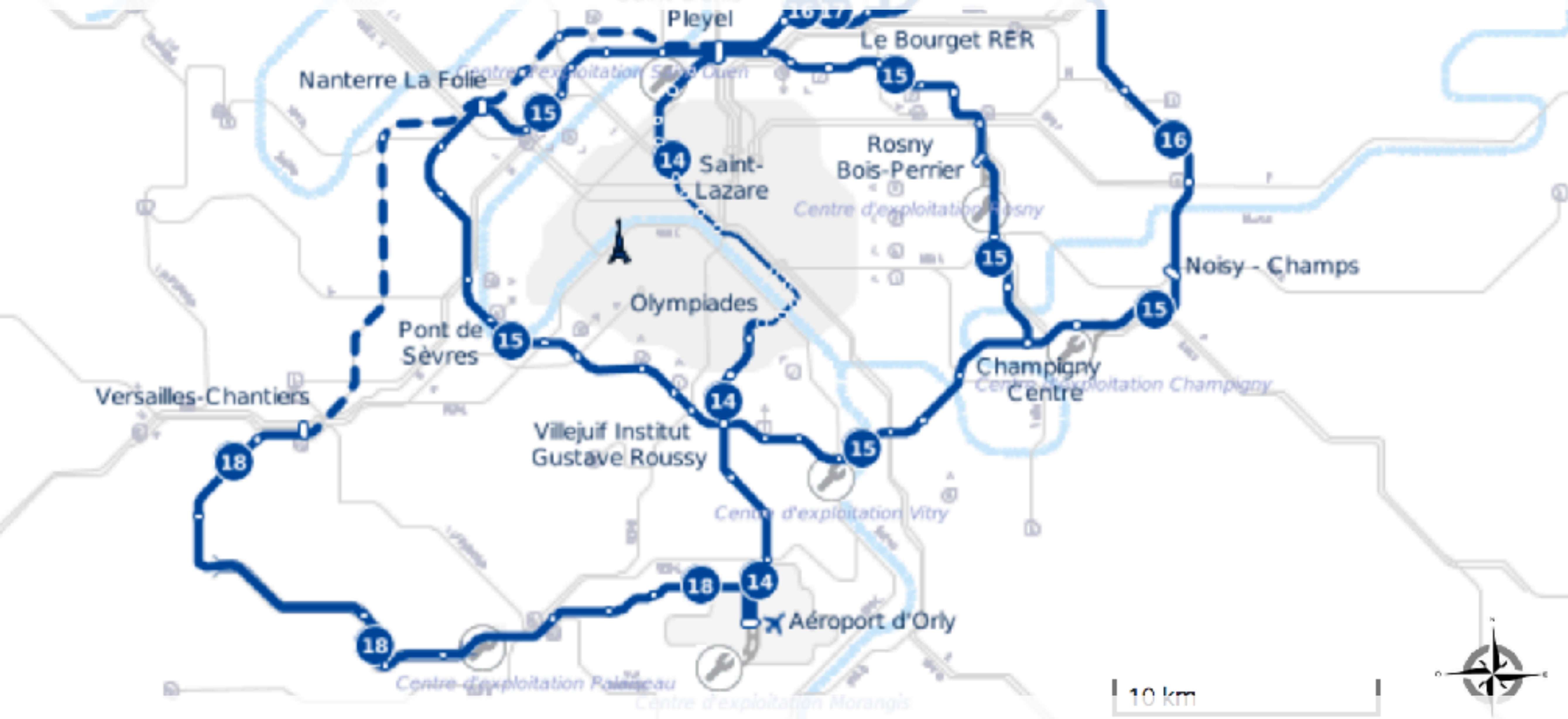
# **Collective** Schedules


Fanny Pascual, **Krzysztof Rzadca**, Piotr Skowron

**Sorbonne Université**

**University of Warsaw, Poland**

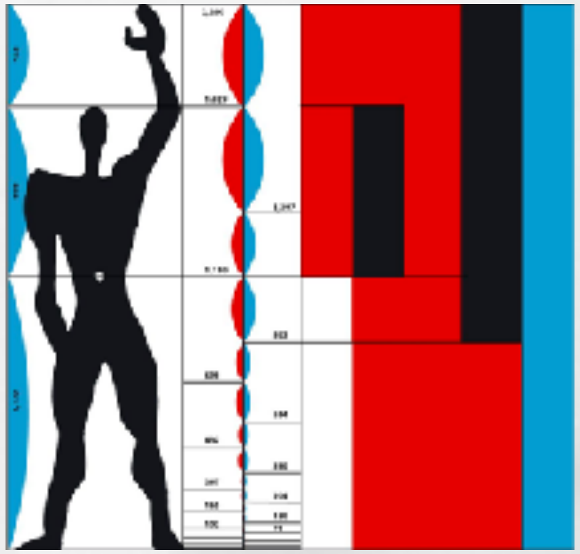


**AAMAS 2018**

**arxiv.org/abs/1803.07484**

# Le Corbusier (1887-1965)

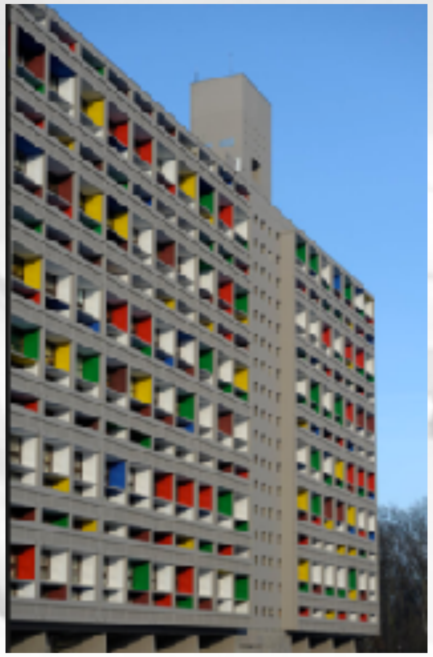
sitelecorbusier.com



models

quality of life, cost
natural light, ...

objectives



solution

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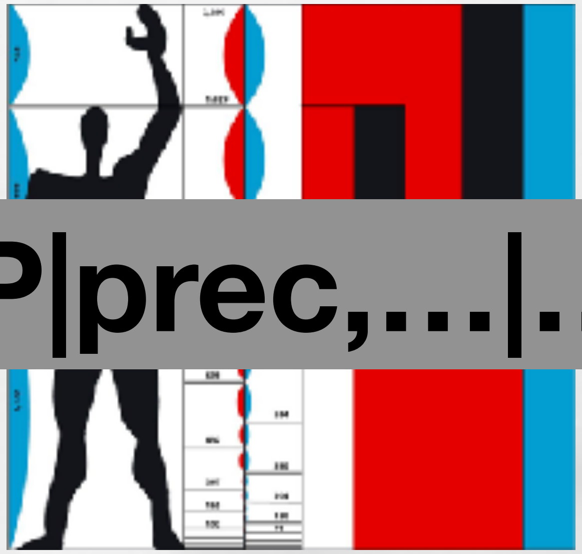


**P|prec,...|...**

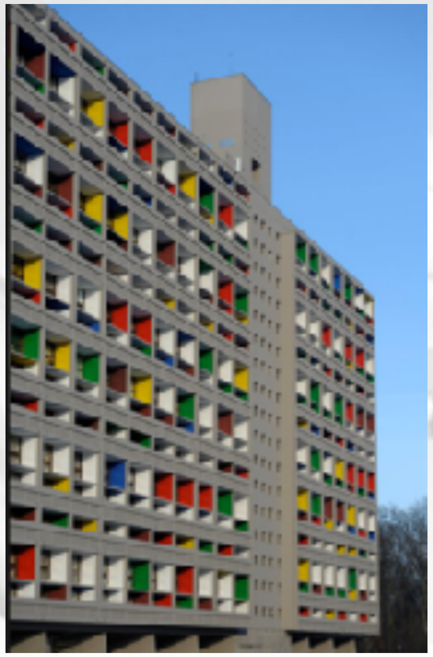
**models**

$C_{max}$ $\Sigma\, C_i$

$\max\, (C_i - r_i)/p_i$

$\Sigma(C_i - r_i)/p_i$

**objectives**



**solution**

Le Corbusier (1887-1965)

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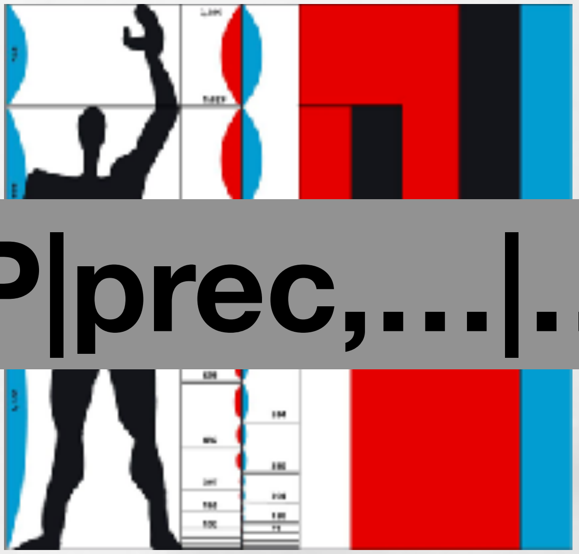


P|prec,....|...

