Exercises-2 (Default Logic)

The following method can be used to find extensions of closed default theories with finite sets of defaults. (For simplicity, we restrict ourselves to single-justification defaults.)

Let T=⟨W,D⟩ be a closed default theory with a finite set of defaults. Let PERM be the set of all permutations of elements from D.

If PERM={}, i.e. D={}, or if W is inconsistent, then return Th(W) as the only extension of T. Otherwise:

while PERM≠ {} **do**

begin

(1) Take any permutation perm=($d_1, \ldots, d_n$) from PERM;
PERM:=PERM–{perm};

(2) [**Initialization**]

BELIEFS:=W; JUSTIFICATIONS:={};

(3) [**Application of defaults+Consistency test**]

repeat

for i:=1 **to** n **do** {We assume that $d_i = A_i : B_i/C_i$.}

If BELIEFS$\vdash A_i$ and BELIEFS$\nvdash \neg B_i$, **then**

begin

BELIEFS:=BELIEFS$\cup\{C_i\}$;

JUSTIFICATIONS:=JUSTIFICATIONS$\cup\{B_i\}$;

If there is A ∈JUSTIFICATIONS such that BELIEFS$\vdash \neg A$, **then** **goto** (5)

end

until the set BELIEFS is *stable* i.e. it has not changed during the “for” loop;

(4) **return** Th(BELIEFS) as an extension;

(5)

end.

Find extensions of the following default theories:

1. $$T = \left\langle \{R(n) \wedge Q(n)\}, \left\{ \frac{R(x) : \neg P(x)}{\neg P(x)}, \frac{Q(x) : P(x)}{P(x)} \right\} \right\rangle.$$

2. $$T = \left\langle \{\neg Sun - Shining \wedge Summer\}, \left\{ \frac{Summer : \neg Rain}{Sun - Shining} \right\} \right\rangle.$$

3. $$T = \left\langle \{\}, \left\{ \frac{: p}{p}, \frac{p \vee q : \neg p}{\neg p} \right\} \right\rangle.$$

4. $$T = \left\langle \{p \vee q\}, \left\{ \frac{: p}{p}, \frac{p \vee q : \neg p}{\neg p} \right\} \right\rangle.$$

5. $$T = \left\langle \{\}, \left\{ \frac{r : \exists x\ P(x)}{\exists x\ P(x)}, \frac{: r \wedge \neg P(x)}{r \wedge \neg P(x)} \right\} \right\rangle.$$

6. $$T = \left\langle \{\}, \left\{ \frac{: p \wedge \neg q}{p}, \frac{: q \wedge \neg s}{q}, \frac{: (r \supset s) \wedge \neg p}{r \supset s} \right\} \right\rangle.$$

7. $T = \langle \mathrm{W},\mathrm{D} \rangle$, where:

$$\begin{aligned} \mathrm{W} = \{ & \forall x.\ Mynah(x) \supset \neg Nests(x) \\ & \forall x.\ Penguin(x) \supset \neg Flies(x) \\ & \forall x.\ Bird(x) \equiv Mynah(x) \vee Penguin(x) \vee Canary(x) \\ & Bird(Tweety) \} \end{aligned}$$

$$\mathrm{D} = \left\{ \frac{Bird(x) : Nests(x)}{Nests(x)}, \frac{Bird(x) : Flies(x)}{Flies(x)} \right\}.$$

Solutions

1. $$T = \left\langle \{R(n) \wedge Q(n)\}, \left\{ \frac{R(x) : \neg P(x)}{\neg P(x)}, \frac{Q(x) : P(x)}{P(x)} \right\} \right\rangle.$$

$$\text{CLOSED}(T) = \left\langle \{R(n) \wedge Q(n)\}, \left\{ \frac{R(n) : \neg P(n)}{\neg P(n)}, \frac{Q(n) : P(n)}{P(n)} \right\} \right\rangle.$$

- perm=(d_1, d_2).

BELIEFS	**JUSTIFICATIONS**	**Consistency test**
$R(n) \wedge Q(n)$	$\{\}$	
$\neg P(n)$	$\neg P(n)$	OK
stable		

- perm=(d_2, d_1).

BELIEFS	**JUSTIFICATIONS**	**Consistency test**
$R(n) \wedge Q(n)$	$\{\}$	
$P(n)$	$P(n)$	OK
stable		

In conclusion, T has two extensions:[1]

$$\text{Th}(\{R(n) \wedge Q(n), \neg P(n)\}) \text{ and } \text{Th}(\{R(n) \wedge Q(n), P(n)\}).$$

2. $$T = \left\langle \{\neg Sun - Shining \wedge Summer\}, \left\{ \frac{Summer : \neg Rain}{Sun - Shining} \right\} \right\rangle.$$

- perm=(d_1).

BELIEFS	**JUSTIFICATIONS**	**Consistency test**
$\neg Sun - Shining \wedge Summer$	$\{\}$	
$Sun - Shining$	$\neg Rain$	FAILS

In conclusion, T lacks an extension.

[1]Note that $L_T = L_{\text{CLOSED}(T)}$.

3. $$T = \left\langle \{\}, \left\{ \frac{: p}{p}, \frac{p \vee q : \neg p}{\neg p} \right\} \right\rangle .$$

- perm=(d_1, d_2).

