ON ORLICZ SPACES SATISFYING THE HOFFMANN-JØRGENSEN INEQUALITY

DOMINIK KUTEK

We say that an Orlicz function Ψ satisfies the Hoffmann-Jørgensen inequality (abbr. H-J inequality), if there exists a constant $C(\Psi) \in (0, \infty)$ such that

$$\Big\| \sum_{k=1}^{n} X_k \Big\|_{L_\Psi(\Omega,\mathcal{F},\mathbb{P},F)} \leq C_\Psi \Big(\Big\| \sum_{k=1}^{n} X_k \Big\|_{L_1(\Omega,\mathcal{F},\mathbb{P},F)} + \Big\| \max_{k\leq n} \|X_k\| \Big\|_{L_\Psi(\Omega,\mathcal{F},\mathbb{P},F)} \Big)$$

for any $n \in \mathbb{N}_+$ and any collection $\{X_k\}_{k\leq n}$ of zero mean and independent random variables taking values in any separable Banach Space $(F, \|\cdot\|)$. Inequality of this type firstly appeared in the work of Hoffmann-Jørgensen (1974) in the case of L_p norms, and was later investigated by Talagrand (1989) in the case of $\Psi(x) = \exp(|x|^a) - 1, a \in (0, 1]$ and sup-exponential Orlicz functions. Moreover, Ziegler (1997) generalised Hoffmann-Jørgensen result to all Orlicz functions satisfying Δ_2 condition. Recently, it has been proved by Chamakh-Gobet-Liu (2021) that the functions of type $\Psi(x) = \exp(\ln^b(1 + x)) - 1, b > 1$ also satisfy the H-J inequality.

The main part of the talk will be devoted to the characterisation of all Orlicz functions Ψ satisfying the forementioned H-J inequality, where the necessary and sufficient condition is obtained in terms of a simple inequality on Ψ. The proof follows very similar reasoning as in the work of Talagrand (1989). Later, we shall discuss some applications concerning concentration inequalities (in the spirit of Adamczak (2008), Klochkov-Zhivotovsky (2020) and Sambale (2020)) and, moreover, $L_\Psi(F)$ boundedness of sums of independent Banach valued random variables (in the spirit of Hoffmann-Jørgensen (1974)). Joint work with R. Adamczak.