Anthony Bak

G - surgery with middle dimensional fixed point sets and applications

We describe for a finite group G, G-surgery on compact smooth $2n$-dimensional manifolds M $(n \geq 3)$, with n-dimensional fixed point sets. Previous accounts of G-surgery have routinely excluded fixed point sets of dimension $\geq n$, under the so called gap hypothesis. This assumption allows one to separate apriori M-immersed n-dimensional spheres from the manifolds in M^H $(1 \neq H \subset G)$. In our more general situation, we can guarantee apriori only that there are a finite number of 0-dimensional overlaps between an immersed n - dimensional sphere and the manifolds in M^H $(1 \neq H \subset G)$. Let $f_1, ..., f_k : S^n \to M$ be a set of immersions generating the homology kernel of a G-surgery problem. We describe in terms of a new surgery obstruction invariant necessary and sufficient conditions when one can simultaneously eliminate all overlaps between the $f_i{}'$s, including self overlaps , and between the $f_i{}'$s and the manifolds in M^H $(1 \neq H \subset G)$. This gives necessary and sufficient conditions when one can perform G-surgery. The resulting G-surgery theory is used to construct smooth G-actions on manifolds M such that M^G is finite with a prescribed order.

Stanisław Betley

K-theory of a square-zero ideal extesion. Homological computations via stable K-theory.

Let $T : \mathcal{C}^{op} \times \mathcal{C} \to S\text{-}mod$ be a functor, where $\mathcal{C}$ is a category of finitely generated free R-modules. Let stable K-groups of R with coefficients in T be defined as

$$K_p^s(R, T) = H_p(F, t)$$

where $t = colim\ T(R^n, R^n)$ and F is a fiber of the "+"-construction $BGL(R) \to BGL(R)^+$.

Conjecture: *$K_p^s(R, .)$ is the p-th derived functor of $K_0^s(R, .)$.*

We prove this conjecture for some R and S. As an easy consequence we get:

Theorem:

$$H_n(GL(R),t) = \bigoplus_{i+j=n} H_i(GL(R), K_j^s(R,T))$$

where the action of $GL(R)$ on $K_j^s(R,T)$ is trivial.

Let now $I \to R \to R/I$ be an extension of rings with a square zero kernel I.

Conjecture:

$$H_q(G) = \bigoplus_{\substack{i+j=q \\ i\neq 0}} K_j^s(R,T_i)$$

where G is a fiber of the map $BGL(R)^+ \to BGL(R/I)^+$ (G is a space of relative K-theory $K(R,I)$) and $T_i = H_i(M(I))$ (where $M(I)$ is a full group of matrices with entries in I).

We prove this conjecture in some cases.

Frank X. Connolly
Spherical Euclidean Space Form Groups and K-theory
(joint work with D. Anderson)

All groups Γ in this discussion are assumed to satisfy: $vcd(\Gamma) < \infty$.

An (m,n)-space form group is a group Γ which acts freely, properly discontinuously and with compact quotient on $\mathbf{R}^n \times S^{n-1}$.

In this work we prove and apply a vanishing theorem for the Wall - Swan finiteness obstructions of the finite subgroups of such Γ. We employ the "continuously controlled K-groups", $K_i^G(\bar{E},\Sigma)$, defined $\forall i \geq 0$, where G is a finite group, and $(\bar{E},\Sigma)$ is a compact G-pair. There is a natural map $tr : K_i(\mathbf{Z}G) \to K_i^G(\bar{E},\Sigma)$. It is not always zero.

Vanishing Theorem 1: *Suppose Γ is a group acting freely, cellularly, and with compact quotient on a CW complex X. Assume X/G is finitely dominated for each finite subgroup $G \subset \Gamma$. Assume Γ has a "continuously controlled classifying pair $(\bar{E},\Sigma)$". Then the finiteness obstruction, $\sigma(X,G)$*

is in the kernel of the map tr : $K_i(\mathbf{Z}G) \to K_i^G(\bar{E}, \Sigma)$, *for each finite subgroup* $G \subset \Gamma$.

