K. Masuda: G-endomorphisms with automorphisms of the quotients

Let G be a complex algebraic group and let $X = \operatorname{Spec} R$ be an affine G-variety. Let φ be a G-endomorphism of X which induces an automorphism of the invariant subring R^G. We show that φ is a G-automorphism when G is unipotent and X is factorial. When G is reductive, we show that φ is a G-automorphism if there exists a closed subset V of $X//G = \operatorname{Spec} R^G$ such that $\operatorname{codim}_X \pi^{-1}(V) \geq 2$ and every fiber of the quotient morphism $\pi : X \to X//G$ over $X//G - V$ consists of one closed orbit. Further, even when $\operatorname{codim}_X \pi^{-1}(V) = 1$, we show that φ is a G-automorphism under additional conditions.