BELIEFS	JUSTIFICATIONS	Consistency test
$\{\}$	$\{\}$	
p	p	OK
stable		

- perm=(d_2, d_1).

BELIEFS	JUSTIFICATIONS	Consistency test
$\{\}$	$\{\}$	
$\{\}$	$\{\}$	
p	p	OK
stable		

In conclusion, T has one extension: $\mathrm{Th}(\{p\})$.

4. $$T = \left\langle \{p \vee q\}, \left\{ \frac{: p}{p}, \frac{p \vee q : \neg p}{\neg p} \right\} \right\rangle .$$

- perm=(d_1, d_2).

BELIEFS	JUSTIFICATIONS	Consistency test
$p \vee q$	$\{\}$	
p	p	OK
stable		

- perm=(d_2, d_1).

BELIEFS	JUSTIFICATIONS	Consistency test
$p \vee q$	$\{\}$	
$\neg p$	$\neg p$	OK
stable		

In conclusion, T has two extensions:

$\mathrm{Th}(\{p\})$ and $\mathrm{Th}(\{\neg p, q\})$.

5. $$T = \left\langle \{\}, \left\{ \frac{r : \exists x\ P(x)}{\exists x\ P(x)}, \frac{: r \wedge \neg P(x)}{r \wedge \neg P(x)} \right\} \right\rangle.$$

$$\text{CLOSED}(T) = \left\langle \{\}, \left\{ \frac{r : \exists x\ P(x)}{P(a)}, \frac{: r \wedge \neg P(a)}{r \wedge \neg P(a)} \right\} \right\rangle.$$

- perm=(d_1, d_2).

BELIEFS	JUSTIFICATIONS	Consistency test
$\{\}$	$\{\}$	
$\{\}$	$\{\}$	
$r \wedge \neg P(a)$	$r \wedge \neg P(a)$	OK
$P(a)$	$\exists x\ P(x)$	FAILS

- perm=(d_2, d_1).

BELIEFS	JUSTIFICATIONS	Consistency test
$\{\}$	$\{\}$	
$r \wedge \neg P(a)$	$r \wedge \neg P(a)$	OK
$P(a)$	$\exists x\ P(x)$	FAILS

In conclusion, T lacks an extension.

6. $$T = \left\langle \{\}, \left\{ \frac{: p \wedge \neg q}{p}, \frac{: q \wedge \neg s}{q}, \frac{: (r \supset s) \wedge \neg p}{r \supset s} \right\} \right\rangle.$$

To restrict the number of permutations to consider, note:

(1) If we start with the first default, we shall be forced to apply the second one, but applying the latter contradicts the justification of the former.

(2) If we start with the third default, and then apply the first one, the consequent of the latter will contradict the justification of the former.

In view of these observations, there are three permutations to analyze:

- perm=(d_2, d_1, d_3).

BELIEFS	JUSTIFICATIONS	Consistency test
$\{\}$	$\{\}$	
q	$q \wedge \neg s$	OK
$r \supset s$	$(r \supset s) \wedge \neg p$	OK
stable		

perm=(d_2, d_3, d_1).

BELIEFS	**JUSTIFICATIONS**	**Consistency test**
$\{\}$	$\{\}$	
q	$q \wedge \neg s$	OK
$r \supset s$	$(r \supset s) \wedge \neg p$	OK
stable		

perm=(d_3, d_2, d_1).

BELIEFS	**JUSTIFICATIONS**	**Consistency test**
$\{\}$	$\{\}$	
$r \supset s$	$(r \supset s) \wedge \neg p$	OK
q	$q \wedge \neg s$	OK
stable		

In conclusion, T has one extension: $\mathrm{Th}(\{q, r \supset s\})$.

7. $T = \langle \mathrm{W}, \mathrm{D} \rangle$, where:

$$\begin{aligned} \mathrm{W} = \{&\forall x.\ Mynah(x) \supset \neg Nests(x) \\ &\forall x.\ Penguin(x) \supset \neg Flies(x) \\ &\forall x.\ Bird(x) \equiv Mynah(x) \vee Penguin(x) \vee Canary(x) \\ &Bird(Tweety)\} \end{aligned}$$

$$\mathrm{D} = \left\{ \frac{Bird(x) : Nests(x)}{Nests(x)}, \frac{Bird(x) : Flies(x)}{Flies(x)} \right\}.$$

$$\mathrm{CLOSED}(T) = \left\langle \mathrm{W}, \left\{ \frac{Bird(Tweety) : Nests(Tweety)}{Nests(Tweety)}, \frac{Bird(Tweety) : Flies(Tweety)}{Flies(Tweety)} \right\} \right\rangle.$$

- perm=(d_1, d_2).

BELIEFS	**JUSTIFICATIONS**	**Consistency test**
Axioms	$\{\}$	
$Nests(Tweety)$	$Nests(Tweety$	OK
$Flies(Tweety)$	$Flies(Tweety)$	OK
stable		

- perm=(d_2, d_1).

BELIEFS	JUSTIFICATIONS	Consistency test
Axioms	{}	
$Flies(Tweety)$	$Flies(Tweety)$	OK
$Nests(Tweety)$	$Nests(Tweety$	OK
stable		

In conclusion, T has one extension:

$$E{=}Th(Axioms \cup \{Nests(Tweety), Flies(Tweety)\}).$$

NOTE: E contains $Canary(Tweety)$!!