PONTRYAGIN DUALITY AND TOPOLOGICAL ALGEBRAS

S.S.Akbarov

All-Russian institute of Scientific and Technical Information,
Moscow, RUSSIA

If you look at Functional Analysis from the point of view of an algebraist, it is quite likely that you don't like this picture. Objects like non-unital algebras, or linear operators which are not defined everywhere, look quite strange for an algebraic eye. As a corollary, the typical questions that algebraists ask specialists in Functional Analysis are: "Why do you like those uglities?", or "Is it really impossible to avoid such un-natural constructions?" If David Hilbert lived in our time, he would ask: "Why this cannot be simpler?" And certainly, he would immediately be surrounded by a group of people convincing him with a great enthusiasm that this is really impossible "because of very different reasons".

But is Nature really so ugly?

It may be unexpected, but there are evidences that this is not so. They come from a quite unexpected side – the Pontryagin duality of topological Abelian groups. It turns out that some proper generalizations of the Pontryagin ideas lead to a category of topological vector spaces that allows to reconstruct Functional Analysis in such a way that many of its uglities disappear. This brings Analysis closer to Algebra, and to Nature (whose laws, of course, cannot have so bulky appearance).

The topological vector spaces we have in mind are called *stereotype spaces* and they are defined by the equality

$$(X^\star)^\star \cong X$$

where every star means the dual space with the *topology of uniform convergence on totally bounded sets*. It turns out that these spaces form a category with amazingly good properties: in many aspects it is even better than the category of Banach spaces.

In our talk we shall discuss some results of the theory of stereotype spaces, and list some examples illustrating our thesis.