Introduction to Combinatorics
Graphs 2 – Hints

Wojciech Nadara, class 6, 2020-04-02

1. First, let us assume that f has value 0 and try decomposing it into cycles. Then let's try reducing general case to this one by pulling out some paths from f. Existence of corresponding paths or cycles should be deduced from analyzing directed graph induced by edges with positive value of flow pushed through them and recalling Kirchhoff's law.

2. Try applying Hall's theorem to this graph.

3. What happens if this graph is regular? Try connecting it with previous exercise. And what happens if this graph is not regular? Try to make it so.

4. This is another problem where we are given two dimensional object with some constraints on rows and columns. Try expressing this setting in terms of bipartite graphs, as we did in previous problems of that type.

After doing so, try relating it to maximum flow.

5. Try looking at sets of edges formed by given perfect matching M and some other perfect matching T. How do they look like in graph H?

6. Try expressing that problem in terms of matchings in bipartite graph and Hall's condition.

   After you did so, if you are given a set breaking Hall's condition, try modifying it in some way which can help you achieve your goal.