

# Zadanie 1

$$i) \quad x^7 - x = x(x^6 - 1) = x(x^3 - 1)(x^3 + 1) =$$

$$= x(x-1)(x^2 + x + 1)(x^3 + 1) =$$

$$= x(x-1)(x^2 + x + 1)(x+1)(x^2 - x + 1)$$

$$x^2 - x + 1 = \left(x - \frac{1 + \sqrt{3}i}{2}\right) \left(x - \frac{1 - \sqrt{3}i}{2}\right)$$

$$\Delta = 1 - 4 = -3 \quad x_1 = \frac{1 + \sqrt{3}i}{2}$$

$$x^2 + x + 1 = \left(x - \frac{-1 + \sqrt{3}i}{2}\right) \left(x - \frac{-1 - \sqrt{3}i}{2}\right)$$

$$\Delta = 1 - 4 = -3 \quad x_1 = \frac{-1 + \sqrt{3}i}{2}$$

$$x^7 - x = x(x-1)(x+1) \left(x - \frac{1 + \sqrt{3}i}{2}\right) \left(x - \frac{1 - \sqrt{3}i}{2}\right) \left(x - \frac{-1 + \sqrt{3}i}{2}\right) \left(x - \frac{-1 - \sqrt{3}i}{2}\right)$$

$$ii) \quad x^4 - 6x^2 + 13 = (x^2 + \sqrt{2\sqrt{13} + 6} + \sqrt{13}) (x^2 - \sqrt{2\sqrt{13} + 6} + \sqrt{13})$$

$$x^2 + \sqrt{2\sqrt{13} + 6} + \sqrt{13} = \left(x - \frac{\sqrt{6 + 2\sqrt{13}} + i\sqrt{2\sqrt{13} - 6}}{2}\right) \left(x - \frac{\sqrt{6 + 2\sqrt{13}} - i\sqrt{2\sqrt{13} - 6}}{2}\right)$$

$$\Delta = 2\sqrt{13} + 6 - 4\sqrt{13} = 6 - 2\sqrt{13} < 0$$

$$\sqrt{\Delta} = \sqrt{6 - 2\sqrt{13}} = i\sqrt{2\sqrt{13} - 6}$$

$$x_1 = \frac{-\sqrt{2\sqrt{13} + 6} \pm i\sqrt{2\sqrt{13} - 6}}{2}$$

$$x^2 - \sqrt{2\sqrt{13} + 6} + \sqrt{13} = \left(x - \frac{\sqrt{6 + 2\sqrt{13}} + i\sqrt{2\sqrt{13} - 6}}{2}\right) \left(x - \frac{\sqrt{6 + 2\sqrt{13}} - i\sqrt{2\sqrt{13} - 6}}{2}\right)$$

PUNKTACJA

Każdy podpunkt:

• Za rozwiązanie na czymila  $\leq 2st$  w  $\mathbb{R}$

0,25 pkt

— // — 1st w  $\mathbb{C}$

0,25 pkt