Topology from differentiable viewpoint.
Exercises 5

SJ

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Zad. 1. Let a (smooth) manifold M be a coproduct $M = \coprod_{i=1}^{\infty} M_i$ of manifolds M_i such that for every i there exists an (smooth) embedding $j_i \colon M_i \hookrightarrow \mathbb{R}^n$. Then there is an (smooth) emebdding $j \colon M \hookrightarrow \mathbb{R}^n$.

Zad. 2 (BJ 6.7.1)**.** Show that, if $M^n \subset \mathbb{R}^p$ is a smooth submanifold, then there exists a hyperplane in $\mathbb{R}^p$, which cuts M^n transversally.

Zad. 3. For an arbitrary smooth manifold M construct a perfect (i.e. inverse images of the compact sets are compact) smooth function $f \colon M \to \mathbb{R}$

Zad. 4 (BJ 7.13.11)**.** Show that there exists a closed embedding of the real line in every connected non-compact smooth manifold.

Zad. 5. Prove that any continuous map $f \colon M \to N$ between smooth manifolds is homotopic to a smooth map and if two smooth maps $f_0, f_1 \colon M \to N$ are homotopic then there exists a smooth homotopy between them.

Zad. 6. Let $(M, \partial M)$ be a smooth manifold with boundary. Prove that its "double" $M \cup_{\partial M} M$ admits a natural structure of a smooth manifold. Is the double of an orientable manifold an orientable manifold?

Zad. 7 (BJ 13.18.2)**.** Show that the double of a compact manifold with boundary certainly bounds (is a boundary of a compact manifold).

Zad. 8 (BJ 13.18.2)**.** Show that on each manifold with boundary $(M, \partial M)$ there exists a smooth function $f \colon M \to \mathbb{R}$ such that $\partial M = f^{-1}(0)$.

Zad. 9. Let $E \to M$ be a smooth vector bundle. Prove that the image of the zero section $s_0 \colon M \to E$, $s_0(x) = 0_x$ is a submanifold diffeomorphic to M and the normal bundle $\nu(M, E)$ is isomorphic to E.

Zad. 10 (BJ 12.14.6)**.** Let $Xn - k \subset M^n$ be a closed submanifold with a trivial normal bundle (if we choose a trivialization then such submanifold is called *framed*). Show that there is a smooth map to the sphere S^k: $f \colon M \to S^k$ such that $X = f^{-1}(p)$ where $p \in S^k$ is a regular value. [cf. J. Milnor *Topology from Differentiable Viewpoint.* Chapter 7.] *Hint:* Construct such map for zero-dimensional submanifolds.