On Determinisation of Good-for-Games Automata

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Success story: **synthesis** (Büchi, Landweber [1968]), ...

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$\forall$ — environment

$\exists$ — system

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$$\left.\begin{array}{rl} \forall : & I_n \\ \exists : & O_n \\ \textbf{det.} : & \text{transition} \end{array}\right\} \text{parity game}$$

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$\varphi \rightsquigarrow \mathcal{A}_{\textbf{det.}}$ } blow-up

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advice — σ accepts whenever possible

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$\exists$: transition $\Big\}$ using σ

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(Colcombet, Löding [2010])

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— Good-for-Trees automata

(Boker, K., Kupferman, S. [2013])

The GFG example (Boker [2013])

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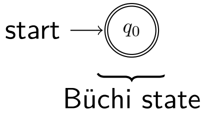

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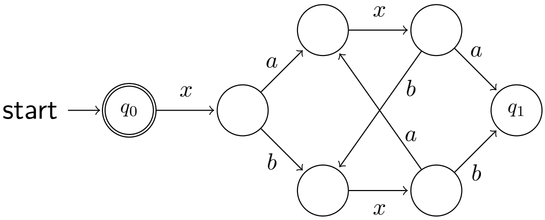

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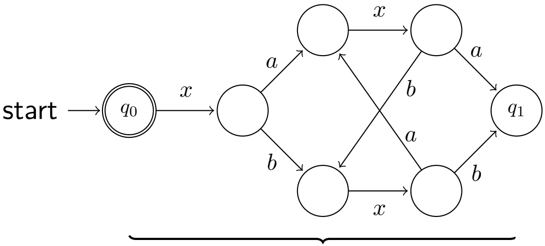

words $\left(x\ \{a, b\}\right)^*$
containing $xaxa$ or $xbxb$

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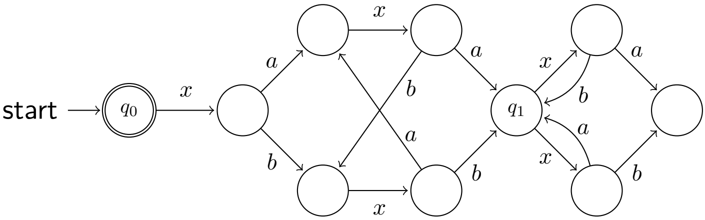

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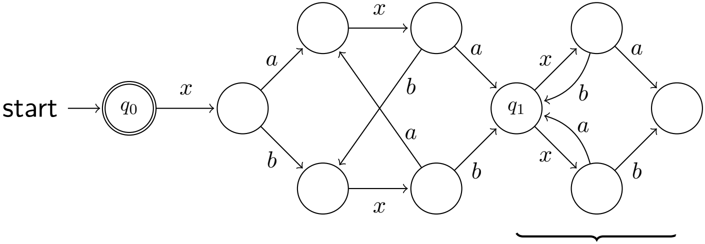

nondeterministically

word xa or xb

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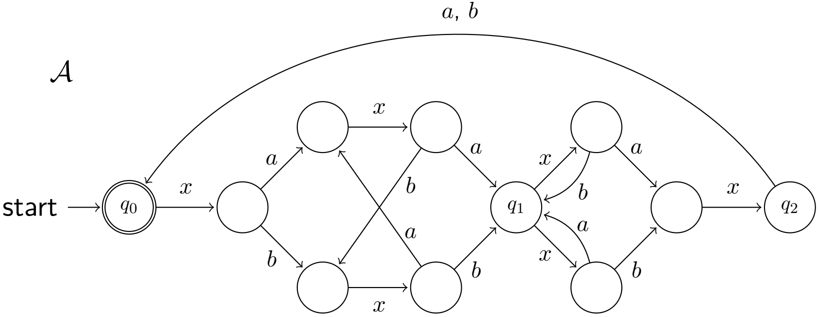

