Zadanie 3 Niech $f \in F(\mathbb{R}, \mathbb{R})$ f-dowolna

- $f_p(x) = \dfrac{f(x)+f(-x)}{2}$

$$f_p(-x) = \frac{f(-x)+f(x)}{2} = \frac{f(x)+f(-x)}{2} = f_p(x)$$

Zatem $f_p(x) \in W_1$

- $f_n(x) = \dfrac{f(x)-f(-x)}{2}$

$$f_n(-x) = \frac{f(-x)-f(+x)}{2} = -\frac{f(x)-f(-x)}{2} = -f_n(-x)$$

Zatem $f_n(+x) \in W_2$

- $f(x) = f_p(x) + f_n(+x) = \dfrac{f(x)-f(-x)}{2} + \dfrac{f(x)+f(-x)}{2} = f(x)$

Zatem $W_1 + W_2 = F(\mathbb{R}, \mathbb{R})$

- Teraz należy sprawdzić że $W_1 \cap W_2 = \{0\}$

Nie wprost: $\bigvee f \in W_1 \cap W_2$

Ale $f(x) \in W_1 \wedge f(x) \in W_2$

Zatem $f(x) = f(-x) = -f(x) \quad \forall x$

$$\forall x \quad f(x) = -f(x) \Rightarrow f(x) = 0$$

Zatem $\dim(V_1 \cap V_2) = 0$ ↯

Zatem $W_1 \oplus W_2 = F(\mathbb{R}, \mathbb{R})$