

## Asymptotic properties of stochastic symbiosis model

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We discuss the influence of various stochastic perturbations on symbiosis system. We consider the following system of stochastic equations

$$\begin{cases} dX(t) = ((a_1 + b_1Y(t) - c_1X(t)) dt + \rho_{11} dW_1(t) + \rho_{12} dW_2(t)) X(t) \\ dY(t) = ((a_2 + b_2X(t) - c_2Y(t)) dt + \rho_{21} dW_1(t) + \rho_{22} dW_2(t)) Y(t), \end{cases} \quad (1)$$

which describes relations between two populations living in symbiosis. We assume that  $a_i, b_i, c_i > 0$  ( $i = 1, 2$ ) are positive constants,  $W_1(t), W_2(t)$  are two independent standard Wiener processes,  $X(t), Y(t)$  are stochastic processes which represent, respectively, the first and the second population. We consider three kinds of stochastic perturbations:

- (i) weakly correlated, i.e.  $\rho_{11}\rho_{22} - \rho_{12}\rho_{21} \neq 0$ ;
- (ii) strongly correlated, i.e.  $\rho_{11} > 0, \rho_{21} > 0, \rho_{12} = 0, \rho_{22} = 0$ ;
- (iii) only one population is stochastically perturbed, by symmetry we assume that the second population is perturbed, i.e.  $\rho_{11} = 0, \rho_{21} > 0, \rho_{12} = 0, \rho_{22} = 0$ .

First we show the existence, uniqueness, positivity and non-extinction property of the solutions of system (1) on the assumption that  $b_1b_2 < c_1c_2$ . Next we prove that the probability distributions of the process  $(X(t), Y(t))$  are absolutely continuous with respect to the Lebesgue measure. Let  $U(x, y, t)$  be the density of the distribution of  $(X(t), Y(t))$ . We give a sufficient and a necessary condition for asymptotic stability of system (1), i.e. the convergence of  $U(x, y, t)$  to an invariant density  $U_*(x, y)$ . In the case when this system is not asymptotically stable, we prove that  $\lim_{t \rightarrow \infty} Y(t) = 0$  a.e. We also show that in this case  $\lim_{t \rightarrow \infty} X(t) = 0$  a.e. or the probability distributions of the process  $X(t)$  converge weakly to some probability measure. We give a biological interpretation of these results.