models

$C_{max}$ $\Sigma C_i$

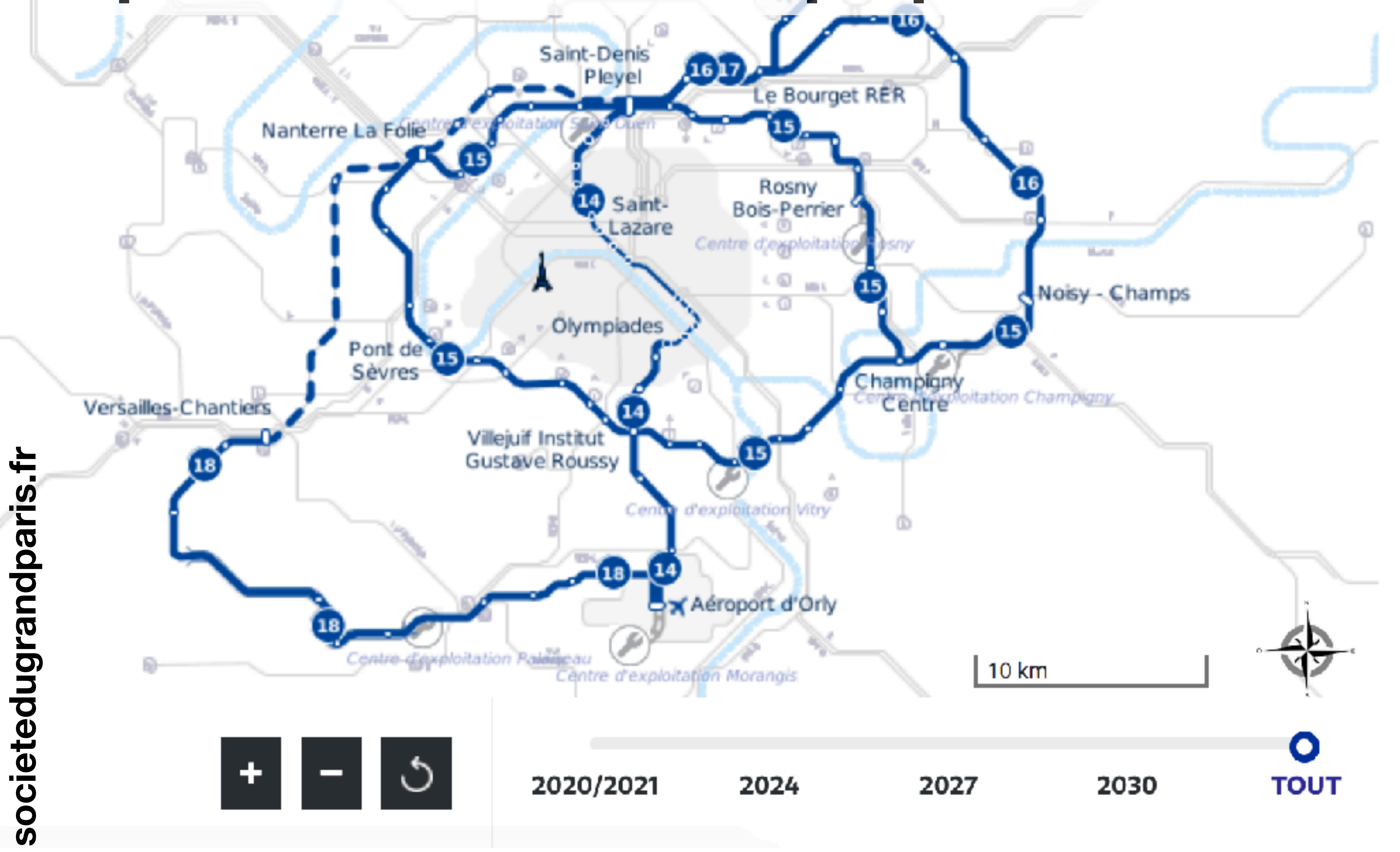
$\max (C_i - r_i)/p_i$

$\Sigma(C_i - r_i)/p_i$

objectives

solution

# How to accommodate preferences of a population?

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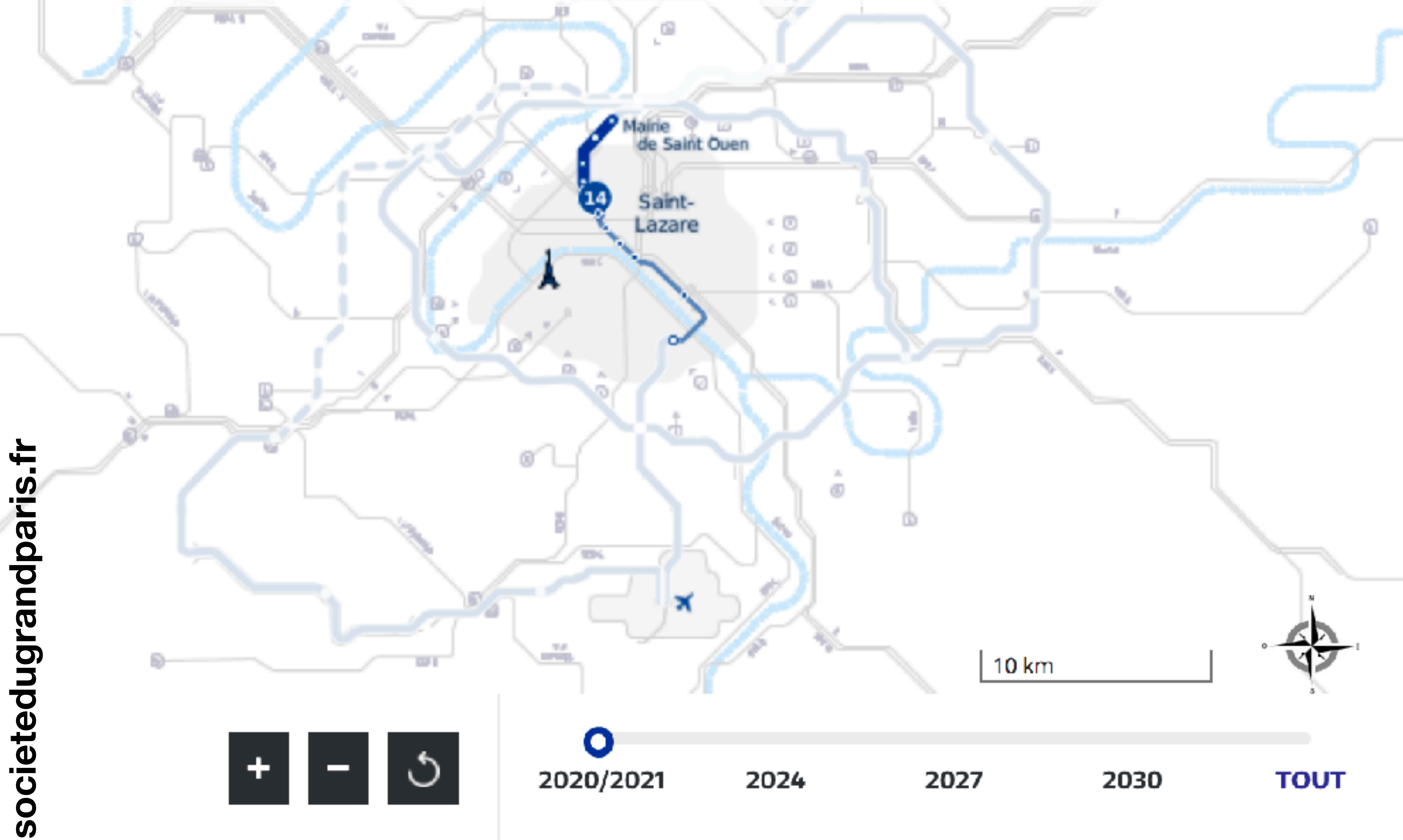


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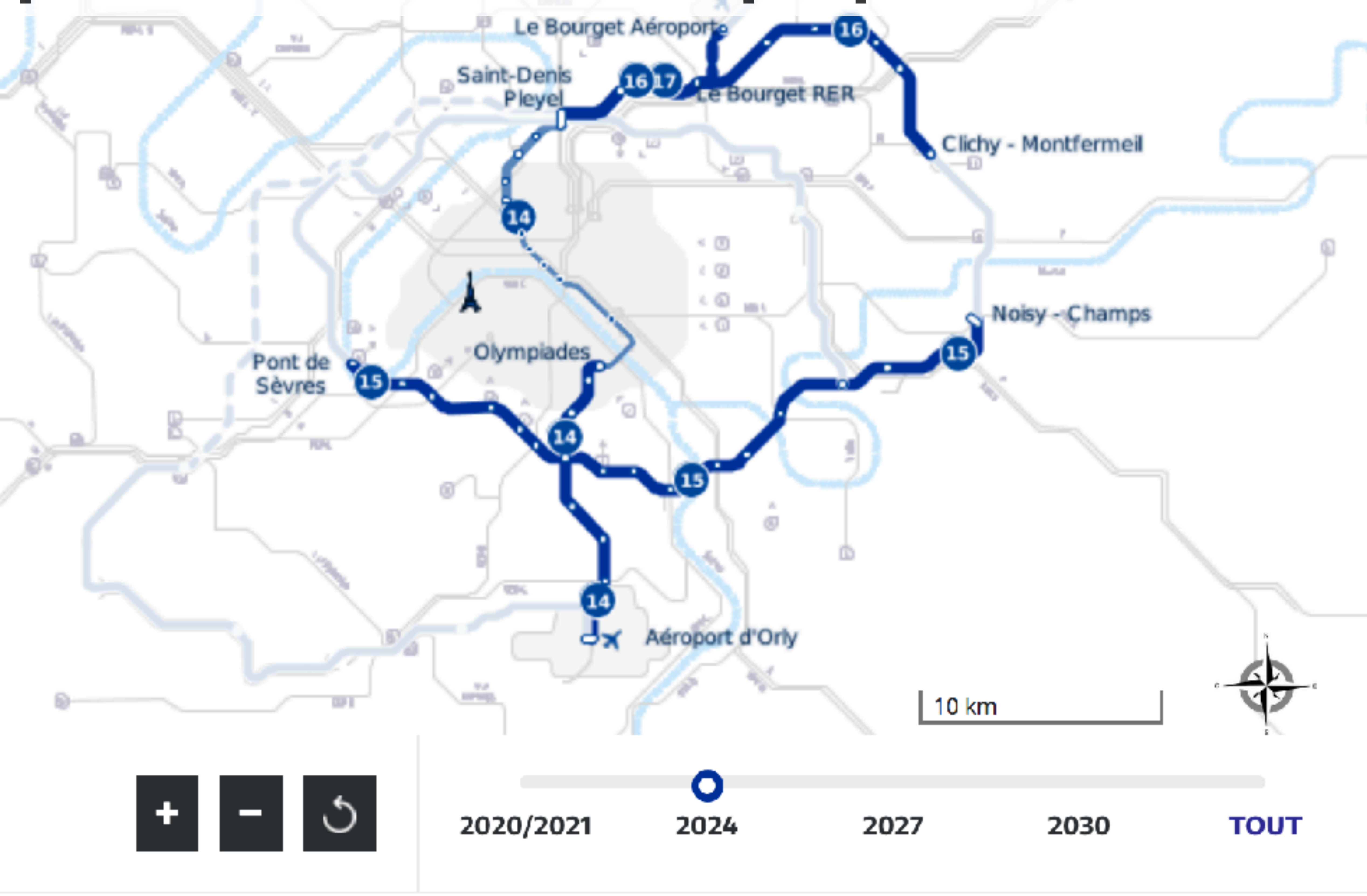


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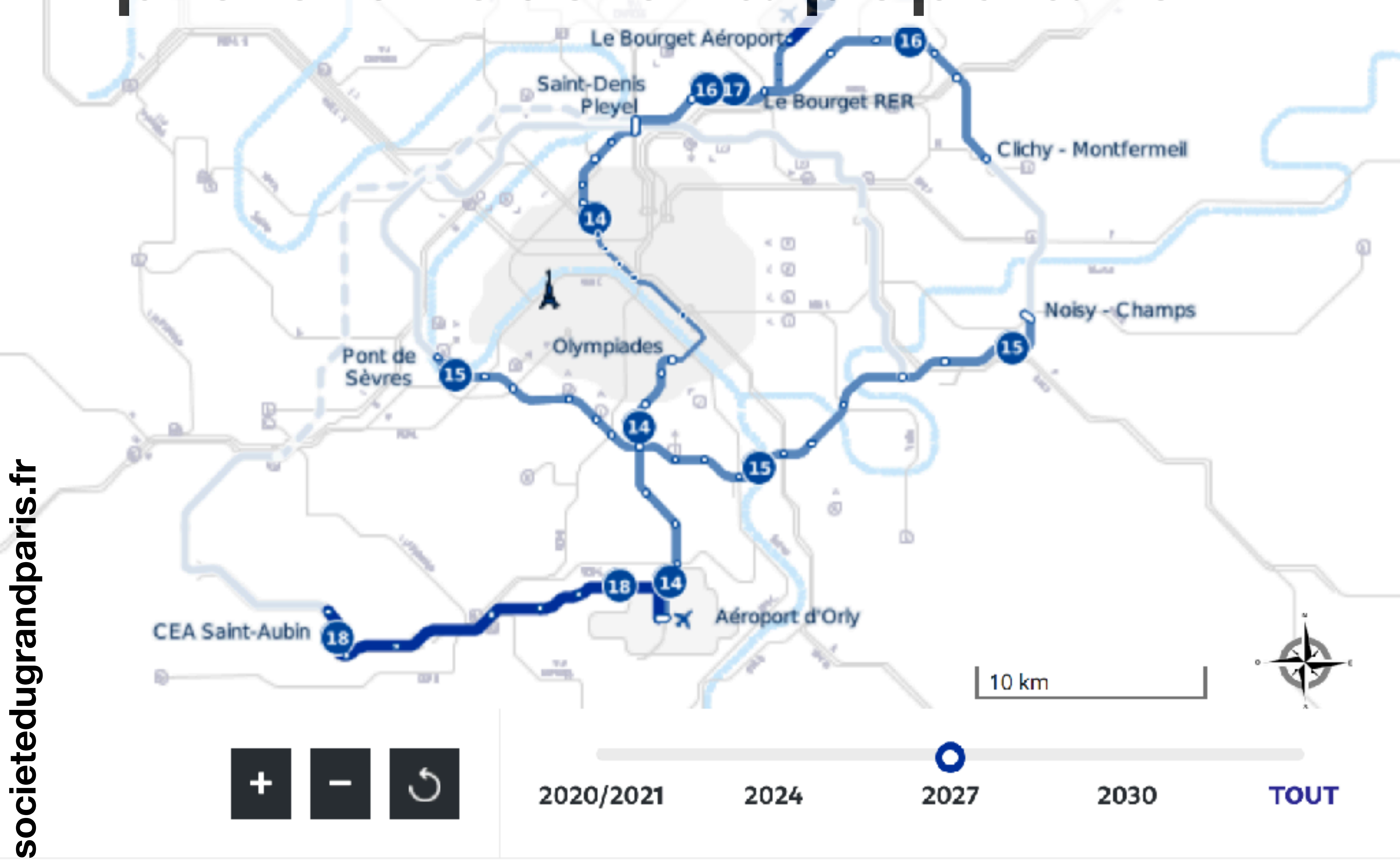


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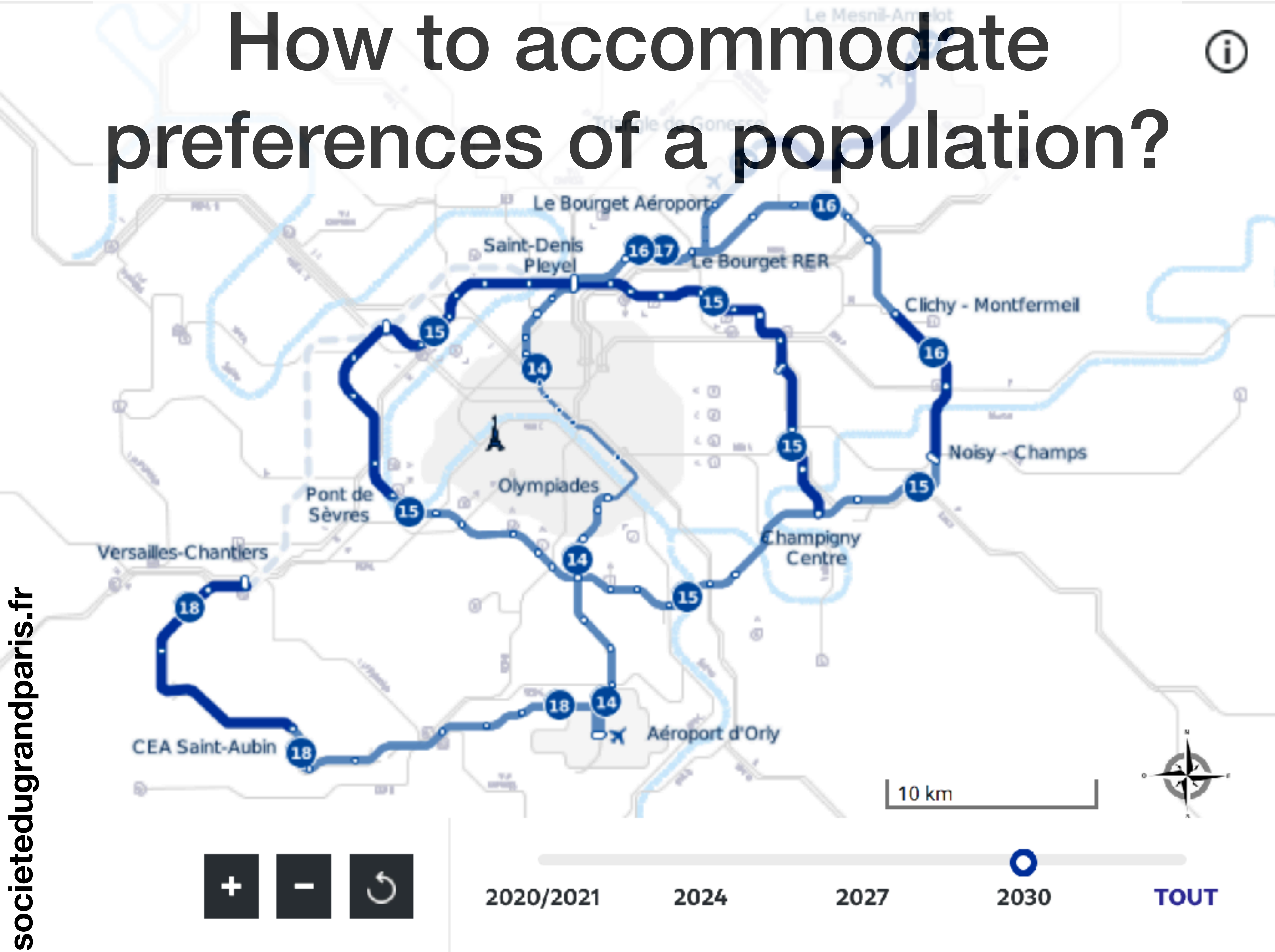


