K-theory of operator algebras - oral examination

E. Easy questions

1. Calculate $K_*(\mathbb{C})$.

2. Define the homotopy equivalence of C^*-algebras and show the homotopy invariance of the K-functors. Argue that $K_*(C(X)) = K_*(\mathbb{C})$ for a contractible compact Hausdorff space X.

3. State all general properties of K_* known to you.

4. Define $K_0(A)$ and $K_1(A)$ for an arbitrary C^*-algebra A.

5. Define graph C^*-algebras and state the theorem about their K-groups. Compute one example.

D. Difficult questions

1. Define the index map.

2. Give examples that demonstrate that the functors K_0 and K_1 are not exact.

3. Assume that $0 \to J \to A \to B \to 0$ is an exact sequence of C^*-algebras, where J is an ideal in A and $J \to A$ is the inclusion mapping. If $u \in U_n(\tilde{B})$ lifts to a partial isometry in $\tilde{A}$, derive a formula for $\partial([u]_1)$.

4. Define the exponential map and prove exactness of the 6-term exact sequence corresponding to $0 \to J \to A \to B \to 0$ at $K_0(B)$.

5. Prove the Whitehead Lemma.

6. Show that every trace on a unital C^*-algebra A induces a homomorphism $K_0(A) \to \mathbb{C}$.

7. Define the map $\theta_A : K_1(A) \to K_0(SA)$ and give an outline of the proof that it is an isomorphism.

8. Define the Bott map $\beta_A : K_0(A) \to K_1(SA)$, first for unital A and then in general.

9. Construct the minimal unitization of a C^*-algebra.

10. Give a sketch of the derivation of the Mayer-Vietoris 6-term exact sequence for the fiber product of two C^*-algebras.