$$\sum_{n=1} n^{\alpha}\left(\sqrt[k]{n^k+1}-n\right)$$

$$(a-b)(a^{k-1}+a^{k-2}b+\dots+b^{k-1})=a^k-b^k$$

$$a=\sqrt[k]{n^k+1} \qquad b=n$$

$$(\dots\dots) = \frac{n^k+1-n^k}{\text{obrzydliwy mianownik}} = \frac{1}{\text{o.m.}}$$

[o.m. ma $\boxed{k \text{ składników}}$ typu] $\sim n^{k-1}$

$$\underbrace{\left(\sqrt[k]{n^k+1}\right)^{j} n^{k-1-j}}_{=s_j(n)}, \quad j=0,\dots,k-1$$

ŁATWO ZAUWAŻYĆ: $\lim_{n\to\infty} \frac{s_j(n)}{n^{k-1}} = 1$, bo:

$$\frac{s_j(n)}{n^{k-1}} = \left(\frac{\sqrt[k]{n^k+1}}{n}\right)^{j} = \left(\sqrt[k]{1+\frac{1}{n^k}}\right)^{j}$$

$$= \left(1+\frac{1}{n^k}\right)^{j/k} = \exp\left[\frac{j}{k}\ln\left(1+\frac{1}{n^k}\right)\right]$$

$\to \ln 1 = 0$

$\to 0$

$\to 1.$

MORAŁ

1) $\frac{\text{o.m.}(n)}{n^{k-1}} \xrightarrow{n\to\infty} k$

2) wyraz szeregu $a_n = n^{\alpha}\left(\sqrt[k]{n^k+1}-n\right)$ spełnia $\frac{a_n}{n^{\alpha-k+1}} \to \frac{1}{k}$)

$$a_n = \frac{n^{\alpha}}{\text{o.m.}} = \frac{n^{\alpha}}{n^{k-1}}\,\frac{n^{k-1}}{\text{o.m.}}$$

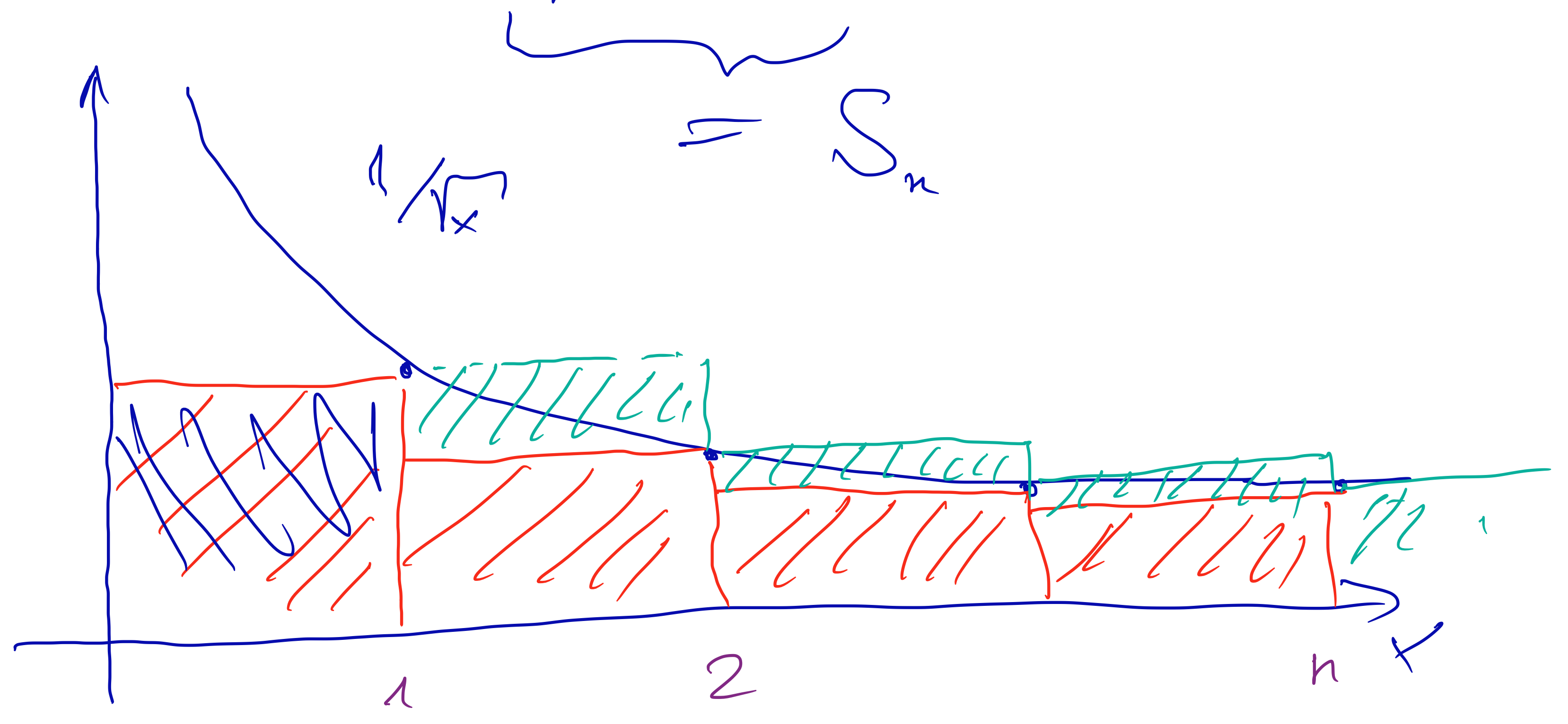
[KRYTERIUM PORÓWNAWCZE, VER. ASYMPTOTYCZNA]

$$\sum_{n=1}^{\infty} a_n \text{ zb.} \iff \sum_{n=1}^{\infty} n^{\alpha-k+1} \text{ zb.}$$

$$\sum_{n=1}^{\infty} n^{p} \text{ zbieżny} \iff p<-1$$

$$\iff \alpha-k+1<-1$$

$c\in\mathbb{R}$
$$\underbrace{\sum_{k=1}^{n}\frac{1}{\sqrt{k}}}_{=S_n} - c\sqrt{n}$$



$$S_n-\frac{1}{\sqrt{n}} \geq \text{Pole pod wykresem } 1/\sqrt{x} \text{ na } [1,n] \geq S_n-1$$

$$\| $$

$$\int_1^n \frac{1}{\sqrt{x}}\,dx = 2\sqrt{x}\Big|_1^n$$

$$= 2\sqrt{n}-2 \approx S_n$$, błąd typu 1