An illustrated story of fast construction of embedded lattice sequences for weighted spaces

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This talk delves deeper into the practical aspects of the construction of embedded lattice sequences as introduced in the talk by Frances Kuo. We optimize for the "best" component-by-component lattice rules with embedded point sets with a number of points equal to 2^m, for a given $m_{\min}$ and $m_{\max}$. Due to the nature of the component-by-component algorithm this lattice rule is also extensible in the dimension, and thus can be used from $d = 1$ up to $d = d_{\max}$ (which can be chosen arbitrarily).

The matrix-vector formulation of the component-by-component construction algorithm enabled us to construct lattice rules in time $\mathcal{O}(d_{\max} n \log(n))$, for n points in d dimensions, $1 \le d \le d_{\max}$. This fast construction was first established for n prime and later extended for general n. For the embedded lattice sequences we exploit the particularly useful embedded structure present in the matrix. This enables us to also construct embedded lattice sequences in a fast way with virtually no additional cost, i.e. in time $\mathcal{O}(d_{\max} 2^{m_{\max}} \log(2^{m_{\max}}))$ for a lattice rule up to dimension $d_{\max}$ with a number of points from $2^{m_{\min}}$ up to $2^{m_{\max}}$. This result holds for function spaces with product weights as well as for order dependent weights of finite order.