WHY THIS PROGRAM LOOPS?

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Abstract. An exercise

1. Question

Consider the following program and answer to the question: will the program stop?

program PF4;
(* This program is to be executed in the domain of nonnegative integers with multi-
precision, without any bounds. The program uses three functions f, g, h. Instead of writing
the bodies of f, g, h we give requirements i.e. a specification of the functions f, g, h

∀x∀y g(f(x, y)) = x,  ∀x∀y h(f(x, y)) = y,
∀x f(g(x), h(x)) = x
*)
unit f: function(x, y: integer): integer; ... end f;
unit g: function(x: integer): integer; ... end g;
unit h: function(x: integer): integer; ... end h;
var z, n, a, b, c, d, i, s1, s2, s3: integer;
begin
z := 1; n := 0;
while z ≠ 0 do
a := g(n);  b := g(h(n));  c := g(h(h(n)));  d := h(h(h(n)));
if (a > 2 & b > 0 & c > 0 & d > 0)
then
  i := 0;  s1 := s2 := s3 := 1;
  while i ≠ a do
    s1 := s1 * b;  s2 := s2 * c;  s3 := s3 * d;  i := i + 1;
  done;
  if s1 + s2 > s3 then z := s1 + s2 - s3 else z := s3 - s1 - s2 fi;
fi;
n := n + 1;
done
end

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