Introduction to Combinatorics
Hints

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1. Let x_i denote the chosen value in i-th vertex. Try expressing fact that for every pair of adjacent vertices their values are different as the fact that some polynomial is nonzero in some point.

2. It should be a good idea to consider two cases. First one when $|A| + |B| \geq p + 1$, and second one when $|A| + |B| \leq p$. Assume by contrary that $|A + B| \leq |A| + |B| - 2$ and try applying Combinatorial Nullstellensatz for a polynomial whose degree will be small thanks to this assumption.

3. Try using Cauchy-Davenport theorem where $|B| = 2$.

4. Polynomial from the problem statement looks similar to the determinant of Vandermonde's matrix...

   By the way this problem doesn't have a direct connection to the Combinatorial Nullstellensatz. You may feel deceived, but it will serve its purpose when the appropriate time comes :).

5. We can express fact that c is a permutation of b in the following way: $c_i \in \{b_1, \ldots, b_k\}$ and $c_i \neq c_j$ for $i \neq j$. Now, condition from statements requires $c_i + a_i \neq c_j + a_j \pmod{p}$ for $i \neq j$. Can all of this be encoded in some polynomial?