Kilka granic:

a) $\lim_{x \to 0^+} x^x$

b) $\lim_{x \to 1} \frac{x^{\frac{2}{3}} - x^{\frac{3}{7}}}{\ln x}$

c) $\lim_{x \to \infty} \frac{\ln(2^x + x^{100})}{\ln(3^x + x^{1000})}$

d) $\lim_{x \to \infty} x^{\frac{1}{x}}$

e) $\lim x^{3/2} (\sqrt{x^3+2} - \sqrt{x^3-2})$

a) $x^x = e^{x \ln x}$ $\quad x \ln x \underset{t \to \infty}{\overset{t=\frac{1}{x}}{=}} \frac{\ln(\frac{1}{t})}{t} = \frac{-\ln t}{t} \xrightarrow[t \to \infty]{} 0$. Czyli

$\lim x^x = e^0 = 1$

b) $t = x - 1$, $x = 1 + t$, $t \to 0$ $\quad \frac{(1+t)^{2/3} - (1+t)^{3/7}}{\ln(1+t)} = \frac{(1+t)^{2/3} - 1}{\ln(1+t)} - \frac{(1+t)^{3/7} - 1}{\ln(1+t)} =$

$= \underbrace{\frac{(1+t)^{2/3} - 1}{t}}_{\to 2/3} \cdot \underbrace{\frac{t}{\ln(1+t)}}_{\to 1} - \underbrace{\frac{(1+t)^{3/7}}{t}}_{\to 3/7} \frac{t}{\ln(1+t)} \xrightarrow[t \to 0]{} \frac{2}{3} - \frac{3}{7} = \frac{5}{21}$

c) ~~lim~~ $\frac{\ln(2^x + x^{100})}{\ln(3^x + x^{1000})} = \frac{\ln(2^x \cdot (1 + \frac{x^{100}}{2^x}))}{\ln(3^x \cdot (1 + \frac{x^{1000}}{3^x}))} =$

$= \frac{x \ln 2 + \ln(1 + \frac{x^{100}}{2^x})}{x \ln 3 + \ln(1 + \frac{x^{1000}}{3^x})} = \frac{x}{x} \frac{\ln 2 + \frac{\ln(1 + \frac{x^{100}}{2^x})}{x}}{\ln 3 + \frac{\ln(1 + \frac{x^{1000}}{2^x})}{x}} \to$

$\to \frac{\ln 2}{\ln 3}$

d) $t = \frac{1}{x}$, $t \to 0^+$ $\quad x^{\frac{1}{x}} = \left(\frac{1}{t}\right)^t = \frac{1}{t^t} \xrightarrow[t \to 0^+]{} 1$ z a).