From trees to rules

- Simple way: one rule for each leaf
- C4.5rules: greedily prune conditions from each rule if this reduces its estimated error
  - Can produce duplicate rules
  - Check for this at the end
- Then
  - look at each class in turn
  - consider the rules for that class
  - find a "good" subset (guided by MDL)
- Then rank the subsets to avoid conflicts
- Finally, remove rules (greedily) if this decreases error on the training data

C4.5rules: choices and options

- C4.5rules slow for large and noisy datasets
- Commercial version C5.0rules uses a different technique
  - Much faster and a bit more accurate
- C4.5 has two parameters
  - Confidence value (default 25%):
    lower values incur heavier pruning
  - Minimum number of instances in the two most popular branches (default 2)

*Classification rules

- Common procedure: *separate-and-conquer*
- Differences:
  - Search method (e.g. greedy, beam search, ...)
  - Test selection criteria (e.g. accuracy, ...)
  - Pruning method (e.g. MDL, hold-out set, ...)
  - Stopping criterion (e.g. minimum accuracy)
  - Post-processing step
- Also: Decision list

  vs. one rule set for each class

*Test selection criteria

- Basic covering algorithm:
  - keep adding conditions to a rule to improve its accuracy
  - Add the condition that improves accuracy the most
- Measure 1: p/t
  - t total instances covered by rule
    p number of these that are positive
  - Produce rules that don't cover *negative* instances, as quickly as possible
  - May produce rules with very small coverage —special cases or noise?
- Measure 2: Information gain $p\,(\log(p/t) - \log(P/T))$
  - P and T the positive and total numbers before the new condition was added
  - Information gain emphasizes positive rather than negative instances
- These interact with the pruning mechanism used

*Missing values, numeric attributes

- Common treatment of missing values: *for any test, they fail*
  - Algorithm must either
    - use other tests to separate out positive instances
    - leave them uncovered until later in the process
- In some cases it's better to treat "missing" as a separate value
- Numeric attributes are treated just like they are in decision trees

*Pruning rules

- Two main strategies:
  - *Incremental* pruning
  - *Global* pruning
- Other difference: pruning criterion
  - Error on hold-out set (*reduced-error pruning*)
  - Statistical significance
  - MDL principle
- Also: post-pruning vs. pre-pruning

Rule based classifiers

- Each *classification rule* is of form

$$r : (Condition) \rightarrow y$$

  - LHS of the rule (*Condition*), called *rule antecedent* or *pre-condition*, is a conjunction of attribute tests
  - RHS, also called the *rule consequent,* is the class label
- *Rule set:*

$$R = \{r_1, r_2, \ldots, r_n\}$$

Classifying Instances with Rules

- A rule r ***covers*** an instance x if the attributes of the instance satisfy the condition of the rule
- Example
  - Rule:

    r : (Age < 35) $\wedge$ (Status = Married) $\rightarrow$ Cheat=No
  - Instances:

    x_1 : (Age=29, Status=Married, Refund=No)

    x_2 : (Age=28, Status=Single, Refund=Yes)

    x_3 : (Age=38, Status=Divorced, Refund=No)
  - Only x_1 is covered by the rule r

Classifying Instances with Rules

- **Rules may not be mutually exclusive**
  - More than one rule may cover the same instance
- **Strategies:**
  - Strict enforcement of mutual exclusiveness
    - Avoid generating rules that have overlapping coverage with previously selected rules
  - Ordered rules
    - Rules are rank ordered according to their priority
  - Voting
    - Allow an instance to trigger multiple rules, and consider the consequent of each triggered rule as a vote for that particular class

- **Rules may not be exhaustive**
- **Strategy:**
  - A *default rule*

    $$r_d : () \rightarrow y_d$$

    can be added
  - The default rule has an empty antecedent and is applicable when all other rules have failed
  - y_d is known as *default class* and is often assigned to the majority class

Advantages of Rule Based Classifiers

- As highly expressive as decision trees
- Easy to interpret
- Easy to generate
- Can classify new instances rapidly
- Performance comparable to decision trees

Definition

- Coverage of a rule:
  - Number (or fraction) of instances that satisfy the antecedent of a rule
- Accuracy of a rule:
  - Fraction of instances that satisfy both the antecedent and consequent of a rule
- Length:
  - Number of descriptors

