal{B}| \leq |\mathcal{A}|^2$

Polynomial determinisation procedure!

*Proof scheme:*

- residual languages and brutal powerset determinisation
- rank signatures of Walukiewicz
- iterative normalization of $\mathcal{A}$
- dependency graph over the automaton