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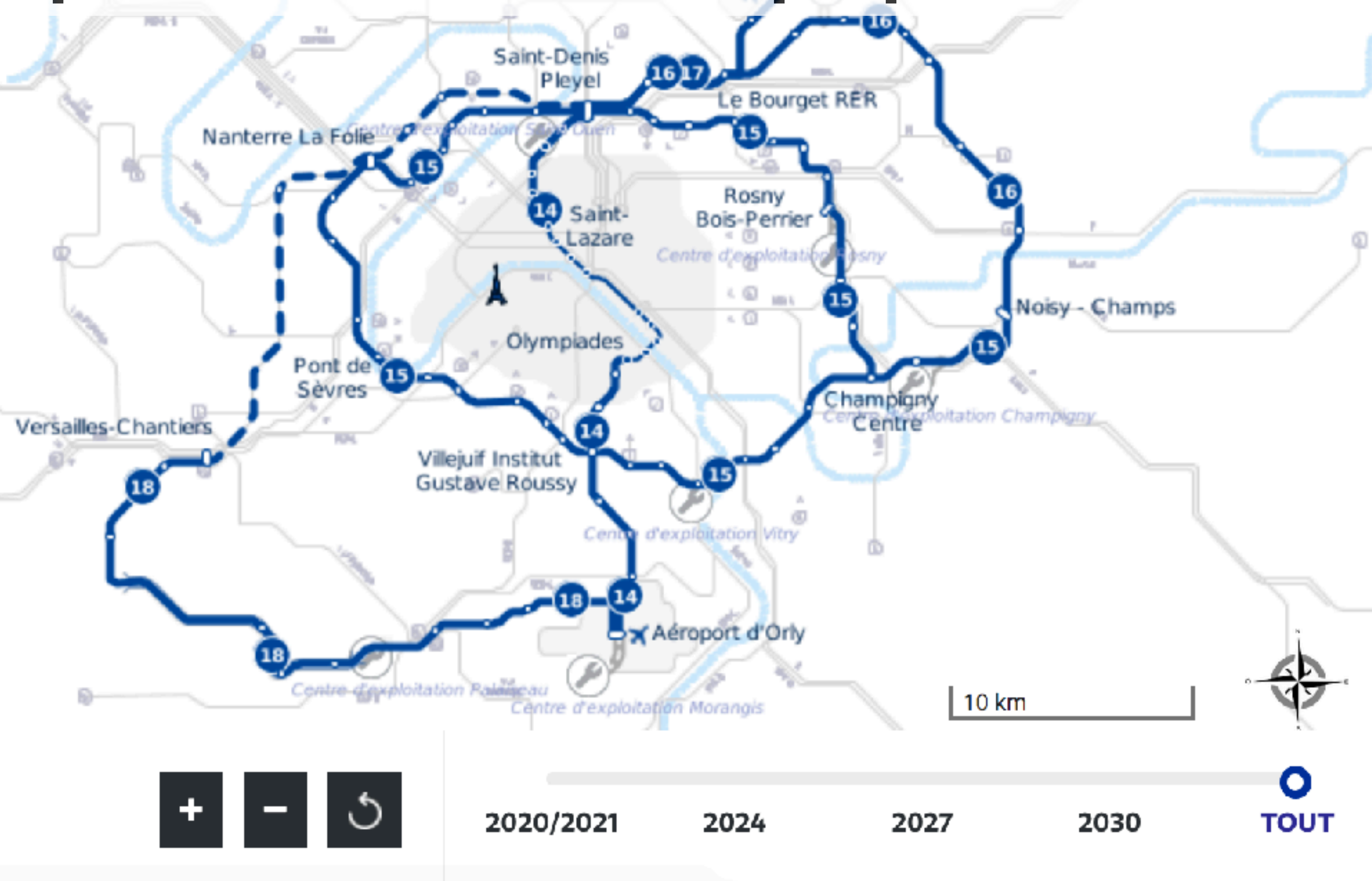


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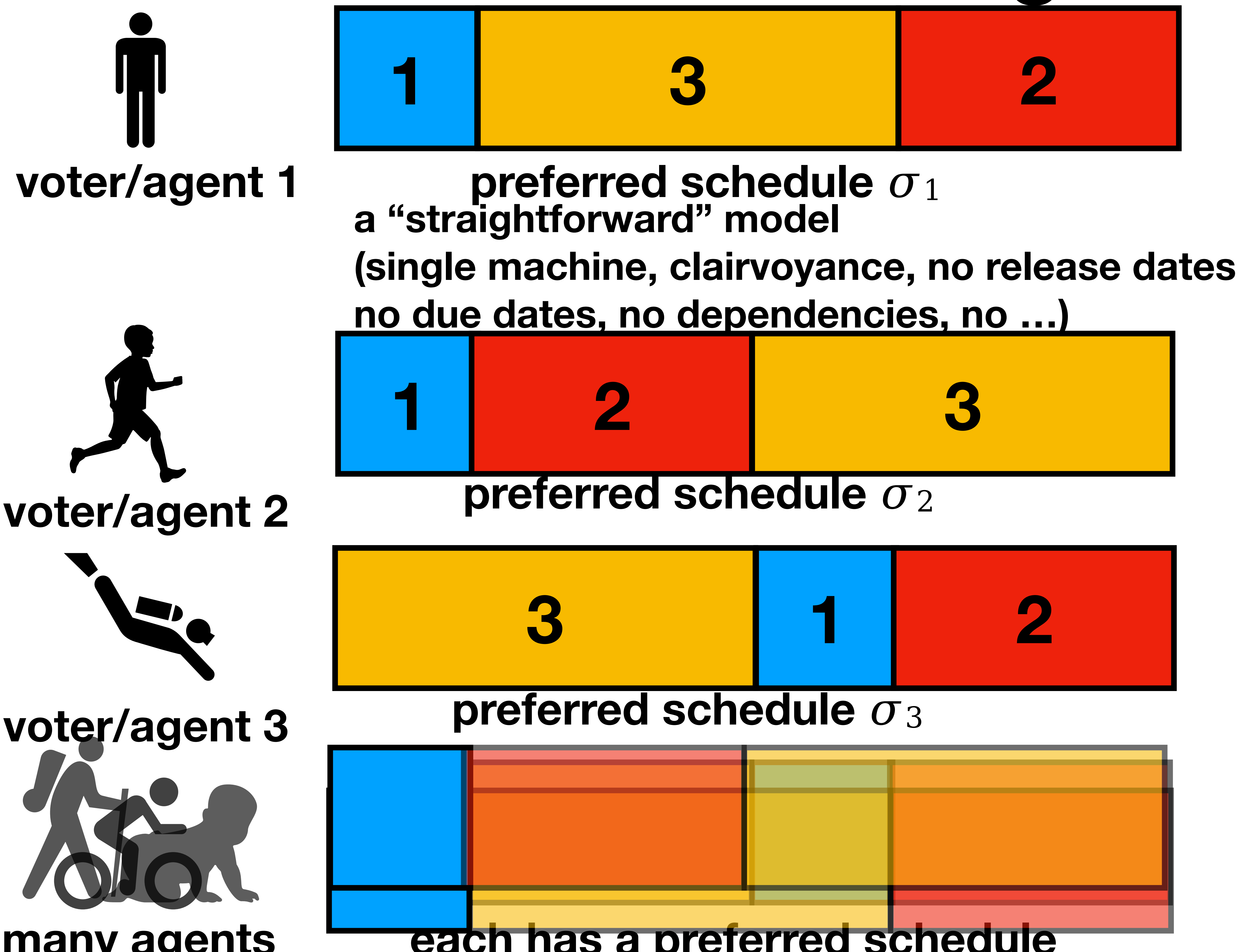
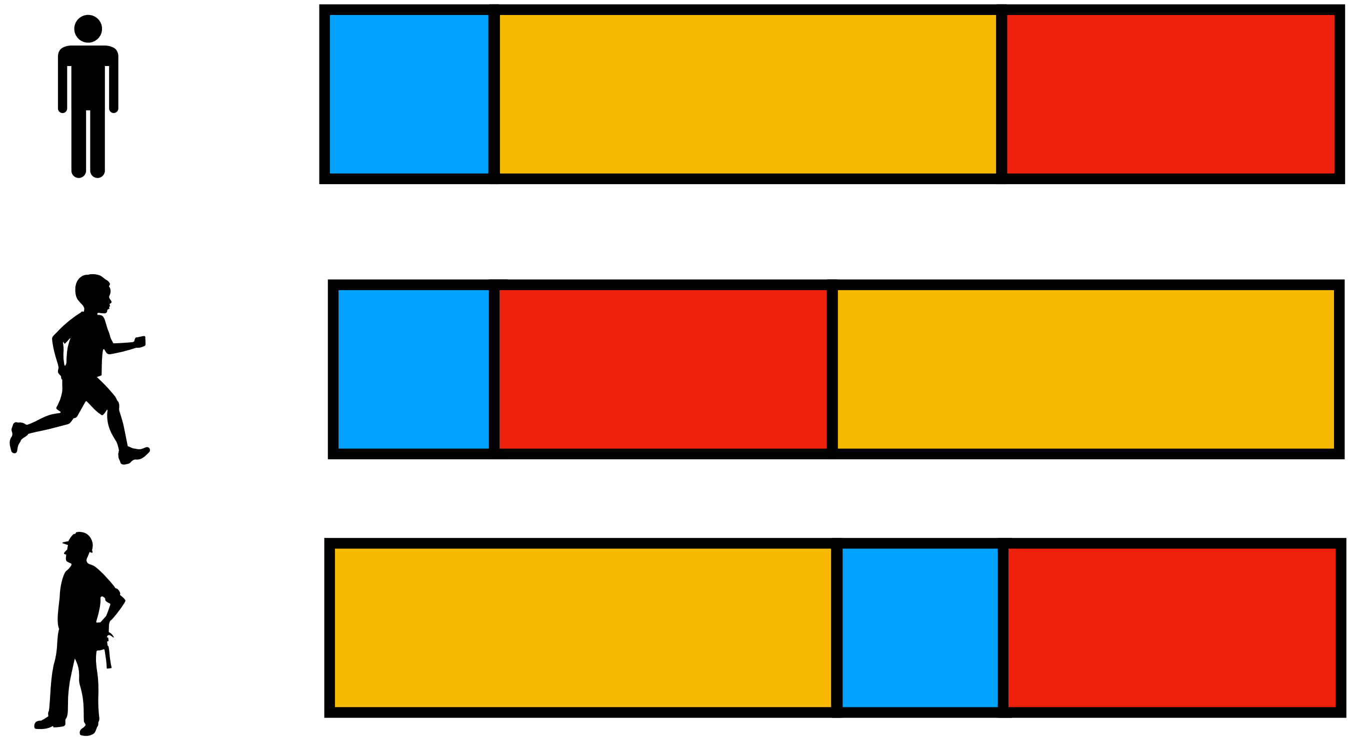
# The collective scheduling model