Example

(Marital Status=Married) → No

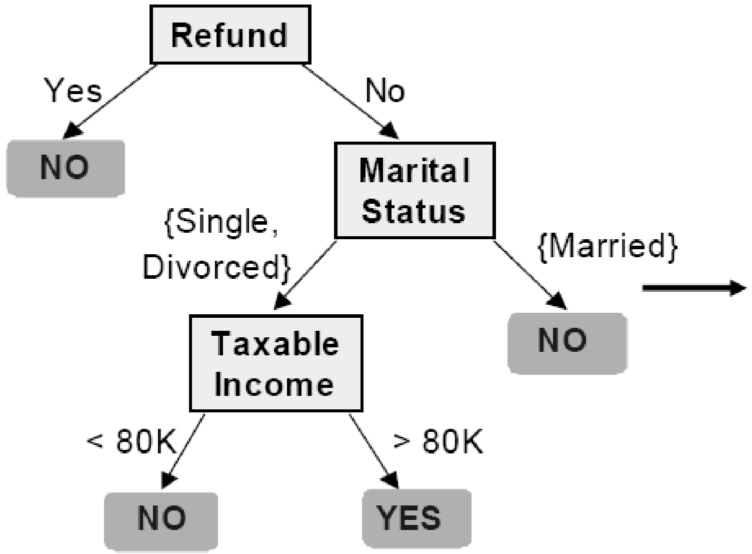
- Coverage = 40%,
- Accuracy = 100%
- Length = 1

ID	Refund	Marital Status	Income	Cheat
1	Yes	Single	125K	No
2	No	**Married**	100K	No
3	No	Single	70K	No
4	Yes	**Married**	120K	No
5	No	Divorced	95K	Yes
6	No	**Married**	60K	No
7	Yes	Divorced	220K	No
8	No	Single	85K	Yes
9	No	**Married**	75K	No
10	**No**	**Single**	**90K**	Yes

Construction of a Rule Based Classifier from data

- Generate an initial set of rules
  - Direct Method:
    - Extract rules directly from data
    - Examples: RIPPER, CN2, Holte's 1R, Boolean reasoning
  - Indirect Method:
    - Extract rules from other classification methods (e.g. decision trees)
    - Example: C4.5 rules
- Rules are pruned and simplified
- Rules can be order to obtain a rule set *R*
- Rule set *R* can be further optimized

Indirect Method:
Conversion from Decision Trees



Classification Rules

(Refund=Yes) ==> No

(Refund=No, Marital Status={Single,Divorced}, Taxable Income<80K) ==> No

(Refund=No, Marital Status={Single,Divorced}, Taxable Income>80K) ==> Yes

(Refund=No, Marital Status={Married}) ==> No

- Rules are mutually exclusive and exhaustive
- Rule set contains as much information as the tree
- Rules can be simplified

(Refund=No) $\wedge$ (Status=Married) $\rightarrow$ No ==> (Status=Married) $\rightarrow$ No

Indirect Method: C4.5 rules

- Creating an initial set of rules
  - Extract rules from an un-pruned decision tree
  - For each rule, $r : A \rightarrow y$
    - Consider alternative rules r': $A' \rightarrow y$, where A' is obtained by removing one of the conjuncts in A
    - Replace r by r' if it has a lower pessimistic error
    - Repeat until we can no longer improve the generalization error
- Ordering the rules
  - Instead of ordering the rules, order subsets of rules
    - Each subset is a collection of rules with the same consequent (class)
    - The subsets are then ordered in the increasing order of