(The terms in quotes will be defined in the lecture.)

Application

Suppose Γ is an (m, n)-space form group. The following facts are known:

A_m) $m = vcd(\Gamma)$. Γ is a virtual Poincaré duality group.

B_n) n is a period for the Farrell cohomology of Γ, $\hat{H}^*(\Gamma; \mathbf{Z})$.

C) For p prime, if $D_{2p} = C_p \tilde{\times} C_2 \subset \Gamma$ then $vcd(Z_\Gamma(C_p)) - vcd(Z_\Gamma(D_{2p})) \leq 2$.

Conversely I.Madsen, C.Thomas and C.T.C.Wall proved:

$A_0 + B_n + C \Rightarrow \Gamma$ is a $(0, n)$ or $(0, 2n)$ space form group.

Ronnie Lee exibited a group Γ and integers n so that Γ satisfies $A_0 + B_n + C$ but Γ is not a $(0, n)$ space form group.

We prove the following application of Theorem 1:

Theorem 2: *For each* $m \equiv 0$ *mod* 48 *and* $n \equiv 4$ *mod* 8 *there is a crystallographic group* Γ *such that :*

Γ *satisfies* $A_m + B_n + C$.

Γ *is* *not* *an* (m, n) *space group.*

Γ *is* *an* $(m, 2n)$ *space group.*

Wojciech Gajda

Divisible elements in K-theory of number fields.

(joint work with Grzegorz Banaszak)

Let F be a number field and $\mathcal{O}_F$ its ring of integers. We discuss certain aspects of K-theory which are related to conjectures of Quillen and Lichtenbaum. It is known that the conjectures in case of $\mathbf{Z}$ imply certain important conjectures concerning arithmethic of cyclotomic fields, e.g.,Vandiver's conjecture.

We use Euler systems of Kolyvagin and Rubin to define new elements in K-groups of number fields. The elements may be applied to bound size of a subgroup in $K_{2m}(\mathcal{O}_F)$ by a number which appears in Quillen - Lichtenbaum conjecture. The subgroup consist of elements which are divisible in the - theory of $\mathcal{O}_F$. The main result of the talk gives a formula in K-theory which resembles a class number formula of Kummer for the second factor of the class number of the cyclotomic field $\mathbf{Q}(\xi_p)$. Here p is a prime number and $\xi_p = e^{\frac{2\pi i}{p}}$.

Karsten Grove

Curvature of singular spaces and their role in geometry.

An Alexandrov space X is a complete finite (Hausdorff) dimensional inner metric space (length space) with curvature bounded below in (local) distance comparison sense.

For each $p \in X$, the tangent space $T_pX = \lim_{\substack{\text{Gromov}-\text{Hausdorff} \\ \lambda \to \infty}} (\lambda X, p)$ is the euclidean cone on the space of directions, S_pX to X at p. S_pX, which in turn is the completion (relative to a canonically notion of angle) of the germs of geodesics emanating at p, is an Alexandrov space itself with $curv\ S_pX \geq 1$ and $dim\ S_pX = dim\ X - 1$ (Burago, Gromov, Perelman, Plaut). Moreover, for each $p \in M$ there is an $\epsilon = \epsilon(p) > 0$ such that the open ϵ-ball centered at p is homeomorphic to T_pX.

The importance of Alexandrov space to riemannian geometry is in part explained by the fact that the class of Alexandrov spaces is closed under the operation of :

(1) Taking Gromov - Hausdorff limits

(2) Taking quotiens by an isometric group action

(3) Taking (spherical) suspension (or more generally joins) of positively curved space.

For example (1) has been used to prove finiteness-, recognition-, and structure theorems, (2) has been used in classification of positively curved manifolds with large isometry groups, and (3) has been used in differentiable packing problems for packing radii. The methods of (1), (2) and (3) are entirely different from one another.

Karl Gruenberg

Relative genus class groups

An old result of Maclane -Whitehead states : if X, Y are m-dimensional finite connected CW-complexes whose universal covers are $m-1$- connected, then X is homotopically equivalent to Y if and only if the fundamental groups are isomorphic and $C(\tilde{X}) \sim^{\pi_1} C(\tilde{Y})$ (the cellular chain complexes of the universal covers are equivariantly homotopically equivalent).