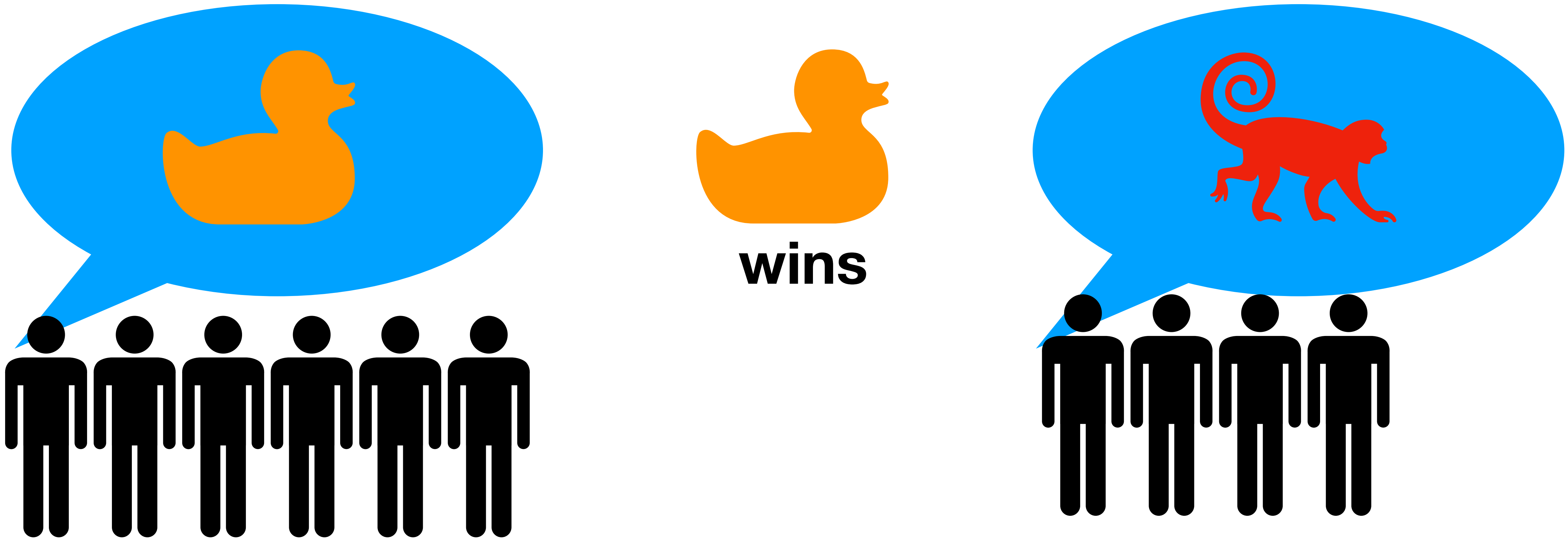
# The collective scheduling model



**Build a single schedule accommodating preferences of all agents!**

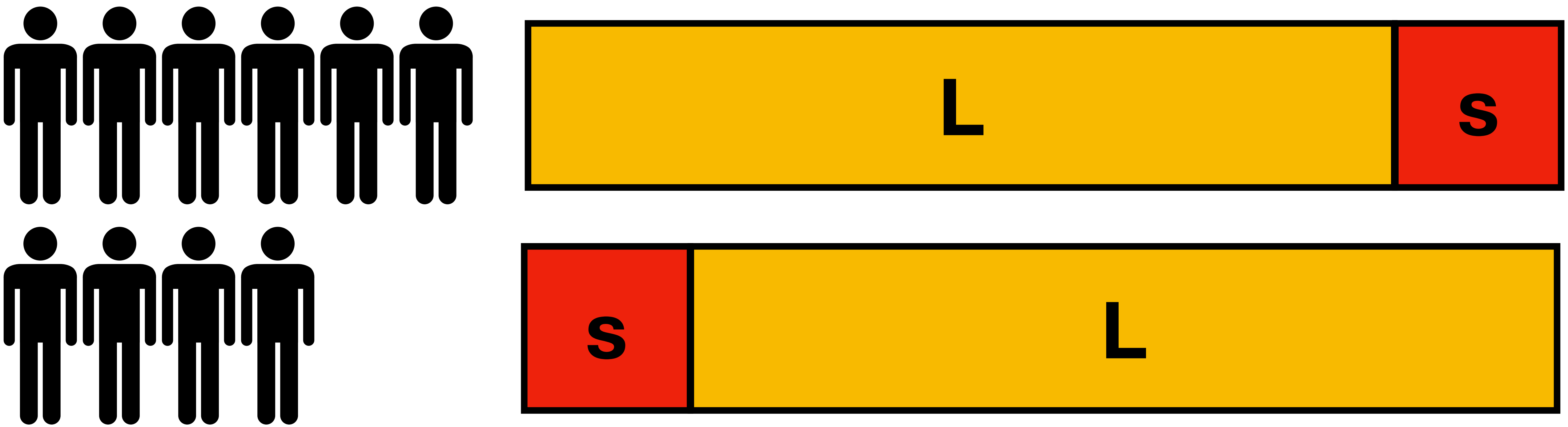
**?**

# social choice:
# how to organize elections




**non trivial in many cases:**
**more than 2 candidates**
**electing a parlament**
**picking a representative committee**
**participatory budgets**

# Social choice cannot be directly applied to collective scheduling




**2 possible collective schedules:**


**preferred by the majority, but delays the red arbitrary long**


**delays the majority by just 1**

# Social choice tools we extend

- Positional scoring rules
- Condorcet
- Kemeny

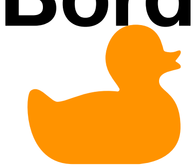
# Positional Scoring Rules

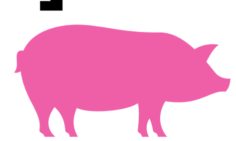
# Positional scoring rules: each ranking position gets a certain amount of points

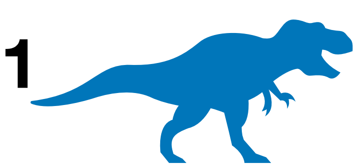
# Winner: highest amount of points

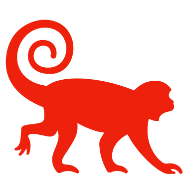
## ranked preferences of voters

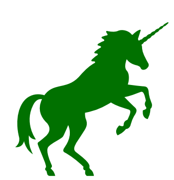

**Borda count [Borda, 1770]: the number of defeated candidates**

 4 + 0 + 2 + 0 + 1 = 7

 2+1+0+4+4=11

 0+3+1+2+0=6

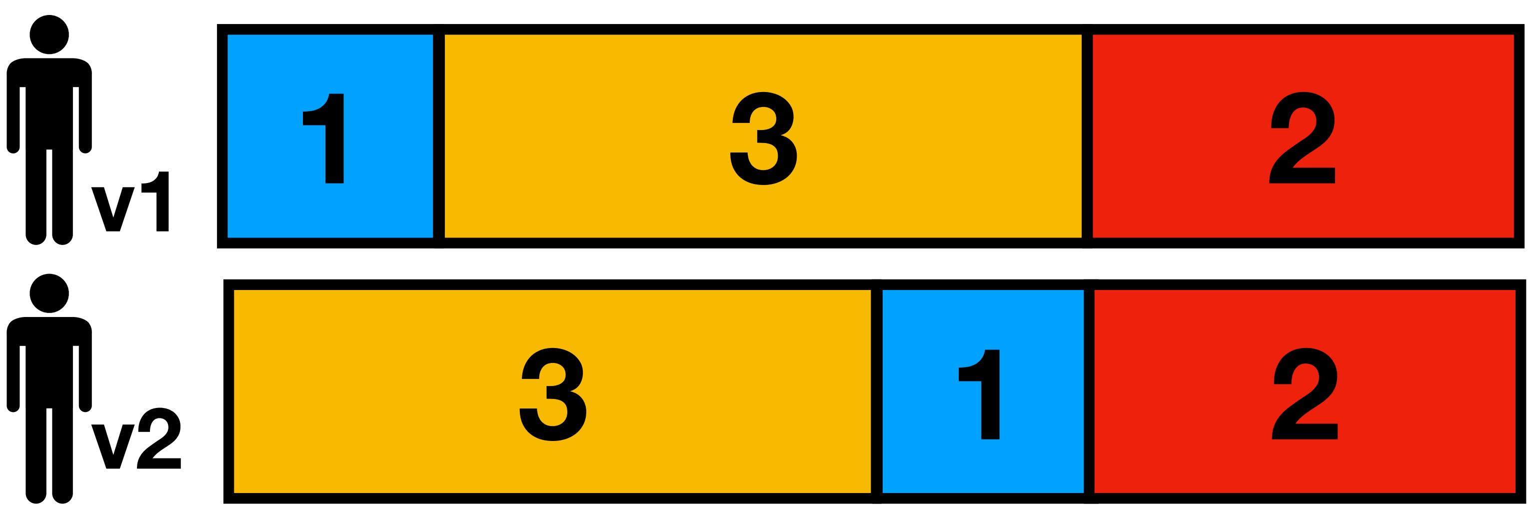
 3 + 2 + 3 + 3 + 3 = 14

 1+4+4+1+2=12

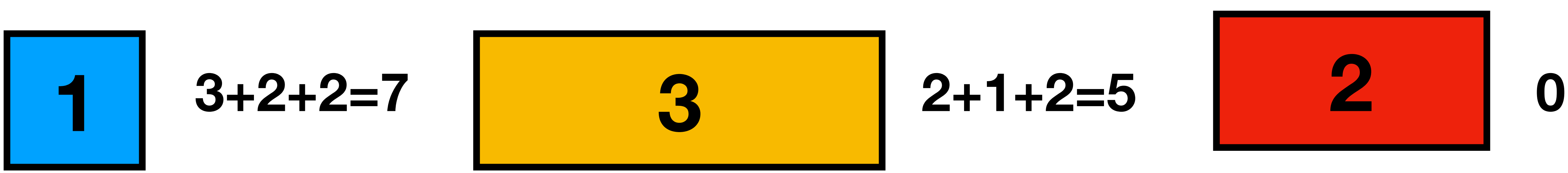
# Extending positional scoring rules by jobs' length

$$h\text{-score}(J) = \sum_{a \in N} f\left(\sum_{J_i :\, J \,\sigma_a\, J_i} p_i\right)$$