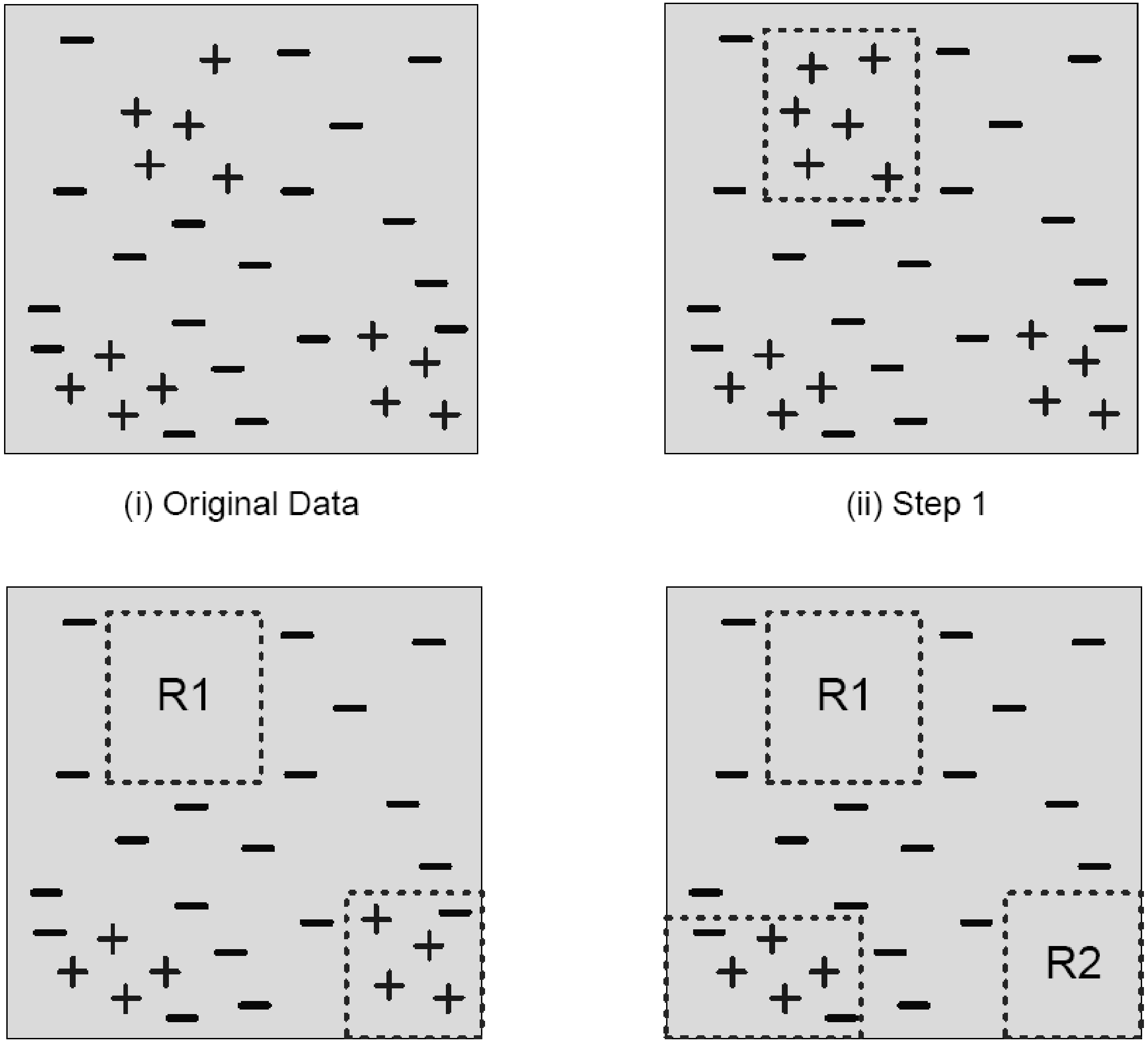
      Description length = L(exceptions) + $g \cdot L$(model)
    - where g is a parameter that takes in to account the presence of redundant attributes in a rule set. Default value is 0.5

Direct method: Sequential covering

(E: training examples, A: set of attributes)

1. Let $R = \{\ \}$ be the initial rule set
2. While stopping criteria is not met
   1. r := Learn-One-Rule (E, A);
   2. Remove instances from E that are covered by r ;
   3. Add r to rule set: $R = R + \{r\}$;

- Ex. Stopping criteria = „E is empty”



(i) Original Data
(ii) Step 1
(iii) Step 2
(iv) Step 3

Learn one rule (1R)

- The objective of this function is to extract the best rule that covers the current set of training instances
  - What is the strategy used for rule growing
  - What is the evaluation criteria used for rule growing
  - What is the stopping criteria for rule growing
  - What is the pruning criteria for generalizing the rule

Learn One Rule: Rule Growing Strategy

- General-to-specific approach
  - It is initially assumed that the best rule is the empty rule, $r: \{\ \} \rightarrow y$, where y is the majority class of the instances
  - Iteratively add new conjuncts to the LHS of the rule until the stopping criterion is met
- Specific-to-general approach
  - A positive instance is chosen as the initial seed for a rule
  - The function keeps refining this rule by generalizing the conjuncts until the stopping criterion is met

Rule Evaluation and Stopping Criteria

- Evaluate rules using rule evaluation metric
  - Accuracy
  - Coverage
  - Entropy
  - Laplace
  - M-estimate
- A typical condition for terminating the rule growing process is to compare the evaluation metric of the previous candidate rule to the newly grown rule

Learn 1R

- Rule Pruning
  - – Each extracted rule can be pruned to improve their ability to generalize beyond the training instances
  - Pruning can be done by removing one of the conjuncts of the rule and then testing it against a validation set
- Instance Elimination
  - – Instance elimination prevents the same rule from being generated again
  - Positive instances must be removed after each rule is extracted
  - Some rule based classifiers keep negative instances, while some remove them prior to generating next rule