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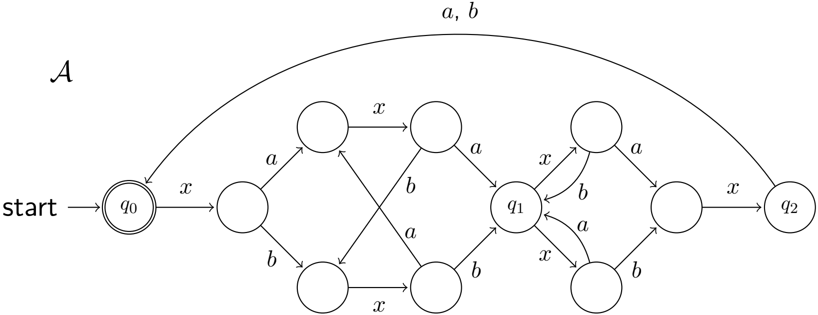

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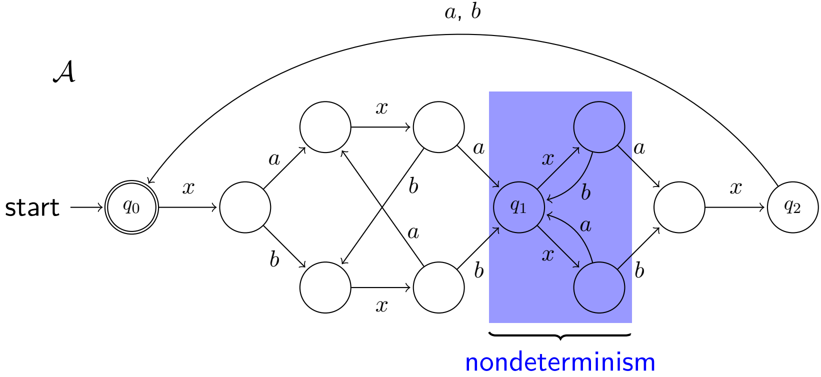

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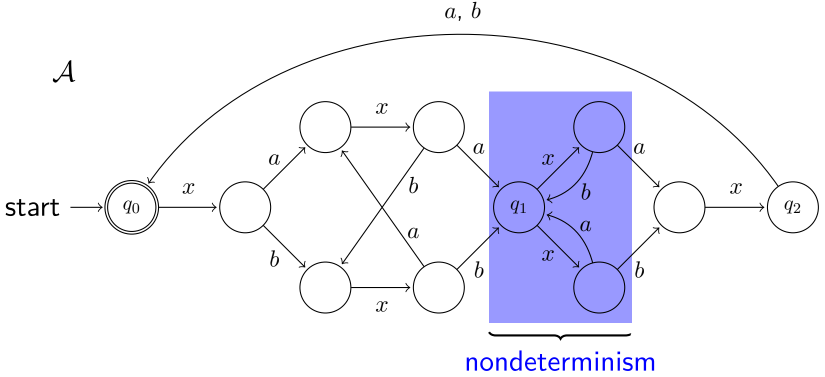

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σ: guess that last a/b will reappear

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[efficient determinisation procedure]

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[efficient determinisation procedure]	[GFG automata are succinct]

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co-Büchi automata:

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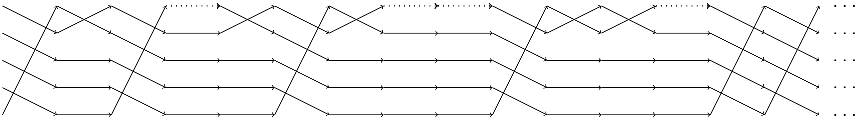

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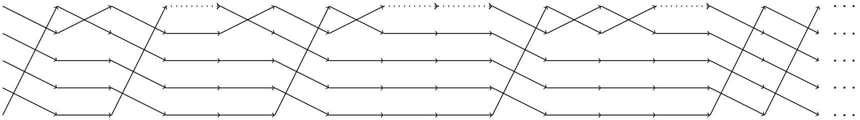

— compactness argument for *limitary pumping*

Summary

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[known algorithm: **NEXPtime** $\cap$ **co-NEXPtime**]

[**EXPtime** for bounded index]

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TODO:

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TODO:

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— what about deciding GFG for higher indices?