Combinatorial Intersection Cohomology: A Survey

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For the characterization of the possible "f(ace)-vectors" $(f_0, ..., f_n) \in \mathbb{N}^{n+1}$ of a simple polytope $P \subset \mathbb{R}^n$ (i.e. the i-th component f_i is the number of i-dimensional faces of P), the h-vector plays a decisive role. Since any simple polytope is combinatorially equivalent to a lattice polytope, we may assume that there is an associated toric variety, and the components of the h-vector then are nothing but the Betti numbers of that variety. The generalized h-vector as introduced by Stanley for any polytope mimics the recursive algorithm for the computation of the intersection cohomology Betti numbers of a toric variety, but now its basic properties do not any longer follow from the theory of toric varieties, since there are non-simple polytopes not combinatorially equivalent to a lattice polytope. Instead one needs a combinatorial intersection cohomology for polytopes respectively fans. In my talk I shall briefly sketch the main features of that construction.