

# Introduction to Combinatorics

## Probabilistic method 2 – Hints

Wojciech Nadara, class 8, 2020-04-30

1. Try expressing  $X$  as a sum of indicators of events  $A_i$ , where event  $A_i$  denotes whether we got head in  $i$ -th toss.

2. Try computing the contribution of a pair of jumps to the final combo score. Or try computing expected difference between scores after  $k$  and  $k + 1$  jumps.

3. Let  $X_n$  be the random variable denoting number of vertices at distance  $n$  from the root that are in the connected component of root. Try bounding  $\mathbb{P}(X_n > 0)$  from below by a constant (possibly depending on  $p$ ). In order to do this compute  $\mathbb{E}X_n$  and  $\mathbb{E}X_n^2$ .

4. Try random assignment and apply Lovasz Local Lemma.

5. Try choosing vertex from each group at random and apply Lovasz Local Lemma.

6. Try coloring every edge at random where edge becomes red with probability  $p$ . Note that we have two types of bad events. Either we can get a red triangle or a blue  $k$ -clique. Try determining which pairs of these events are dependent. Apply asymmetric Lovasz Local Lemma.