Fronts for Periodic KPP Equations

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We consider the KPP equation in a periodic environment:

$$u_t(t,x) = \nabla(A(x)\nabla u(t,x)) + b(x) \cdot \nabla u(t,x) + f(u,x)$$

where $A(x), f(u,x)$ are L-periodic w.r.t. all x_i. The nonlinearity $f(x,u)$ is of KPP type w.r.t. u. A periodic travelling wave in direction e and velocity c has the form $u(t,x) = U(\xi,x)$, $\xi = x \cdot e + ct$, $U(\xi,x)$ is 1-periodic w.r.t. all x_i. It is well known that a travelling wave exists iff the speed satisfies $c \geq c^*(e)$, where $c^*(e)$ is obtained from a nonselfadjoint linear eigenvalue problem. We will derive a new saddle point characterization of $c^*(e)$. Dualizing yields a maximization problem. This allows for a quantative analysis of the dependence of $c^*(e)$ on the parameters A, b, f, L, e of the problem. Examples are:

In the limit $L \to 0$ the minimal speed converges to the speed of the homogenized equation. If $b(x)$ equals to zero, then $c^*(e)$ is nondereasing w.r.t. the period L. Also the limit $L \to \infty$ can be studied.

$c^*(e)$ as a function of e is the support function of a convex set. The support function of its convex polar set is the inverse of the closely related concept of the asymptotic spreading speed.

A drift term enhances the speed due to enlargement and mixing of the reaction zone. Asymptotics for a large drift $b(x)$ can be derived.

Generalizations for perforated domains und time periodic coefficients are possible.