Monday's Nonstandard Seminar 31

15:00

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Title: **Some regularity results on gradients; p-Laplacian versus one-Laplacian**

Abstract: In this talk we would like to discuss two local reglarity results on weak solutions to a singular elliptic equation given by

$$-\Delta_1 u - \Delta_p u = f(x) \in L^q(\Omega) \quad \text{for } x \in \Omega \subset \mathbb{R}^n$$

for some fixed $1 < p < \infty$, $n < q \leq \infty$. Our interest is the following question; "When it comes to regularity of weak solutions, does the p-Laplace operator Δ_p overcome the one-Laplace operator Δ_1?" This problem seems difficult to handle, due to strong diffusivity of the one-Laplacian over a facet, which contains the degenerate points of solutions.

On Lipschitz regularity (i.e., boundedness of gradients), a recent work [1] reveals that

$$f \in L^q \, (n < q \leq \infty) \Rightarrow \nabla u \in L^\infty_{\text{loc}}.$$

This result is obtained by avoiding analysis over facets precisely. More regularity results of ∇u (e.g., $\nabla u \in C^\alpha_{\text{loc}}$ for some $0 < \alpha < 1$) have not been studied yet, since we will probably face to make analysis over facets. For convex weak solutions, we have established a C^1-regularity result (i.e., $\nabla u \in C$) by appealing to convex analysis. We would like to demonstrate our strategies for these two results.

This talk is based on a joint work with Yosikazu Giga (University of Tokyo).

References

[1] S. Tsubouchi, "Local Lipschitz bounds for solutions to certain singular elliptic equations involving the one-Laplacian", Calc. Var. Partial Differ. Equ. **60**: 33 (2021), 35 pages.