RIPPER

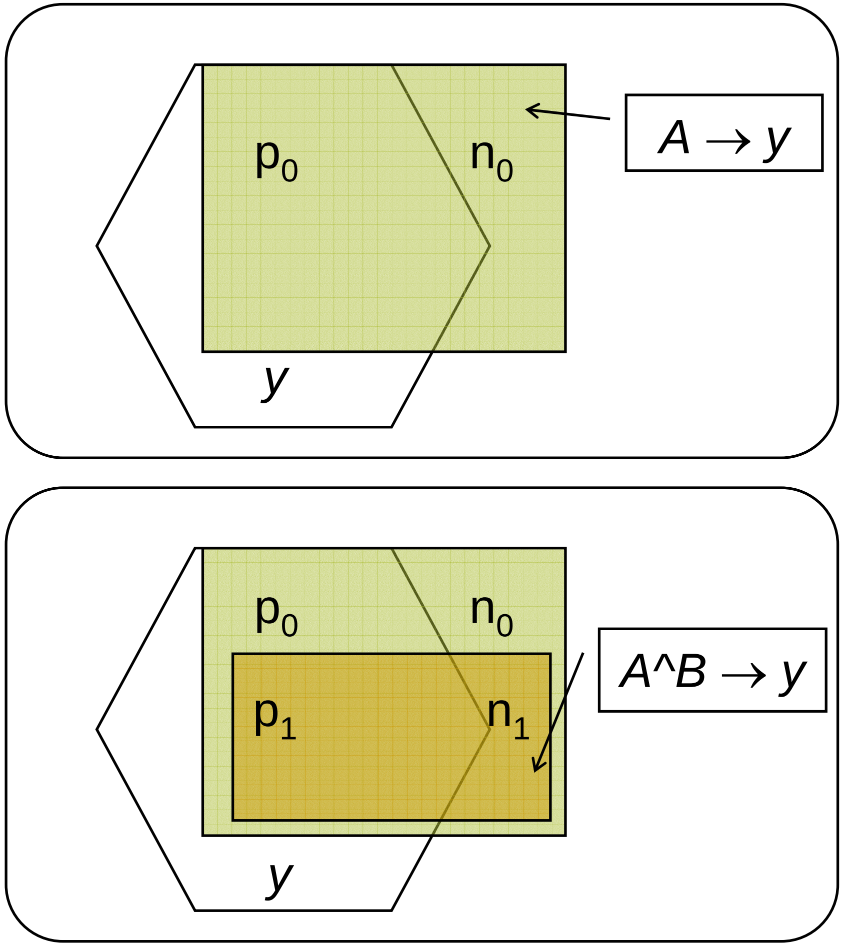
- For 2-class problem, choose one of the classes as positive class, and the other as negative class
  - Learn rules for positive class
  - Negative class will be default class
- For multi-class problem
  - Order the classes according to increasing class prevalence (fraction of instances that belong to a particular class)
  - Learn the rule set for smallest class first, treat the rest as negative class
  - Repeat with next smallest class as positive class

Foil's Information Gain

- Compares the performance of a rule before and after adding a new conjunct
- Foil's information gain is defined as

  $= t \cdot [\, log_2(p_1/(p_1 + n_1)) - log_2(p_0/(p_0 + n_0))\,]$

  where t is the number of positive instances covered by both r and r'



Direct Method: RIPPER

- Growing a rule:
  - Start from empty rule
  - Add conjuncts as long as they improve Foil's information gain
  - Stop when rule no longer covers negative examples
  - Prune the rule immediately using incremental reduced error pruning
  - Measure for pruning: $v = (p - n) / (p + n)$
    - p: number of positive examples covered by the rule in the validation set
    - n: number of negative examples covered by the rule in the validation set
  - Pruning method: delete any final sequence of conditions that maximizes v