Based on (unpublished) works of W.Browning, I shall show how the chain homotopy classes of truncated projective resolutions of $\mathbf{Z}$ for a given finite group G can be parametrised by the elements of a naturally occuring group in the K-theory of G. This provides a K-theoretic parametrisation of the spaces considered by MacLane and Whitehead.

There are other applications of the K-theoretic framework.

Hans - Werner Henn

The Borel construction of the singular locus and localizing away from finite unstable modules.

Let G be a compact Lie group. Jackowski and McClure showed that there is a mod p equivalence

$$hocolim_{\mathcal{A}(G)^{op}} EG \times_G G/C_G(E) \to BG$$

Here $\mathcal{A}(G)$ denotes Quillen's category of nontrivial elementary abelian p -subgroups of G.

If $\mathcal{A}(G)$ is a G-space let X_{p-sing} consist of all elements in X whose isotropy group contains a nontrivial element of order p.

Theorem: *The natural map*

$$hocolim_{\mathcal{A}(G)^{op}} EG \times_G (G \times_{C_G(E)} X^E) \to EG \times_G X_{p-sing}$$

is a mod p equivalence.

If X is a finite G-CW complex we show that the cohomology spectral sequence of this hocolim almost collapses. This depends in a crucial way on studying $H^*(EG \times_G X; \mathbf{F}_p)$ as an unstable modul over the Steenrod algebra.

Finally we apply these results to studying $H^*(SL(3, \mathbf{Z}[\frac{1}{2}]; \mathbf{F}_2)$.

Soren Illman

Every proper smooth action of a Lie group is equivalent to a real analytic action : A contribution to Hilbert's fifth problem

We prove that if a Lie group G acts on a smooth manifold M by a proper and smooth action then there exists a real analytic structure β on M, compatible with the given smooth structure on M, such that the action of G on M_β is real analytic. By smooth we here mean C^∞, but we could as well consider the C^r case, $r \geq 1$, and the result still holds by essentially the same proof.

We shall now discuss Hilberts fifth problem, following the discussion in Montgomery - Zippin. We quote: "Let us now considers the following questions, the second and third of which are asked by Hilbert:

If a locally compact group G acts effectively on a manifold M (locally-euclidean space) then

1. is G necessarily locally euclidean,

2. if the group G is locally euclidean, is it a Lie group in some appropriate coordinates.

3. If G is a Lie group, can coordinates be chosen in G and M so that the transforming functions are analytic?

The answer to 2. which can be asked, of course, without mentioning transformation groups, is yes and was first solved by the contents of two papers one by Gleason, the other by Montgomery and Zippin."

At the time of the appearance of the book by Montgomery and Zippin the answer to 1. was unknown, except in some special cases, and as far as I know the answer to 1. is still an open question, in any case there has not appeared in print any paper containing a complete solution of 1.

Concerning 3. Montgomery and Zippin give the following discussion. "The answer to 3. is no. For example a group of reals can act on $\mathbf{R}^2$ by having fixed $x^2 + y^2 \leq 1$ and slowly rotating the rest of $\mathbf{R}^2$. This can not be analytic since if it where the existence of an open set of fixed points would imply that all points were fixed. The answer to 3. is no even if G is compact as was first shown by Bing by an example of the cyclic group of order two acting on $\mathbf{R}^3$ in a way that could not be differentiable."

More recently it has been shown that the answer to 3. is no also in the case of actions of connected compact Lie groups. For example there exists a compact smoothable 12-dimensional manifold M^{12} which admits an effective, continuous S^1-action, but which admits no nontrivial smooth S^1-action in any differentiable structure, see Bredon. Here one can use the powerful theorem of Atiyah and Hirzebruch, which says that if a compact connected oriented smooth manifold M, of dimension $4m$ and with $w_2(M) = 0$, admits a non-trivial smooth action of S^1, then $\hat{A}(M) = 0$.

