Topology from differentiable viewpoint.
Exercises 6

SJ

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Zad. 1 (BJ 9.6.1)**.** Let M be a connected manifold with $\dim(M) > 1$. Let $\{x_1, \dots x_k\}$ be distinct points of M, and let $\{y_1, \dots y_k\}$ also be distinct points of M. Show that there exists a diffeomorphism (with compact support) $\phi\colon M \to M$ such that $\phi(x_i) = y_i$ for $i = 1, \dots k$.

Zad. 2 (BJ 9.6.2)**.** Let $M \subset N$ be a submanifold of the connected manifold N such that $\dim N - \dim M \geqslant 2$, and $p, q \in N \setminus M$. Show that there exists a diffeomorphism $h\colon N \to N =$ such that $h(p) = q$ and $h|N = id$.

Zad. 3 (BJ 10.11.1)**.** Let M be an oriented connected manifold, $p, q \in M$ and $\phi\colon TM_p \to TM_q$ an orientation preserving isomorphism. Show that there exists a diffeomorphism $f\colon M \to M$ such that $f(p) = q$ and $Df_p = \phi$.

Zad. 4. Prove that a smooth manifold is orientable if and only if the normal bundle to any immersion (embedding) $f\colon S^1 \to M$ is trivial.

Zad. 5. *Let*$p\colon E \to M$ be a smooth vector bundle. Prove that the zero section $s_0\colon M \to E$ is an embedding and the normal bundle $\nu(M, E)$ is isomorphic to $p\colon E \to M$.

Zad. 6 (BJ 11.7.1,2)**.** *Let*$p\colon E \to M$ be a smooth vector bundle and $Dp\colon TE \to TM$ be its derivative. We identify $M = s_0(M)$. Prove that $(Dp)|M\colon (TE)|M \to TM$ is an epimorphism of vector bundles and $\ker(Dp)|M$ is isomorphic to the bundle $p\colon E \to M$.

Zad. 7 (BJ 11.7.2)**.** Let $p\colon E \to M$ be a smooth vector bundle. Prove that $TE \simeq p^*(E \oplus TM)$.

Zad. 8. Prove that any vector bundle over the real line $\mathbb{R}$ (or more general $\mathbb{R}^n$) is trivial.

Zad. 9. Prove that for any manifold M the normal bundle to the diagonal $\Delta_M \subset M \times M$ is isomorphic to the tangent bundle $TM \to M$.

Zad. 10. Prove that if M is a connected non-compact manifold then there exists an embedding $j\colon \mathbb{R} \to M$ such that $j(\mathbb{R})$ is a closed subset.