**workload scheduled later**
**(preference for shorter jobs)**



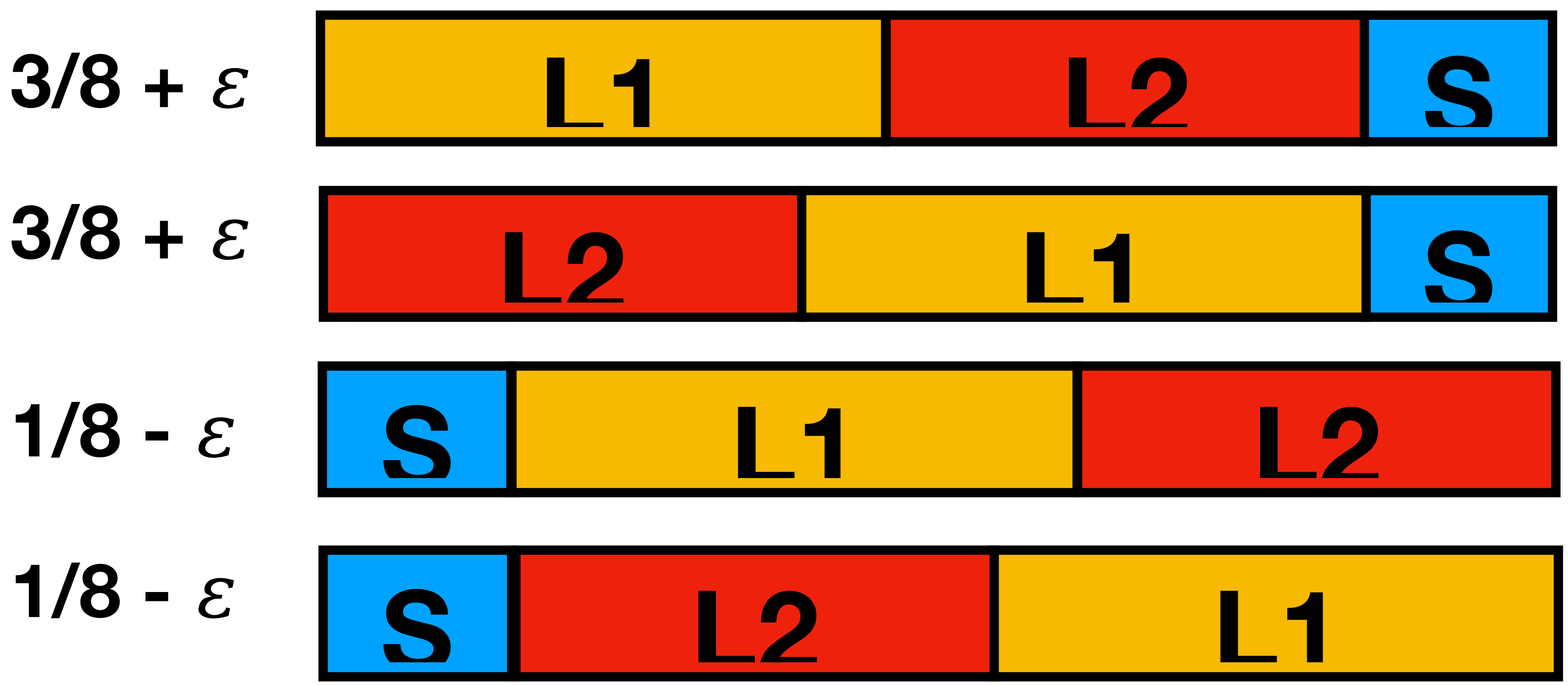

**scores**




**collective schedule:**

# Positional scoring rules don’t really work well

**fraction of votes**

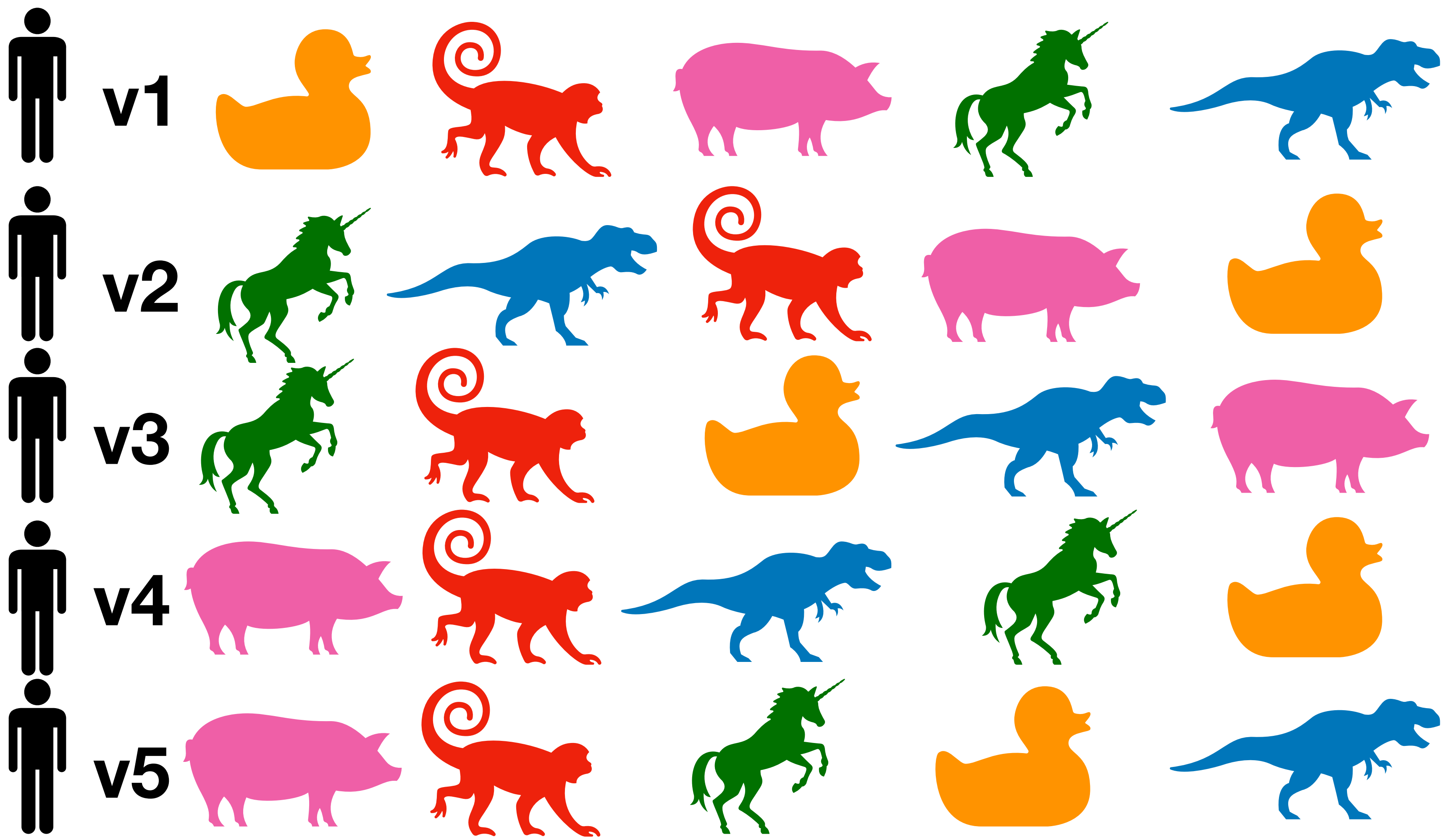



**collective schedule:**


**s voted as first by ~1/4 of agents, but**
**s is delayed by arbitrary large L1+L2**

# The Condorcet Principle

# The Condorcet Principle: if an object preferred by a majority, it should be selected as the winner




ranked preferences of voters

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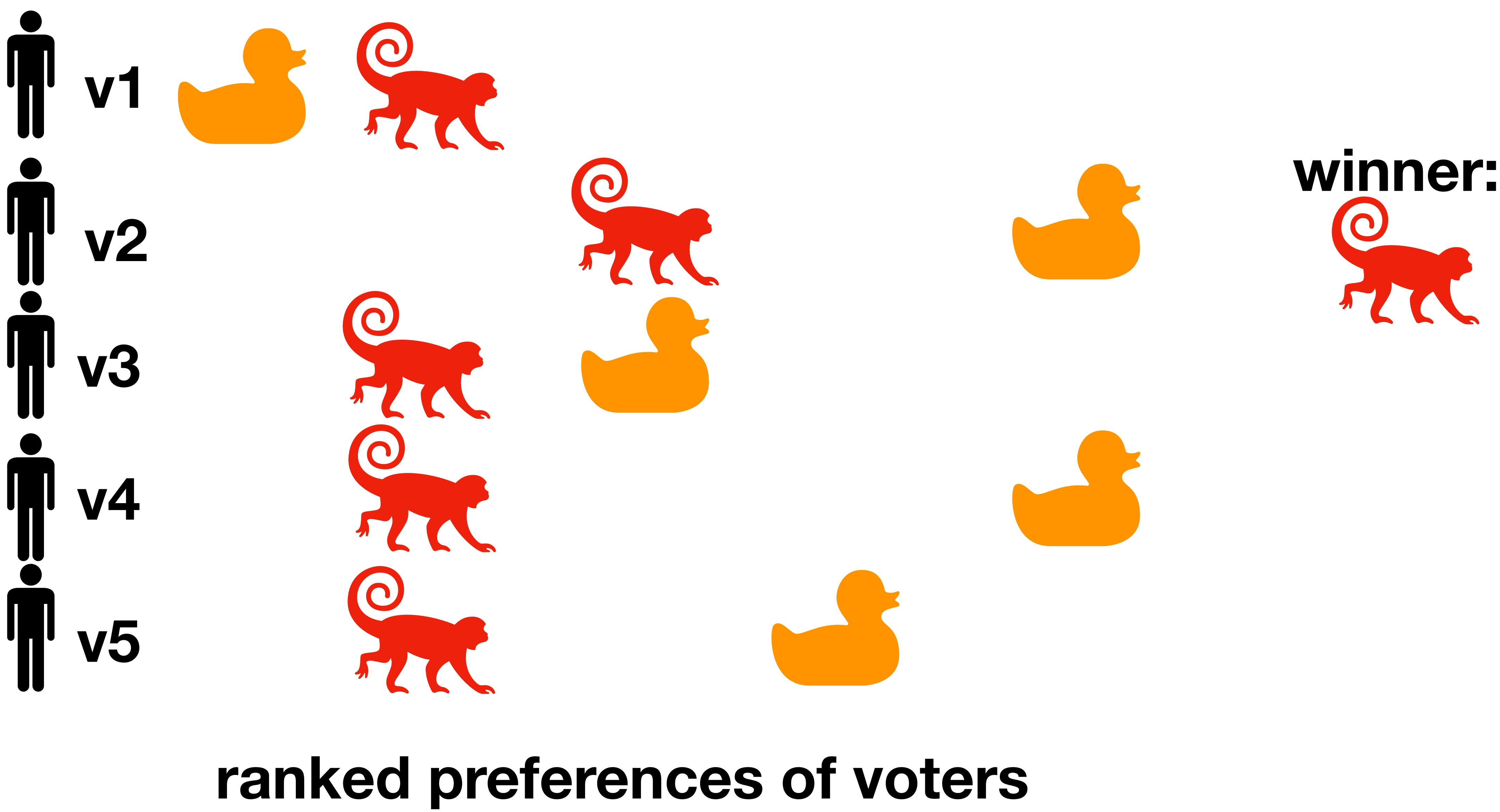

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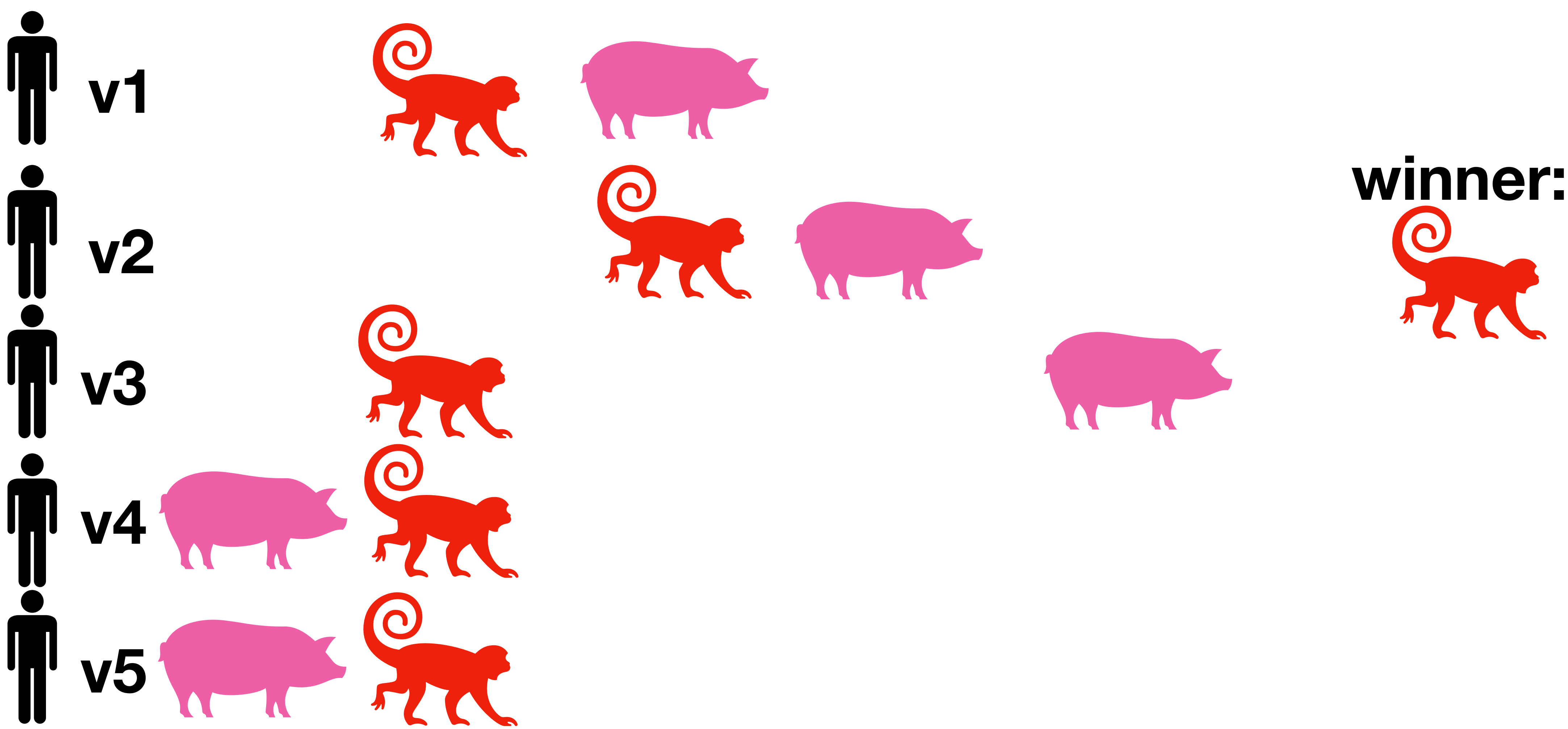


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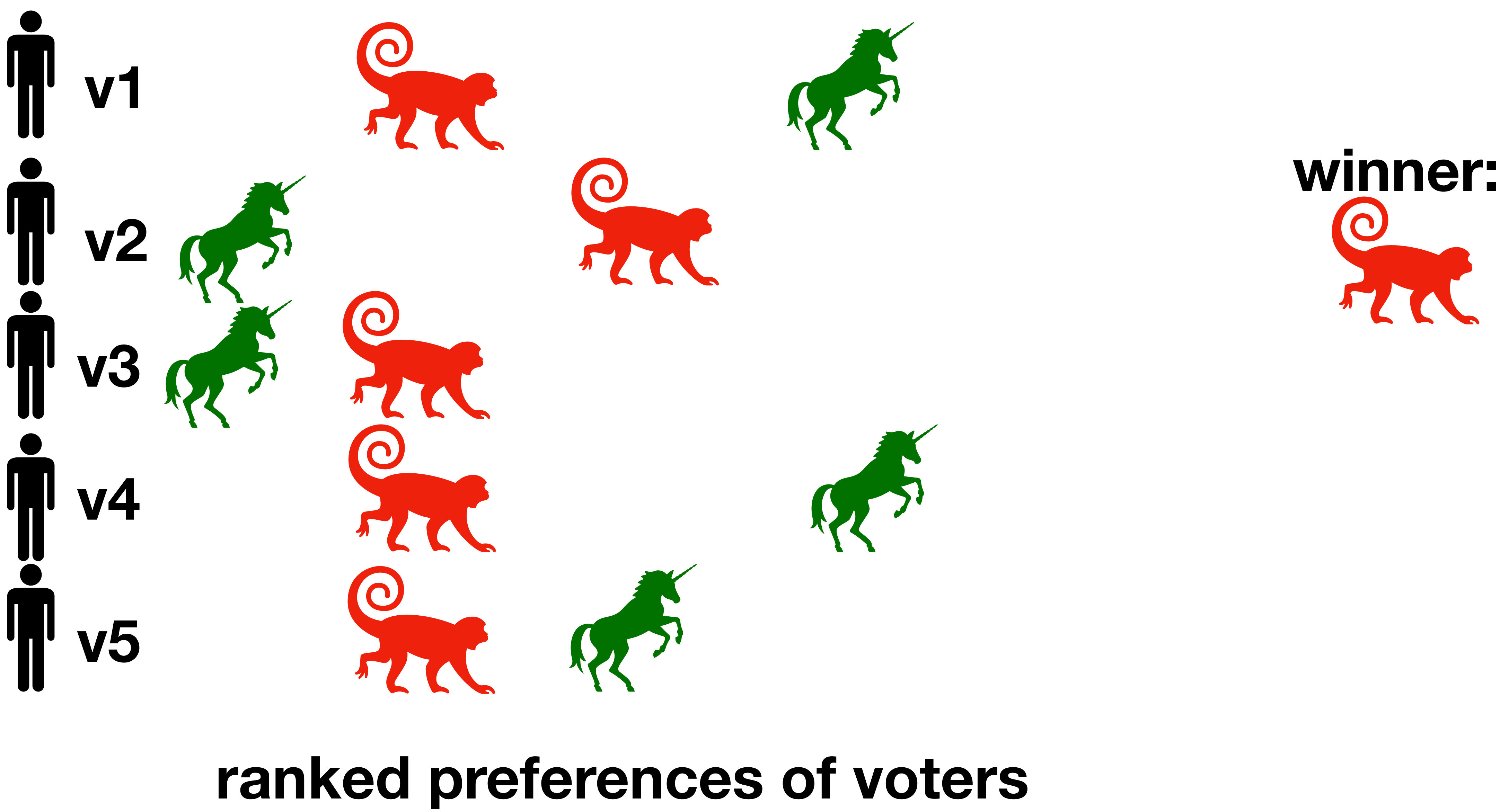

