

## Research Statement

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### Overview

My main research interests lie in theoretical computer science with a primary focus on randomness in computation. I prefer to study basic algorithmic problems, which are in the same time well-motivated by applications in practice. Among others I have been focusing on applications in physics. The examples include finding matchings in graphs, randomized approximation schemes for counting perfect matchings, transitive closure and shortest distances in graphs. I have been also studying the possibility of solving these problem in dynamic setup.

I am also pursuing research in solid state theory. I am working on modeling basic spintronics devices. Spintronics is concerned with building electronics that takes advantage of the spin of electron.

### Past and Current Research

#### Algebraic Graph Algorithms

The algebraic methods had turned out to be very useful in many graph applications, starting from transitive closure computations and ending on counting perfect matchings. The constructed algorithms use matrix operations such as multiplication or computing determinant as a basic building block. Through this the algorithms usually gain on clearness. Also in many cases the algebraic approach yields the asymptotically fastest solutions. The basic example is the transitive closure problem.

The main topic of my research is the application of the algebraic methods to a wider spectra of problems. In one of my main results I was able to show that also in dynamic setup the algebraic approach is very useful. I have developed the asymptotically fastest algorithms for dynamic transitive closure [1]. This result is based on the first algorithms for dynamic matrix problems (i.e., dynamic determinant, dynamic matrix inverse) and a novel reduction of the transitive closure problem to the matrix inverse. In the paper [2] I have extended the algorithms for the case of polynomials. This extension was needed in order to get first subquadratic dynamic algorithms for computing shortest distances in graphs. The further results concerning algorithms with lookahead are submitted for publication. The reduction from transitive closure to matrix inverse was developed especially for dynamic case, but astonishingly it can also be used in static case in order to solve single source shortest path problem in graphs with integer weights from the set  $\{-W, \dots, 0, \dots, W\}$  in  $\tilde{O}(Wn^\omega)$  time [3] ( $\omega < 2.376$  is the matrix multiplication exponent). For dense graphs with small edge weights, this result improves upon the previous known results.

Using algebraic methods we (with M. Mucha) were able to show faster algorithms for the perfect matching problem. We have resolved a long-standing open question of whether

Lovász's randomized technique of testing graphs for perfect matching in time  $O(n^\omega)$  can be extended to an algorithm that actually constructs a perfect matching [4]. This technique applies also to planar case and gives an  $O(n^{\frac{\omega}{2}})$  time algorithm for finding perfect matchings in planar graphs [5]. This year I was also able to show that the same work is enough to solve the matching problem in parallel [6] and that the algebraic approach can be used to solve weighted matchings in bipartite graphs (the assignment problem) in  $O(Wn^\omega)$  time [7], where  $W$  is maximum integer edge weight in the graph.

## Randomized Approximation Schemes

Most of the simulation problems arising in statistical physics can be stated as a counting problem, i.e., the monomer-dimer system can be restated as a problem of counting matchings in some graph. Most of the arising problems in this way are  $\#P$ -complete. In my research I was able to give some new ideas of counting matchings [8]. In [9] I have presented a new general construction of almost uniform samplers, which can be also used for counting.

I am also working on using the algebraic approach to counting perfect matchings in graphs. The idea is very simple and is hoped to lead to the first efficient algorithms for counting perfect matchings in general graphs.

## Future Research in the Near Term

Between others, I intend to explore the following specific questions which are extensions of my past and ongoing research:

- The weighted matching in bipartite graphs can be solved using matrix multiplication. I am working on extending this method to general graphs.
- Can the matrix multiplication be used to improve the complexity of the algorithms for the network flow problem?
- The algebraic algorithm for the shortest paths problem work in subquadratic time. However, there is still a gap between dynamic transitive closure and the distance problem. I am working on closing this gap.
- The algebraic approach leads to  $O(Wn^\omega)$  algorithm for the single source shortest paths problem with edge weights in the set  $\{-W, \dots, W\}$ . Can it be used for computing all pairs shortest paths?
- Can the algebraic approach to matchings be used to efficiently count perfect matchings in general graphs?

## Future Research in the Long Term

In the long term I intend to extend my knowledge on representations and certificates of the graph properties. Nowadays, the field of sparse distance representations is intensively

studied, the examples include, spanners and approximate distance oracles. These studies give deep insight into the hardness of the problems, and can often lead more efficient solutions. A number of very interesting questions arise, e.g., can fast matrix multiplication algorithms be viewed as a sparse representation, or can such techniques be used in case of network flows. The answer to the first question could give us an idea how to construct combinatorial algorithm for fast matrix multiplication.

My second field of research is spintronics, i.e., spin electronics. Two years ago I finished the master thesis in solid state theory and I am continuing my research on this topic. In summer 2006 I plan to finish my Ph.D. thesis in physics. Currently, I am working on computer simulations of basic spin devices. The spin devices I am concerned with include spin injectors, spin detectors and magnetic tunneling junctions. The research in this topic is hoped to lead to better and cheaper RAM memories, and it already found applications in hard disk read heads.

Spintronics is between other things concerned with actual building of quantum computers based on the spin of electron. As a computer scientist I think that quantum computers are very interesting model of computation worth further investigations and better understanding. During the next year I am going to broaden my knowledge in this field. As a physicist I doubt that one will be able to build a working quantum computer in near future. However, I do believe that we will be able to realize classical computer that will benefit from using quantum phenomena pretty soon. Whatever the future shall be, the interdisciplinary knowledge of the technology and the theory of quantum computers will be very handy.

## References

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