Tutorial 7: Cointegration

1. Consider the monthly U.S. 1-year and 3-year Treasury constant maturity rates from April 1953 to March 2004. The data is found in `m-gs1n3-5304.txt`. Use the interest rates directly (without taking a log transformation). Set $\underline{X}_t = \begin{pmatrix} X_{t1} \\ X_{t2} \end{pmatrix}$ where X_{t1} denotes the 1-year maturity rate and X_{t2} denotes the 3 year maturity rate.

   (a) Identify a VAR model for the bivariate interest rate series. Write down the fitted model.

   (b) Compute the *impulse response functions* of the fitted VAR model for the first six lags. The impulse response function is defined by Definition 6.8. It is simply $g_{ij}(s) := \Psi$ in the expansion of the linear time series

   $$\underline{X}_t = \sum_{s=-\infty}^{\infty} g(s)\underline{Z}_{t-s} \qquad \{\underline{Z}_t\} \sim \mathrm{WN}(0, \Sigma)$$

   and provides a measure of the effect of component j of the innovation on component i of the process s time units forward.

   (c) Use the fitted VAR model to produce 1-step to 12-step ahead forecasts of the interest rates, assuming that the forecast origin is March 2004.

   (d) Are the two interest rates cointegrated? Use a 5% significance level to perform the test.

   (e) If the series are cointegrated, build an ECM for the series. Write down the fitted model.

   (f) Use the fitted ECM to produce 1-step to 12-step ahead forecasts of the interest rates, assuming that the forecast origin is March 2004.

   (g) Compare the forecasts produced by the VAR model and the ECM.

2. This example deals with pairs trading. The data sets `d-bhp0206.txt` and `d-vale0206.txt` contain the data for the stock prices of two companies; Billiton Ltd. of Australia and Vale S.A. of Brazil, with stock symbols BHP and VALE respectively. Both are multinational companies belonging to the natural resource industry and encounter similar risk factors. Use the adjusted closing prices from July 1 2002 to March 31 2006.

   (a) Plot the two stocks price processes. Do they look stationary? Or is there a linear trend? Check the cointegration of their stock prices. Make a simple regression model: $p_{1t} = \beta_0 + \beta_1 p_{2t} + \epsilon_t$ where $p_{1.}$ and $p_{2.}$ denote BHP and VALE respectively. Fit an ARMA model to the residuals $(\widehat{\epsilon}_t)$. You should find that the AR(2) model fits. Try a Dickey - Fuller test. You should find that the residuals do not have a unit root.

   (b) Pair trading involves simultaneously buying a quantity of one stock and selling another stock for the same value. You do this when you believe that one of them is underpriced and the other overpriced, hoping that this will correct itself in future. Let Δ denote the standard error of $\widehat{\epsilon}_t$. Let $w_t = \widehat{\beta}_0 + \widehat{\epsilon}_t$. Make a time series plot of w_t superimpose the lines $w_t \pm \Delta$. Do you see opportunities for pair trading here?

(c) Try a cointegration test and decide which trend variable is appropriate for the cointegration. Fit an error correction model.

3. This question uses the data in `stockmarket.dat`, found in the course data directory, which contains stock market data for seven cities for the period January 6, 1986 to December 31, 1997.

   (a) Use an appropriate statistical test to test whether the London and/or the New York series have unit roots. Does the evidence from the statistical tests suggest the series are stationary or non-stationary?

   (b) Let $\{x_t\}$ represent the London series (Lond) and $\{y_t\}$ the New York series (NY). Fit the following VAR(1) model, giving a summary output containing the fitted parameters and any appropriate statistical tests:

$$\begin{cases} x_t = a_0 + a_1 x_{t-1} + a_2 y_{t-1} + Z_{1,t} \\ y_t = b_0 + b_1 x_{t-1} + b_2 y_{t-1} + Z_{2,t} \end{cases}$$

   (c) Which series influences the other the most? Why might this happen?

   (d) Test the London and New York series for cointegration.

   (e) Fit the model below, giving a summary of the model parameters and any appropriate statistical tests.

$$x_t = a_0 + a_1 y_t + Z_t$$

   (f) Test the residual series for the previous fitted model for unit roots. Does this support or contradict the result in part (d)? Explain your answer.