Lucy Moser-Jauslin: Embeddings of certain affine surfaces in complex three-space

Abstract : This talk describes work in collaboration with P.-M. Poloni on embeddings of surfaces in complex three-dimensional space.

A Danielewski surface is an affine complex total space of a locally \mathbf{A}^1 -fibration over the scheme X_m , which is a copy of the affine line with a multiple number of zeros. Some Danielewski surfaces can be realized as hypersurfaces in \mathbf{C}^3 . For example for any n and m, the hypersurface in C^3 defined by $x^n y = z^m - 1$ is Danielewski surface over X_m . I will describe some of the properties of these embeddings.

We will start with a family of examples of inequivalent embeddings of Danielewski surfaces in \mathbb{C}^3 . We will show that there are examples of inequivalent embeddings of hypersurfaces where all fibers of the two embeddings are isomorphic. Moreover, we show that there are inequivalent embeddings which are stably equivalent. We will then give some generalizations of this family. The main tool used in this work, is that these surfaces have a non-trivial Makar-Limanov invariant.

The families described above give rise to interesting examples of automorphisms of $\mathbf{C}[x, y, z]$ which fix the variable x.