Introduction to Combinatorics
Problems

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1. Try to recall the proof of Lemma 2 from lecture notes on Borsuk's conjecture and mimic it here.

2. It is a very similar problem to the previous one, but it needs some twists in the solution. We need to somehow make up for the condition that $|A| \notin L$ that is no longer here. After you have some idea how to fix this it should simpler to solve this under the assumption that if $A, B \in \mathcal{F}$ and $A \neq B$ then $A \not\subseteq B$, so you may firstly think about such case and then how to handle the general case.

3. Try using the induction on the number of variables by singling out one of them.

4. We need to count the number of different monomials we can get. Monomial $x_1^{c_1} \dots x_n^{c_n}$ can be identified with sequence of its exponents $c_1, \dots, c_n$. These exponents cannot sum to more than d and any such sequence of nonnegative integers gives a valid monomial, so in fact the answer is number of such sequences.

5. Does $\binom{n+q-2}{n}$ ring a bell? As proved in the previous exercise it is the dimensions of linear space of polynomials in n variables with degree at most $q-2$. And as you can expect from the hint next to the problem statement, we should focus on mapping from space of these polynomials to functions on set B.

6. Try to find a connection between points that we are able to color and roots of polynomials in variables x, y, z.