# Introduction to Combinatorics
## Problems

Wojciech Nadara, class 4, 2020-03-19

1. In every vertex of a $2n$-gon there is a set of two real numbers. Prove that we can choose one number in every vertex so that no two adjacent vertices have the same number chosen.

2. (Cauchy-Davenport theorem) Let $p$ be a prime number and let $A$ and $B$ be two subsets of $\mathbb{Z}_p$. Prove that $|A+B| \geq \min(p, |A|+|B|-1)$, where $A+B = \{a+b : a \in A, b \in B\}$.

3. (Erdős-Ginzburg-Ziv theorem — special case) Let $p$ be a prime number. Prove that for every set of $2p-1$ integers we can find $p$ of them whose sum is divisible by $p$.

4. Let $k$ be a positive integer. Determine the coefficient of $x_1^{k-1} \ldots x_k^{k-1}$ in $\prod_{1 \leq i < j \leq k} (x_i - x_j)^2$.

5. Let $p$ be a prime number and $k$ a positive integer such that $k < p$. Let $a_1, \ldots, a_k, b_1, \ldots, b_k$ be integers so that $b_1, \ldots, b_k$ are pairwise different modulo $p$. Prove that there exists a permutation $c_1, \ldots, c_k$ of $b_1, \ldots, b_k$ so that $a_1 + c_1, \ldots, a_k + c_k$ are pairwise different modulo $p$.