RIPPER: Building a Rule Set

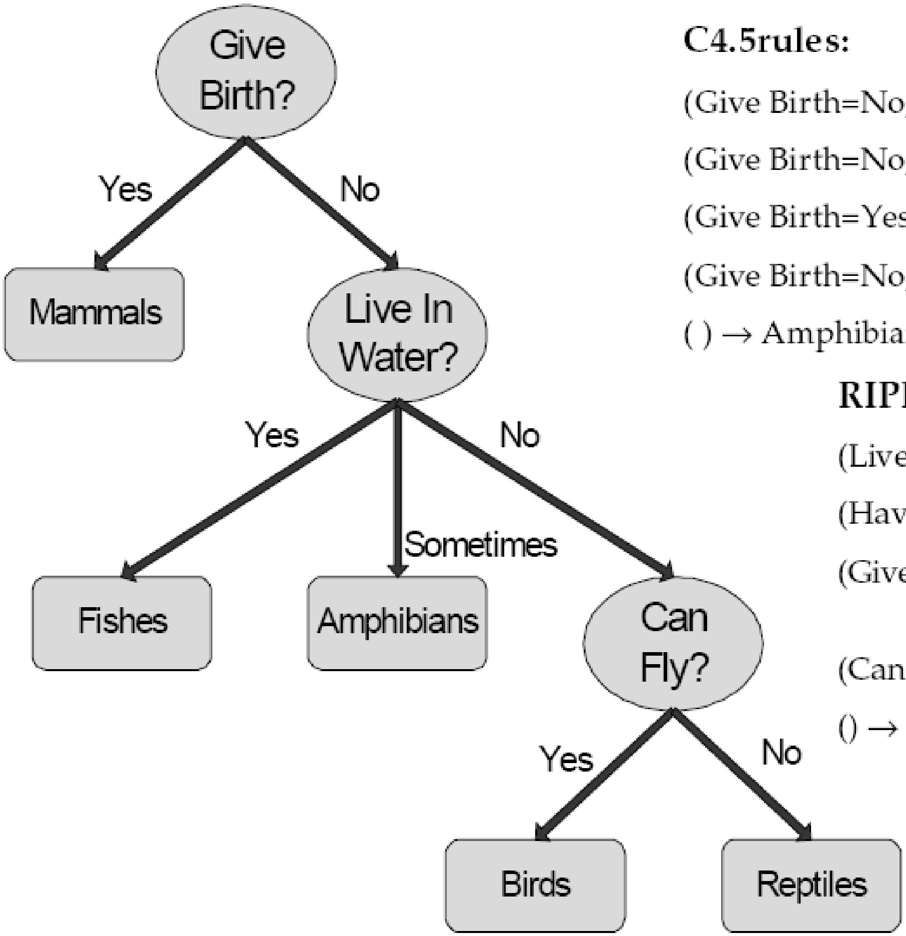
- Use sequential covering algorithm
  - Finds the best rule that covers the current set of positive examples
  - Eliminate both positive and negative examples covered by the rule
- Each time a rule is added to the rule set, compute the description length
  - Stop adding new rules when the new description length is *d* bits longer than the smallest description length obtained so far. *d* is often chosen as 64 bits

RIPPER: Optimize the rule set:

- For each rule *r* in the rule set ***R***
  - Consider 2 alternative rules:
    - Replacement rule (*r**): grow new rule from scratch
    - Revised rule (*r'*): add conjuncts to extend the rule *r*
  - Compare the rule set for *r* against the rule set for *r** and *r'*
  - Choose rule set that minimizes MDL principle
- Repeat rule generation and rule optimization for the remaining positive examples

Name	Give Birth	Lay Eggs	Can Fly	Live in Water	Have Legs	Class
human	yes	no	no	no	yes	mammals
python	no	yes	no	no	no	reptiles
salmon	no	yes	no	yes	no	fishes
whale	yes	no	no	yes	no	mammals
frog	no	yes	no	sometimes	yes	amphibians
komodo	no	yes	no	no	yes	reptiles
bat	yes	no	yes	no	yes	mammals
pigeon	no	yes	yes	no	yes	birds
cat	yes	no	no	no	yes	mammals
leopard shark	yes	no	no	yes	no	fishes
turtle	no	yes	no	sometimes	yes	reptiles
penguin	no	yes	no	sometimes	yes	birds
porcupine	yes	no	no	no	yes	mammals
eel	no	yes	no	yes	no	fishes
salamander	no	yes	no	sometimes	yes	amphibians
gila monster	no	yes	no	no	yes	reptiles
platypus	no	yes	no	no	yes	mammals
owl	no	yes	yes	no	yes	birds
dolphin	yes	no	no	yes	no	mammals
eagle	no	yes	yes	no	yes	birds

C 4.5 rules vs. RIPPER



C4.5rules:

(Give Birth=No, Can Fly=Yes) → Birds

(Give Birth=No, Live in Water=Yes) → Fishes

(Give Birth=Yes) → Mammals

(Give Birth=No, Can Fly=No, Live in Water=No) → Reptiles

( ) → Amphibians

RIPPER:

(Live in Water=Yes) → Fishes

(Have Legs=No) → Reptiles

(Give Birth=No, Can Fly=No, Live In Water=No) → Reptiles

(Can Fly=Yes,Give Birth=No) → Birds

() → Mammals