Our theorem shows that the answer to 3. is yes if the action of G on M is proper and smooth. The example given by Montgomery and Zippin with the group of reals acting in $\mathbf{R}^2$ can be chosen to be smooth, but it is not a proper action.

The fact that every smooth manifold has a compatible real analytic structure was first proved by Whitney. It is also known that if two real analytic manifolds are smoothly diffeomorphic then they are also real analytically diffeomorphic. The proof of this fact requires the use of Grauert's imbedding theorem, which says that every real analytic manifold has a real analytic manifolds then $C^\omega(M, N)$ is dense in $C^r(M, N)$, $0 \leq r < \infty$. (Here the function spaces $C^r(M, N)$ and $C^\omega(M, N)$ have the Whitney topology, i.e., the strong topology.)

In the case of smooth manifolds with symmetries there are earlier known results about raising the differentiability in the case when the group of symmetries is a compact Lie group, see Palais and Matumoto - Shiota. Palais proves that every C^r, $0 \leq r < \infty$, action of a compact Lie group on a compact manifold is C^r-equivalent to a C^∞-action. Matumoto and Shiota use essentially the same method as Palais and their Theorem 1.3 gives the result that every C^r, $0 \leq r < \infty$ action on a C^r manifold is C^r-equivalent to a C^ω action, i.e., a real analytic action, on a C^ω manifold. Matumoto and Shiota also show that if M and N are two real analytic H-manifolds, where H is a compact Lie group, such that the number of H-isotropy types occuring in M and N is finite, then they are real analytically H-diffeomorphic if they are C^1 H-diffeomorphic.

In the case of proper actions of a non - compact Lie group G, the question whether the obtained proper real analytic G-manifold M_β is unique up to a real analytic G-equivariant diffeomorphism is a very intriguing question. We are only able to prove that the G-manifold M_β is unique up to a sub-analytic G-equivariant C^∞ smooth diffeomorphism. (This result is however good enough for some applications that we have in mind.)

Our main result implies that we can use the subanalytic equivariant triangulation theorem for proper real analytic G-manifolds, also in the case of proper smooth actions. In this context our uniqueness result is also sufficiently strong.

Tadeusz Januszkiewicz

Nonclassical aspherical manifolds

I will discuss manifolds of nonpositive curvature (A.D.Alexandrov sense, like in K. Groves lecture).

In several instances their behaviour is very different from that of smooth riemannian manifolds of nonpositive curvature.

I will show how to construct $K \leq 0$ manifolds which carry no $C^{\infty} K \leq 0$ metric.

This also suggests some (interesting?) questions about smooth $K \leq 0$ manifolds.

Jan Jaworowski

Antipodal Coincidence for Maps of Spheres into Complexes

This paper gives a partial answer to the question whether there exists a Borsuk - Ulam theorem for maps of S^n into lower-dimensional spaces which are not necessarily manifolds. The following facts can be proved :

(1) For each k and $n \leq 2k-1$, there exists a map f of S^n into a contractible k-dimensional complex K such that $fx \neq f(-x)$, for all $x \in S^n$.

(2) If $n > 2k-1$, then for every map f of S^n into a k-dimensional complex K such that $H_k(K; \mathbf{Z}_2) = 0$, there exists a point $x \in S^n$ with $fx = f(-x)$.

It is not hard to see that, if $n > 2k$, then for every map f of S^n into an arbitrary k-dimensional complex K there exists a point $x \in S^n$ with $fx = f(-x)$.

The qestion whether there exists a map f of S^3 into a 2-di- mensional complex K such that $fx \neq f(-x)$, for all x in S^3, was raised by Conner and Floyd (P.E. Conner and E.E. Floyd, Differentiable Periodic Maps, Springer - Verlag 1964, p.89). Thus (1) gives a construction of a contractible complex with this property. Moreover, $n = 3$ is the lowest dimension of the sphere for which such an example can be constructed.

