(iii)

Chcemy znaleźć $M(\psi \circ \varphi)^{st}_{st}$

$$M(\psi\circ\varphi)^{st}_{st} = M(id)^{st}_{\mathcal{A}} \, M(\psi\circ\varphi)^{\mathcal{A}}_{\mathcal{A}} \cdot M(id)^{\mathcal{A}}_{st}$$

$$M(id)^{st}_{\mathcal{A}} = \begin{bmatrix} 1 & 0 & 0 & -1 \\ 0 & 1 & 1 & 0 \\ 1 & 0 & -1 & 1 \\ 0 & 1 & 0 & 0 \end{bmatrix}$$

$$M(id)^{\mathcal{A}}_{st} = \left[M(id)^{st}_{\mathcal{A}} \right]^{-1}$$

$$\left[\begin{array}{cccc|cccc} 1 & 0 & 0 & -1 & 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 & 1 & 0 & 0 \\ 1 & 0 & -1 & 1 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 1 \end{array}\right] \sim \left[\begin{array}{cccc|cccc} 1 & 0 & -1 & 0 & 0 & 0 & 1 & -2 \\ 0 & 1 & 0 & -1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 & 0 & -1 & 1 \\ 0 & 0 & 0 & 1 & 0 & 1 & 0 & 0 \end{array}\right] \sim \left[\begin{array}{cccc|cccc} -\frac{1}{2} & \frac{1}{2} & \frac{1}{2} & -\frac{1}{2} & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & -1 & 0 & 0 & 1 & 0 \\ 0 & 1 & 1 & -1 & 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 & 1 & 0 & 0 \end{array}\right]$$

$$\sim \left[\begin{array}{cccc|cccc} \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & -\frac{1}{2} & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & -1 & 0 & 0 & 1 & 0 \\ -\frac{1}{2} & \frac{1}{2} & \frac{1}{2} & -\frac{1}{2} & 0 & 0 & 0 & 1 \end{array}\right]$$

$$M(\psi\circ\varphi)^{st}_{st} = \begin{bmatrix} 1 & 0 & 0 & -1 \\ 0 & 1 & 1 & 0 \\ 1 & 0 & -1 & 1 \\ 0 & 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} -1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & -\frac{1}{2} \\ 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & -1 \\ -\frac{1}{2} & \frac{1}{2} & \frac{1}{2} & -\frac{1}{2} \end{bmatrix} = \begin{bmatrix} -\frac{1}{2} & -\frac{1}{2} & -\frac{1}{2} & \frac{1}{2} \\ 0 & 0 & 0 & -1 \\ -\frac{1}{2} & -\frac{1}{2} & -\frac{1}{2} & \frac{1}{2} \\ 0 & 0 & 0 & -1 \end{bmatrix}$$

Zatem

$$(\psi\circ\varphi)(x_1, x_2, x_3, x_4) = \left(-\frac{1}{2}x_1 - \frac{1}{2}x_2 - \frac{1}{2}x_3 + \frac{1}{2}x_4,\ -x_4,\ -\frac{1}{2}x_1 - \frac{1}{2}x_2 - \frac{1}{2}x_3 + \frac{1}{2}x_4,\ x_4\right)$$