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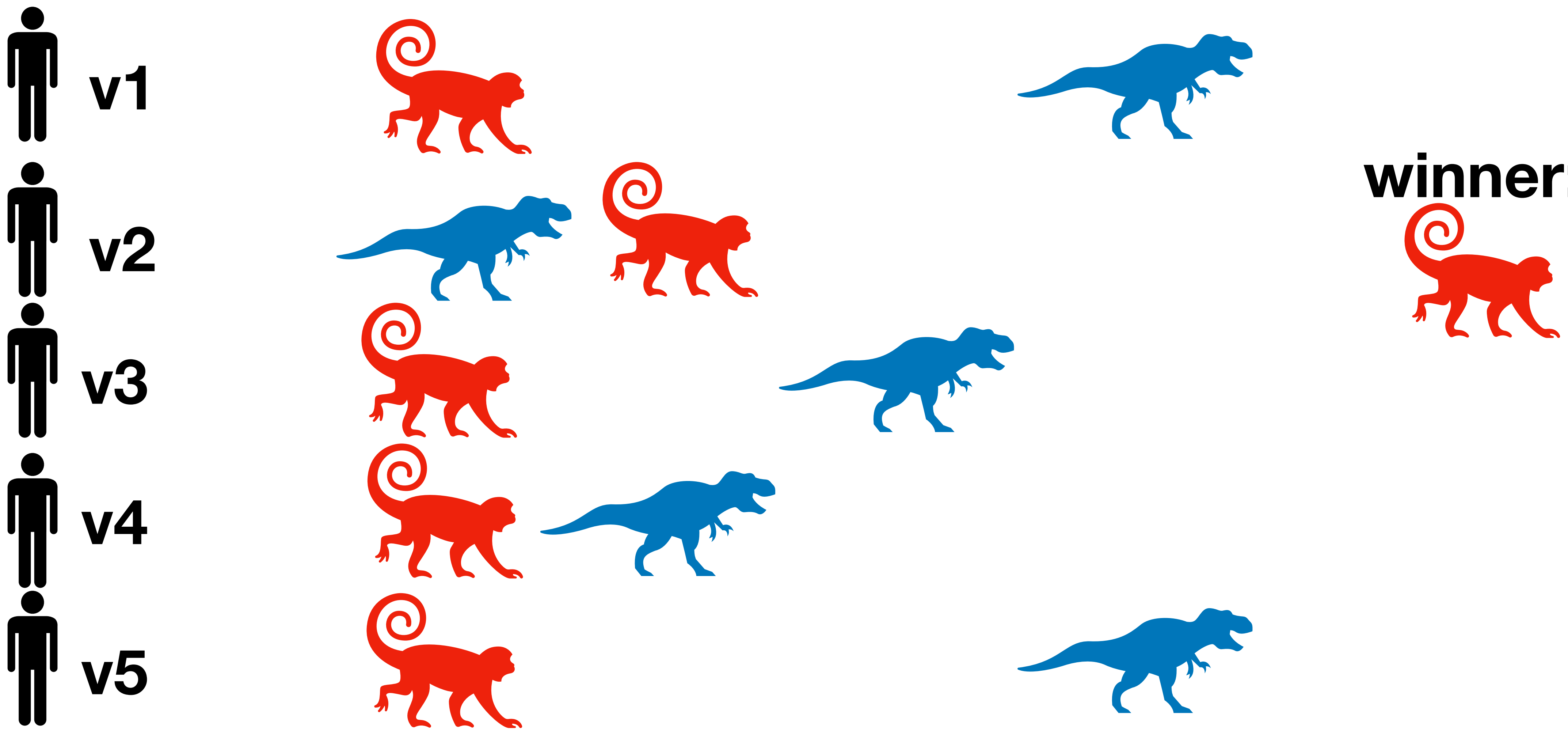
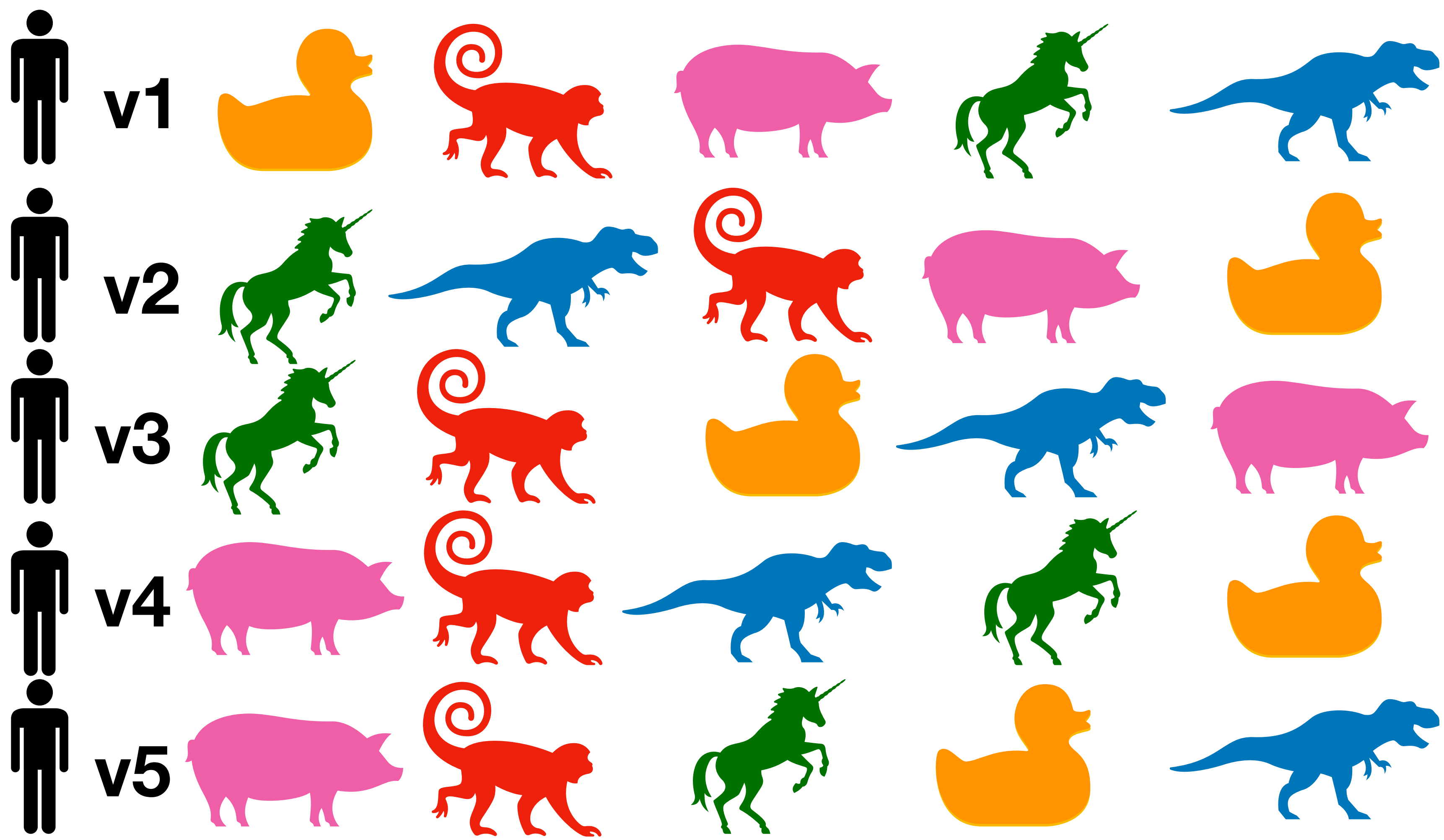
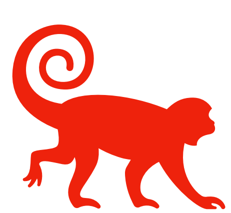



ranked preferences of voters

# Extending Condorcet to the whole ranking is easy…




**collective ranking:**



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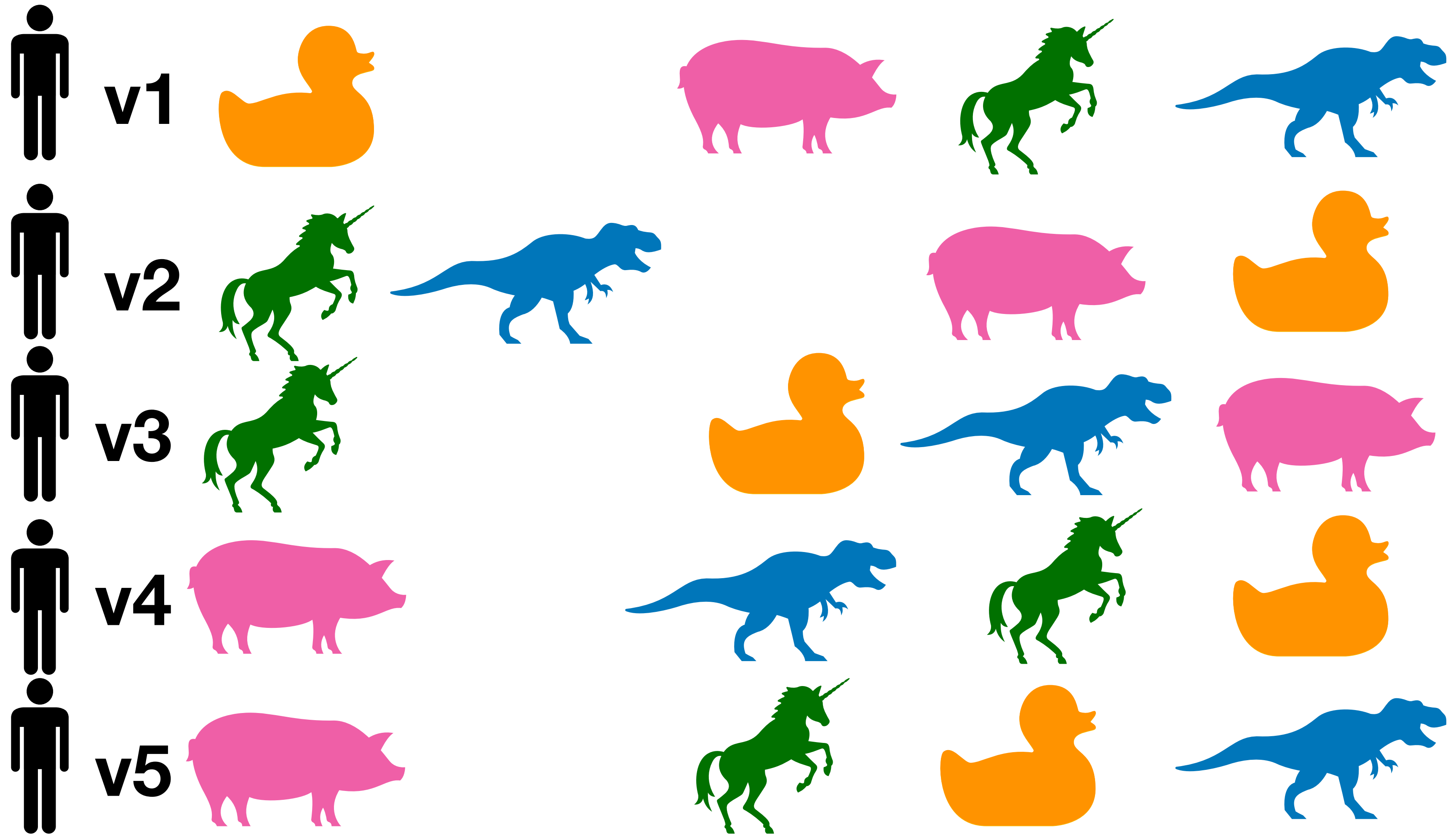


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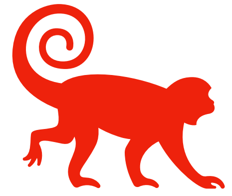