Tadeusz Kożniewski

On the Nil groups in K-theory and surgery

(joint work with Frank Connolly)

We study the Nil groups: $Nil(R,\alpha)$ of Bass and Farell, $Nil(R;\mathcal{B}_1,\mathcal{B}_{-1})$ of Waldhausen and the $UNil$ groups $UNil^h_{2n}(R;\mathcal{B}_1,\mathcal{B}_{-1})$ of Cappell. The talk will contain:

1. A short review of some geometric application of these groups to the splitting problems and to rigidity conjectures.

2. A description of the Nil and $UNil$ groups in terms of a polynomical extension category $\mathbf{A}_\alpha[t]$. The description implies that all Nil groups above are special cases of the same construction. We also show that in geometric cases:

$$UNil^h_{2n}(R;\mathcal{B}_1,\mathcal{B}_{-1}) \cong L_\epsilon(\mathbf{A}_\alpha[t])$$

where $L_\epsilon(\ )$ is the Wall - Ranicki L-group.

3. Computations of the $UNil$ groups in the universal case $UNil^h_{2n}(R) := UNil^h_{2n}(R;R,R)$. These will use a sequence of operations

$$F_a : UNil^h_{2n}(R) \to UNil^h_{2n}(R)$$

for all $a \in \mathbf{Z}, a$ odd, $a > 0$ and a construction of an Arf invariant

$$A : UNil^h_0(R) \to coker(\Psi_2 - 1)$$

where R is a field of characteristic 2 and $\Psi_2 : R[t] \to R[t]$ is the Frobenius homomorfizm. The computations include:

Theorem: *Let R be a division ring with involution.*
(i) If the characteristic of R is $\neq 2$, then $UNil^h_{2n}(R) = 0$.
(ii) Let R be a perfect field of characteristic 2. Then
if the involution on R is nontrivial then $UNil^h_{2n}(R) = 0$,
if the involution on R is trivial then $UNil^h_{2n}(R) \cong \sum_{k=0}^{\infty} R$.

Theorem: *Let R be a PID of characteristic $\neq 0$ with trivial involution. Assume $R/2R$ is a perfect field. Then*
(i) $UNil_0^h(R) = 0$
(ii) The natural map $UNil_2^h(R) \to UNil_2^h(R/2R)$ *is an isomorphism.*

Nick Kuhn

On topologically realizing modules over the Steenrod algebra

$H^*(X; \mathbf{Z}_p)$ is a module over the Steenrod algebra A. A simple conjecture one can make is :

Realization Conjecture: *If* $H^*(X; \mathbf{Z}_p)$ *is a finitely generated* A*-module, then* $H^*(X; \mathbf{Z}_p)$ *is of finite total dimension.*

Here X must be a space or a spectrum if we also assume that $H^*(X; \mathbf{Z}_p)$ is an unstable module. Towards the conjecture, I can show

Theorem: *The conjecture is true if we also assume that the Bockstein* β *is zero on* $H^*(X; \mathbf{Z}_p)$ *in all high degrees.*

I prove this theorem by using "$\mathcal{U}$-technology", where $\mathcal{U}$ is the category of unstable A-modules. Featured tools in the proof are:

(1) the nilpotent filtration of $\mathcal{U}$ introduced by Lionel Schwarz:

$$... \subseteq \mathcal{U}il_3 \subseteq \mathcal{U}il_2 \subseteq \mathcal{U}il_1 = \mathcal{U}il \subseteq \mathcal{U}il_0 = \mathcal{U}$$

(2) the equivalence $\mathcal{U}/\mathcal{U}il \simeq \mathcal{F}_w$ (due to Henn, Lannes, Schwartz), where $\mathcal{F}_w$ is the locally finite subcategory of the category of all functors

$$F : \text{finite } \mathbf{F}_p\text{−vector spaces} \ \to \ \mathbf{F}_p\text{−vector spaces}$$

(3) the polynomial filtration of $\mathcal{F}_w$ (essentially introduced in the 1950's by Eilenberg - MacLane)

(4) Adams' work on Hopf invariant 1.

