1 Praca domowa I

Termin oddawania: 22.03.2016.

Zadanie 1. Find an explicit formula for $u : \mathbb{R}^2 \to \mathbb{R}$ solving the problem

$$\begin{aligned} \partial_t u + x u_x + 4 u_y &= u \quad \text{in } \mathbb{R}^d \times (0,\infty), \\ u(0,x,y) &= e^{-(x^2+y^2)}. \end{aligned}$$

Zadanie 2. Modifying the proof of the mean value property, prove that if $d \geq 3$ and u is a solution of the problem

$$\begin{aligned} -\Delta u &= f \quad \text{in } \mathrm{int} B(0,r), \\ u &= g \quad \text{on } \partial B(0,r), \end{aligned}$$

then

$$u(0) = \frac{1}{|\partial B(0,r)|} \int_{\partial B(0,r)} g d\sigma + \frac{1}{d(d-2)\omega(d)} \int_{B(0,r)} \left(\frac{1}{|x|^{d-2}} - \frac{1}{r^{d-2}} \right) f dx.$$

By $\mathrm{int} A$ we denote the interior of A and by $\omega(d)$ we denote the measure of a unit ball in $\mathbb{R}^d$.

Zadanie 3. Let U be an open and bounded subset of $\mathbb{R}^d$ and let V be its connected subset such that $V \subset\subset U$ (i.e. the closure of V is a compact subset of U). Prove that there exists a constant C depending only on V such that

$$\sup_V u \leq C \inf_V u$$

for all nonnegative and harmonic in U functions u.

Zadanie 4. Let g be a continuous and bounded function. Provide an explicit formula for the solution to the problem

$$\begin{aligned} \Delta u &= 0 \qquad \text{in } \mathrm{int} B(0,1), \\ u &= g \qquad \text{on } \partial B(0,1). \end{aligned}$$

Hint: Proceed similarly to the case of the half-space. Consider the mapping $x \mapsto \frac{x}{|x|^2}$ to 'remove' the singularity.