Excercise sheet 2.

In all of the below exercises, $\Omega \subset \mathbb{R}^n$, $n \geq 2$, is an open connected bounded set with a smooth boundary.

Ex. 1 Let $u : \Omega \to \mathbb{R}$ be such that its weak gradient is 0 a.e. in Ω. Prove that then u is constant on Ω.

Ex. 2 Let $u : \Omega \to \mathbb{R}$ be an element of $W^{1,1}(\Omega)$, $u^+(x) := \max\{u(x), 0\}$ and $u = u^+ - u^-$. Prove that the support of $Tr(u^+)$ is disjoint with a support of $Tr(u^-)$, where Tr is the trace operator.

Ex. 3 Let $u : \Omega \to \mathbb{R}$ be a nonnegative element of $W^{1,1}(\Omega)$, show that $Tr(u) \geq 0$.

Ex. 4 Let $u_{k+1} \geq u_k : \Omega \to \mathbb{R}$ be a monotone sequence of harmonic functions in Ω. Prove that if there exists $x \in \Omega$ such that $u_k(x) \to u(x)$, then u_k converges uniformly to a harmonic function u.