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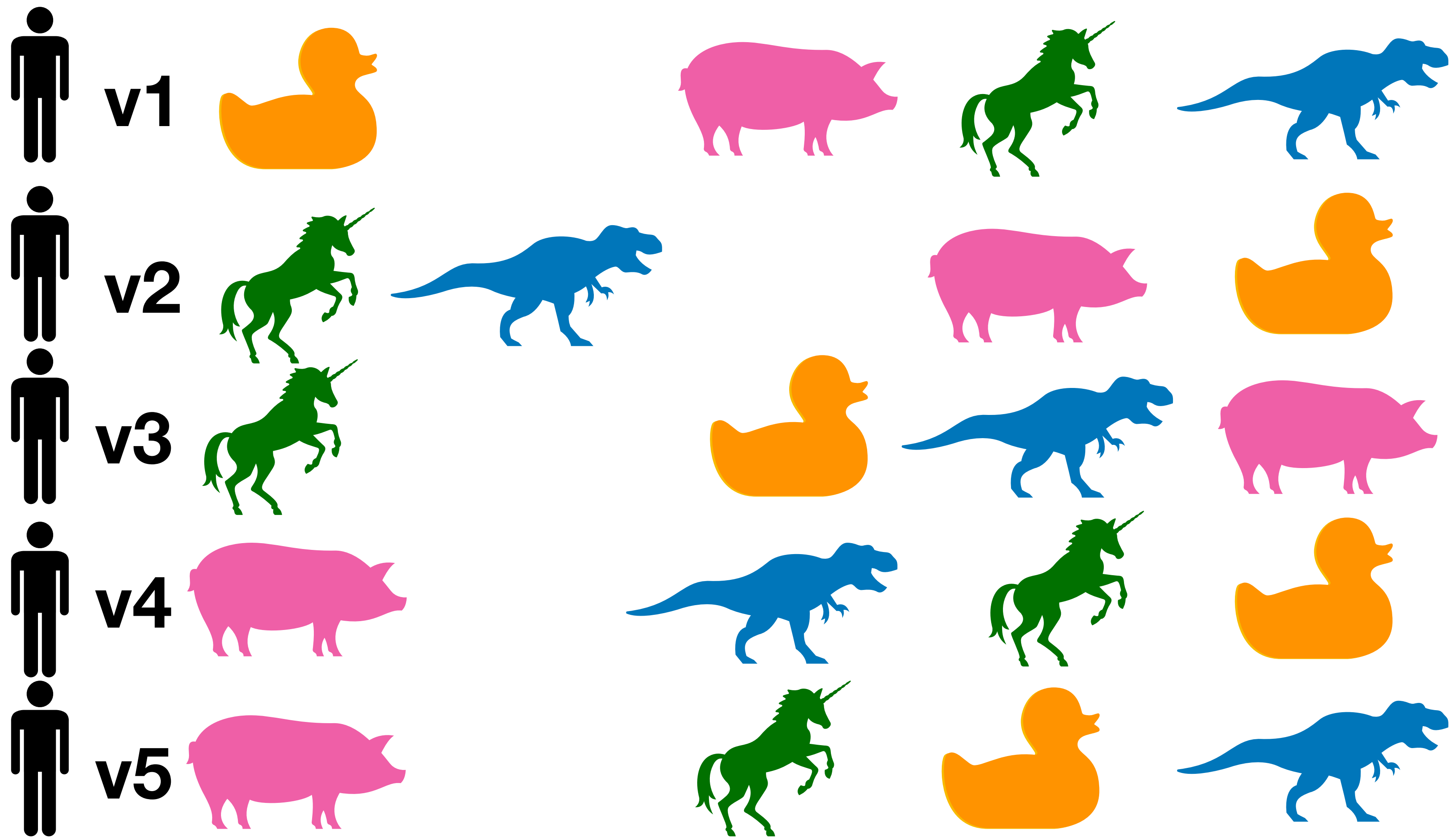


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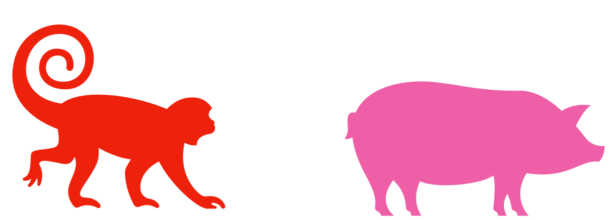
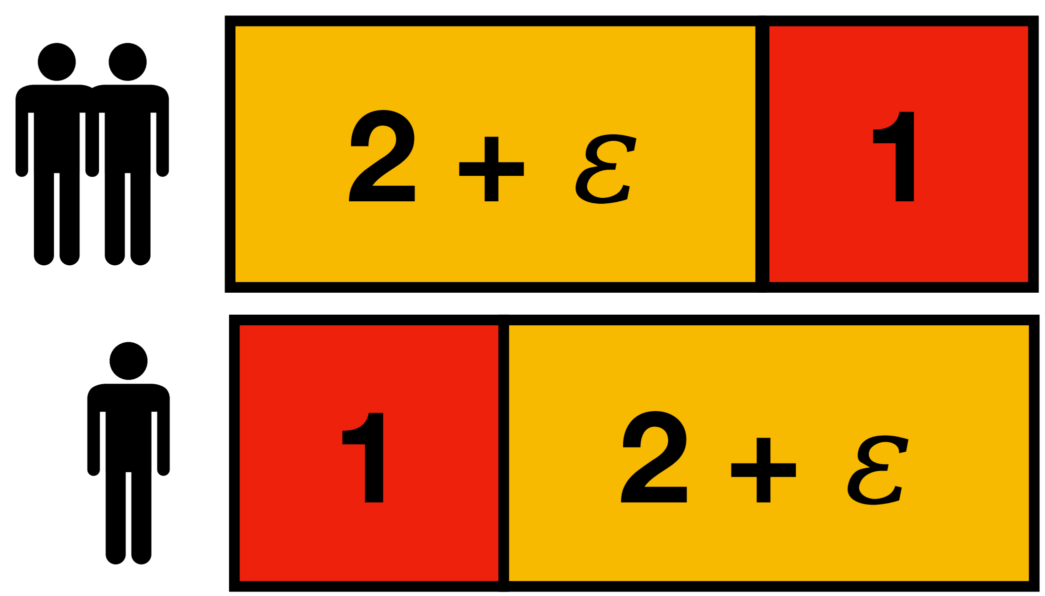
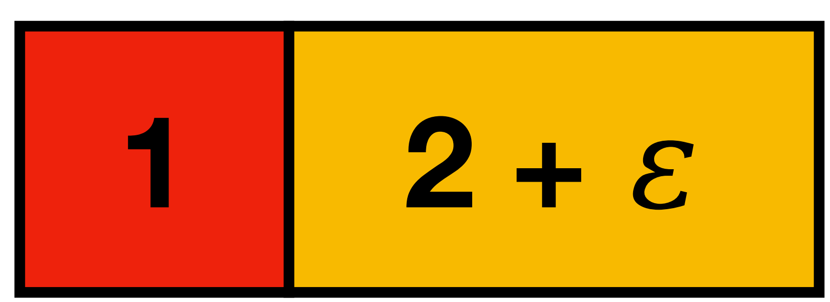


# Extending the Condorcet to processing times: PTA Condorcet

**Job k before job l if at least** $n\frac{p_k}{p_k + p_\ell}$ **voters put k before l**




**PTA Condorcet schedule:**



# Why the ratio? $n\frac{p_k}{p_k+p_\ell}$
# The utilitarian dissatisfaction

**$N_k$: agents who prefer k to l**

**Assume:**

$$|N_k| > n\frac{p_k}{p_k+p_\ell}$$

**If we start with k before l and then swap, k delayed by $p_l$**

**utilitarian dissatisfaction is $|N_k|p_l$**

**If we start with l before k and then swap, l delayed by $p_k$**

$$\begin{aligned}\mathrm{dis}(N_\ell) = |N_\ell|p_k &< \left(n - \frac{p_k}{p_k+p_\ell}n\right)p_k \\ &= n\cdot\frac{p_kp_\ell}{p_k+p_\ell} < |N_k|\cdot p_\ell = \mathrm{dis}(N_k).\end{aligned}$$

# PTA-Condorcet on the short-long example

| | | | |
|---|---|---|---|
| $3/8 + \varepsilon$ | L1 | L2 | S |
| $3/8 + \varepsilon$ | L2 | L1 | S |
| $1/8 - \varepsilon$ | S | L1 | L2 |
| $1/8 - \varepsilon$ | S | L2 | L1 |
| **Borda schedule:** | L2 | L1 | S |

**PTA Condorcet:**

S **before** L2 **in 1/4-$\varepsilon$ votes, thus**

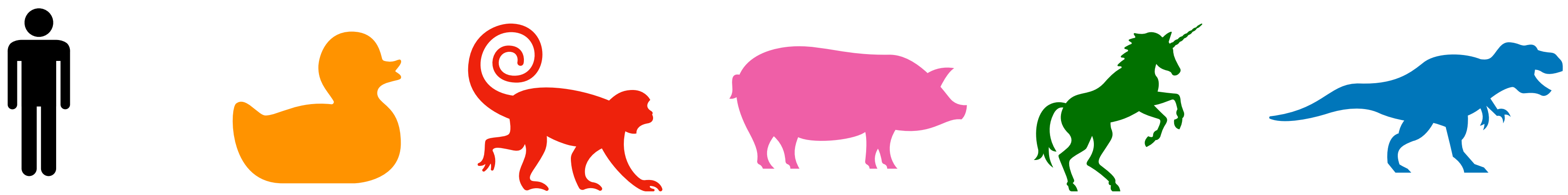
S L2 **if 1/4-$\varepsilon$ > s/(s+L2)**

