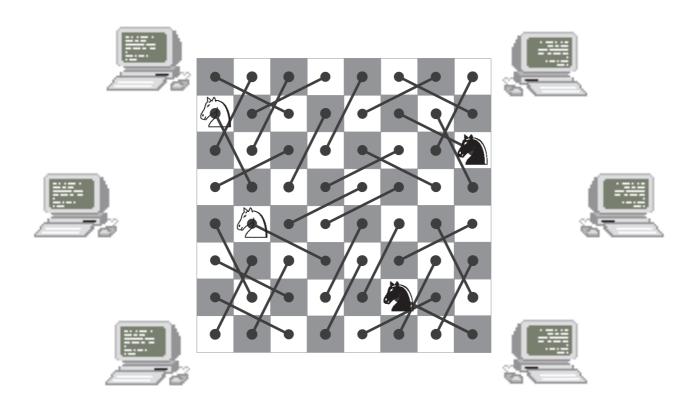
# Fast Parallel Algorithms for Graph Matching Problems

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