Introduction to Combinatorics

Graphs 1 – Hints

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1. Try considering the longest path in G.

2. The answer is $f(n) = \binom{n-1}{2} + 1$. Showing an example with that many edges should be easy, the harder part is that if a graph has more edges than it contains a Hamiltonian cycle. Try proving that inductively on the number of vertices. If you remove one vertex, what conditions do you need on it for the induction argument to go through? What happens if that vertex doesn't satisfy it?

3. That problem is basically pushing Ore's Theorem to its limits. You should start in a similar fashion by adding as many edges as you can while maintaining that there is no Hamiltonian cycle and use Ore's condition later.

4. As suggested in the hint, we should introduce here some bipartite graph. Let's create vertices $r_1, \ldots, r_n$ and $c_1, \ldots, c_n$ and if there is a pawn in i-th row and j-th column put an edge between r_i and c_j. How does this graph relate to the problem?

5. Try finding here some planar graph.

6. This looks like the statement that there are no cliques on 5 vertices in planar graphs, doesn't it? Try relating these two statements by applying operation of *edge contraction*.