**thus, for long L1, L2, PTA Condorcet schedule is**

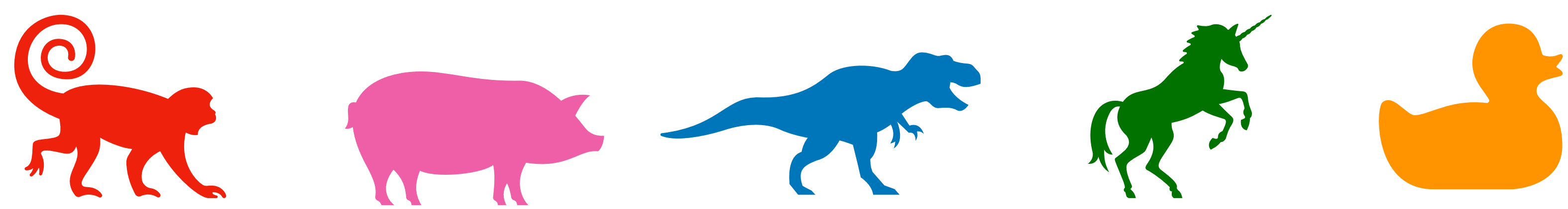
S L2 L1

# The Kemeny Rule

# Find a ranking minimizing the distance to voters' preferences



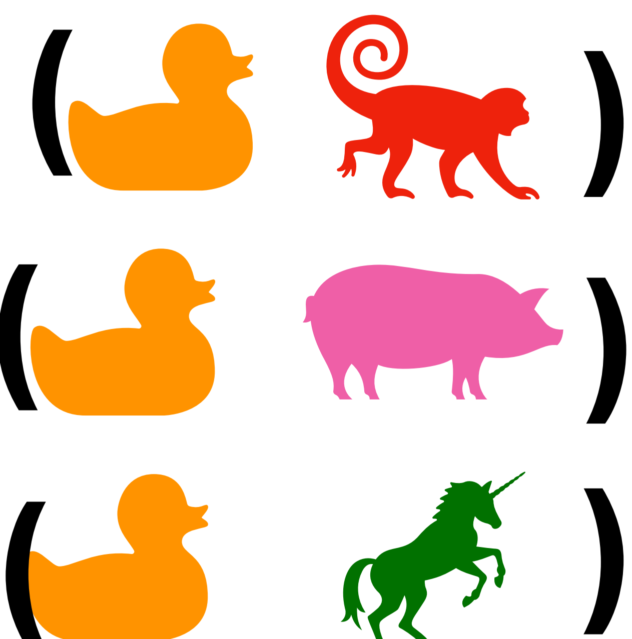
**the proposed ranking:**

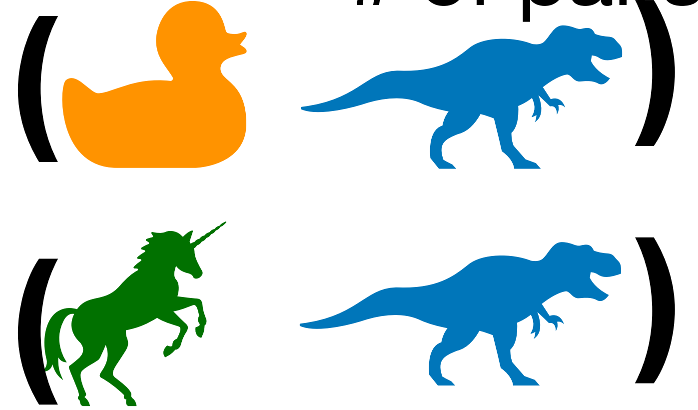


**The Kendall swap distance:**

# of swaps between neighbors to convert proposed to preferred

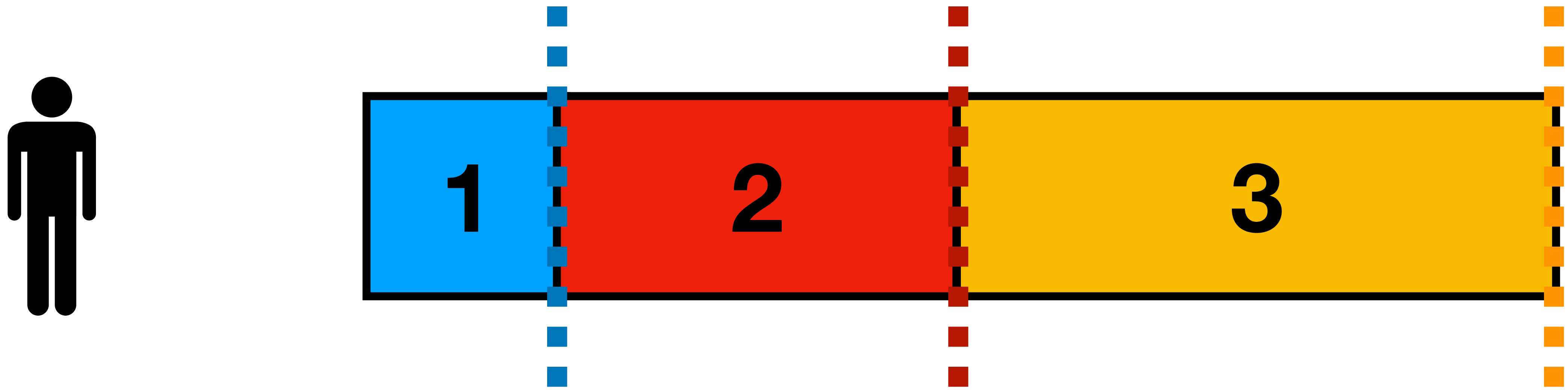
# of pairs in non-preferred order





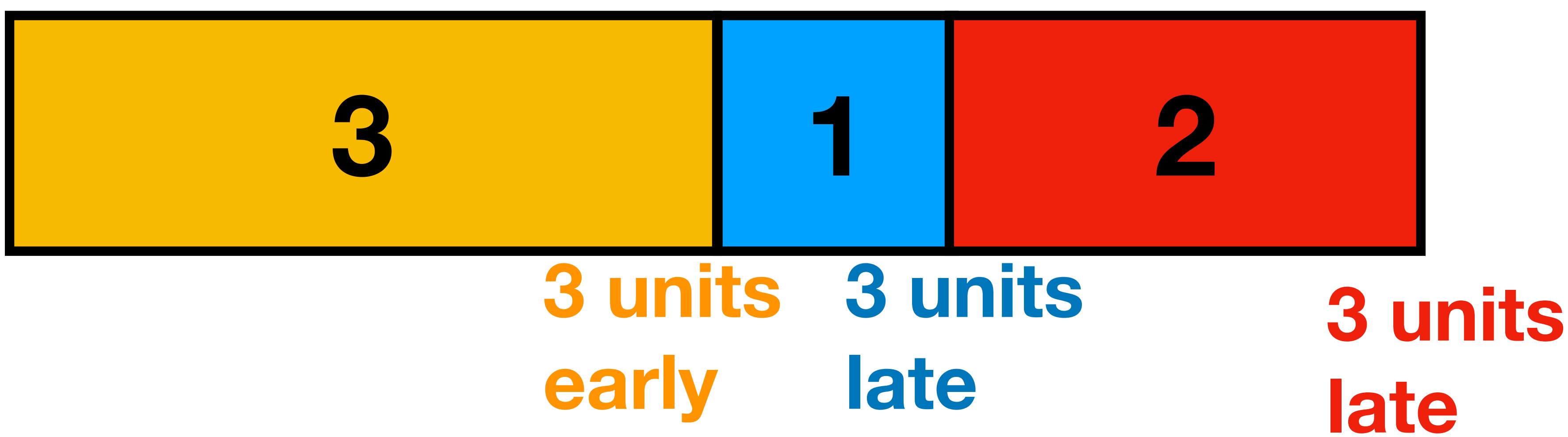
**Kendall distance is 5**

# Meaningful distances between two schedules



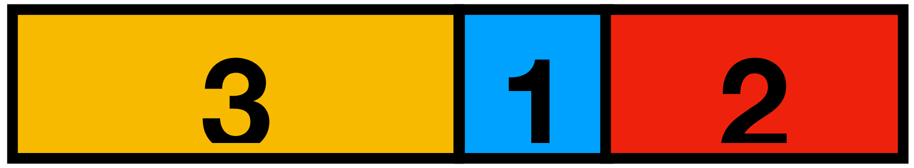

**The preferred schedule defines due dates for jobs**

**The proposed schedule:**



# Meaningful distances between two schedules


**The proposed schedule:**




**Quantifying the difference for each job by standard measures:**

**Tardiness (T)** : $T(c_i, d_i) = \max(0, c_i - d_i)$.

**Unit penalties (U)** : measure how many jobs are late:

$$U(c_i, d_i) = \begin{cases} 1 & \text{if } c_i > d_i \\ 0 & \text{otherwise.} \end{cases}$$

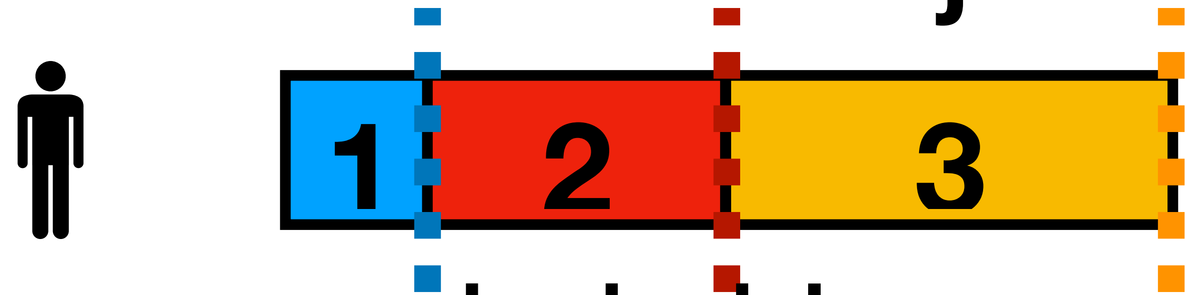
**Lateness (L)** : $L(c_i, d_i) = c_i - d_i$.

**Earliness (E)** : $E(c_i, d_i) = \max(0, d_i - c_i)$.

**Absolute deviation (D)** : $D(c_i, d_i) = |c_i - d_i|$.

**Squared deviation (SD)** : $SD(c_i, d_i) = (c_i - d_i)^2$.

# Aggregating distances over jobs and voters




**The proposed schedule:**


**aggregating over jobs: sum**

E.g. tardiness T: 3 + 3 + 0

**aggregating over voters:**

**The sum ($\Sigma$):** $\sum_{a\in N} f(\tau, \sigma_a)$, a utilitarian aggregation.

**The max:** $\max_{a\in N} f(\tau, \sigma_a)$, an egalitarian aggregation.

**The $L_p$ norm ($L_p$):** $\sqrt[p]{\sum_{a\in N} \left(f(\tau, \sigma_a)\right)^p}$,

# Our complexity results

| aggregation of voters' preferences | cost function | job sizes | complexity |
|---|---|---|---|
| $\sum$ | **L (lateness)** | arbitrary | poly<br>(SPT ordering!) |
| $\sum$ | **T (tardiness)** | arbitrary | strongly NP-hard |
| $\sum$ | **U**<br>**(# of late jobs)** | arbitrary | strongly NP-hard |
| $\sum$ | **T, U, L,**<br>**E, D, SD** | unit | poly<br>(assignment) |
| $\sum$ | **K,S**<br>**(Kemeny, Spearman)** | unit | NP-hard for 4 agents<br>[Dwork 2001] |

# Our complexity results

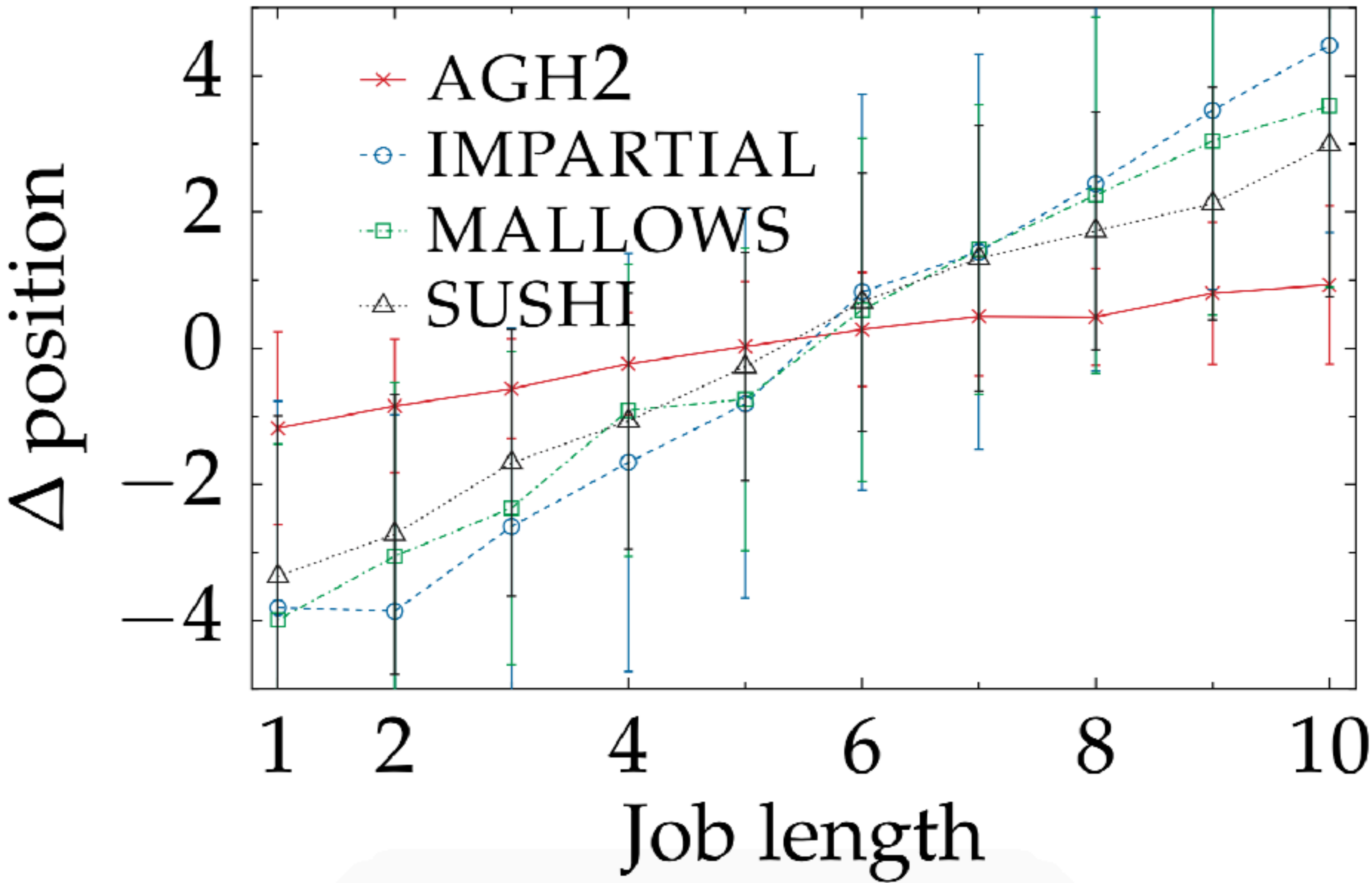
| aggregation of voters' preferences | cost function | job sizes | complexity |
|---|---|---|---|
| Lp norm (also max) | **T, E, D** | arbitrary | NP-hard for 2 agents (similar to [Agnetis04]) |
| max | **T, E, D, SD** | unit | NP-hard (from closest string) |

# Experimental evaluation

# Settings

- agents preferences from PrefLib
- Tardiness (T) as the cost function (strongly NP-hard, easy to interpret)
- Jobs' sizes random between 1 and $p_{max}$ (uniform, but we also tested normal and exponential)
- Optimal solutions computed by the Gurobi solver (a schedule encoded by binary precedence variables)
- 20 jobs, 5000 voters take minutes; 30 jobs doesn't finish in an hour

On the average, if jobs' lengths picked randomly, the short jobs are indeed advanced compared to a length-oblivious schedule



# PTA-Condorcet and Kemeny schedules are not that different

**# of job pairs executed in non-PTA-Condorcet order**

**relative difference of PTA vs Kemeny schedules**

| Dataset | PTA C. Paradox | | PTA Copeland $\cdot/\cdot$ | |
|---|---|---|---|---|
| | $\Sigma$-$T$ | max-$T$ | $\Sigma$-$T$ | max-$T$ |
| AGH1 | 6% | 15% | 1.03 | 1.23 |
| AGH2 | 5% | 18% | 1.03 | 1.28 |
| SUSHI | 7% | 24% | 1.02 | 1.22 |
| IMPARTIAL | 3% | 8% | 1.00 | 1.01 |
| MALLOWS | 10% | 24% | 1.03 | 1.21 |

# Collective Schedules

Fanny Pascual, **Krzysztof Rzadca**, Piotr Skowron

**AAMAS 2018** **arxiv.org/abs/1803.07484**

- How to take into account preferences of large population over possible schedules
- Each voter presents her preferred schedule
- Positional Scoring Functions may delay short jobs with significant support
- Processing Time Aware Condorcet is polynomial
- Kemeny-based methods are (mostly) NP-hard, but feasible for realistic instances