Zadanie 4.

$A=\{\alpha_1,\alpha_2,\alpha_3\}$ $\quad$ $\Phi: R^3 \longrightarrow R^2$

$B=\{\beta_1=\beta_2\}$ $\quad$ $\Phi((x,y,z)) = (x-y+2z,\ 3x+y+z)$

$M(\Phi)_A^B = \begin{bmatrix} 1 & 1 & 1 \\ 2 & 0 & 1 \end{bmatrix}$

1pkt

Wyznaczam ker Φ

$\begin{cases} x-y+2z=0 \\ 3x+y+z=0 \end{cases} \Rightarrow \begin{cases} x=-\frac{3}{4}z \\ y=\frac{5}{4}z \end{cases}$

$\ker\Phi = \mathrm{lin}(-\frac{3}{4}, \frac{5}{4}, 1)$

$M(\Phi)_A^B = \begin{bmatrix} 1 & 1 & 1 \\ 2 & 0 & 1 \end{bmatrix}$

$\Phi(\alpha_1) = \beta_1 + 2\beta_2$

$\Phi(\alpha_2) = \beta_1$

$\Phi(\alpha_3) = \beta_1 + \beta_2$

$\Rightarrow \begin{cases} \Phi(\alpha_1) = \Phi(\alpha_2) + 2\beta_2 \\ \Phi(\alpha_2) = \beta_1 \\ \Phi(\alpha_3) = \Phi(\alpha_2) + \beta_2 \end{cases}$

$\begin{cases} \Phi(\alpha_1) = \Phi(\alpha_2) + 2[\Phi(\alpha_3) - \Phi(\alpha_2)] \\ \Phi(\alpha_2) = \beta_1 \\ \Phi(\alpha_3) - \Phi(\alpha_2) = \beta_2 \end{cases}$

$\Phi(\alpha_1) = \Phi(\alpha_2) + 2[\Phi(\alpha_3) - \Phi(\alpha_2)]$

$\Phi(\alpha_1) = \Phi(\alpha_2) + \Phi(2\alpha_3 - 2\alpha_2)$

$\Phi(\alpha_1) = \Phi(2\alpha_3 - \alpha_2)$

$\Phi(\alpha_1 + \alpha_2 - 2\alpha_3) = 0$

Zatem $\alpha_1 + \alpha_2 - 2\alpha_3 \in \ker\Phi$

niech $\alpha_1 + \alpha_2 - 2\alpha_3 = (-3,5,4)$ $\quad$ (bo $\ker\Phi = \mathrm{lin}(-\frac{3}{4}, \frac{5}{4}, 1)$

$\alpha_1 = (1,0,0)$, $\alpha_2 = (0,1,0)$, ~~$\alpha_3 = (0,0,1)$~~ $\alpha_3 = (a,b,c)$

$\begin{cases} 1-2a = -3 \\ 1-2b = 5 \\ -2c = 4 \end{cases} \Rightarrow \begin{cases} a = 2 \\ b = -2 \\ c = -2 \end{cases} \Rightarrow \alpha_3 = (2,-2,-2)$

$\beta_1 = (-1,1)$, $\beta_2 = (1,1)$

$\alpha_1, \alpha_2, \alpha_3$ – są l.-niez. oraz β_1, β_2 są lin niezależne

Zatem baza $A = \{(1,0,0), (0,1,0), (2,-2,-2)\}$

baza $B = \{(-1,1), (1,1)\}$