Tomasz Maszczyk

Foliations with Rigid Godbillon - Vey Class

For any foliation $\mathcal{F}$ of codimension one on a compact oriented 3-fold (such $\mathcal{F}$ exists if and only if $\chi(X) = 0$) there is well defined the Godbillon - Vey number $gv(\mathcal{F}) \in \mathbf{R}$ which is an invariant of concordance and cobordism equivalence of foliated manifolds. However gv is not an invariant of homotopy (that is it can change under 1 - parametric deformations of $\mathcal{F}$).

We consider foliations for which $gv(\mathcal{F})$ is rigid in the sesne of infinitesimal deformation and prove that it happens if and only if $\mathcal{F}$ arises by gluing level sets of local functions with fractional linear transition maps. So briefly speaking in this situation we obtain the following equivalence:

$$\text{(topological rigidity)} \iff \text{(algebraization)}.$$

J. Peter May

Commutative algebra is stable homotopy theory

E_∞ ring spectra are the closest analogs to commutative rings that are possible in stable homotopy theory. Examples include the sphere spectrum S, the Eilenberg-MacLane spectra HR of commutative rings R, the Thom spectra MO and MU, the connective K-theory spectra kO and kU, the spectra KR of algebraic K-theory, and many others.

The talk described joint work with Elmendorf and Kriz in which we have developed stable homotopy categories of E_∞ module spectra over E_∞ ring spectra, including smash products $M \wedge_R N$ and function modules $F_R(M,N)$. One immediate application is a purely homotopical construction of spectra such as Morava K-theory spectra that previously could only be constructed by the Bass - Sullivan theory of manifolds with singularities.

For an E_∞ ring spectrum R and R-modules M and N, we define

$$Tor_n^R(M,N) \equiv \pi_n(M \wedge_R N) \equiv M_n^R(N)$$

$$Ext_R^n(M,N) \equiv \pi_{-n}(F_R(M,N)) \equiv N_R^n(M).$$

Applied to $R = S$, these recover the usual homology and cohomology theories on spectra. Applied to Eilenberg - MacLane spectra, they recover the usual Tor and Ext of groups. There are Eilenberg - Moore type spectral sequences

$$Tor_{*,*}^{R_*}(M_*, N_*) \Longrightarrow Tor_*^R(M,N)$$

$$Ext^{*,*}_{R_*}(M_*, N_*) \Longrightarrow Ext^*_R(M, N)$$

that specialize to give universal coefficients and Ḱunneth spectral sequences in generalized homology and cohomology.

During the conference, a remarkably clean new way of setting up the definitions evolved. The category of S-modules has, it seems, a strictly associative and commutative smash product (but not strictly unital). E_∞ ring spectra are defined by maps $R \wedge_S R \to R$ and R-module spectra are defined by maps $R \wedge_S M \to M$. The analogy with commutative algebra, always close, has now become transparent and easy to work with.

Jim McClure

Obstruction theory for E_∞ ring spectra

The definition of E_∞ ring spectrum was given in May's lecture. There is a weaker, up to homotopy, version of this definition; such spectra are called H_∞ ring spectra. H_∞ ring structures can often be constructed by purely homotopy - theoretic methods, as opposed to E_∞ structures which require exact commutativity of certain diagrams.

In the lecture I will describe an obstruction theory for lifting H_∞ structures to E_∞ structures. In the special case where the spectrum in question is p-adic topological K-theory, the obstruction groups turn out to be André - Quillen homology groups, and these vanish by results of Miller and Quillen. Therefore p-adic K-theory has an E_∞ structure.

Hopkins and Miller have shown that the spectra $E(n)$ also have H_∞ structures and Hopkins and Strickland have shown how to carry out the above argument to show that the $E(n)$ are E_∞ ring spectra.

Masaharu Morimoto

Construction of smooth one fixed point actions on spheres

Through the study of "fixed-point-free actions on disks and of one-fixed-point actions on spheres" (by e.g. Oliver, Stein, Petrie, Laitinen, Traczyk, Pawalowski, Bak, Furuta , Buchdahl, Kwasik, Schultz, myself,...), the following were gradually conjectured.

Conjecture A: *There exists a smooth one fixed point action on S^n from* *some finite group if and only if* $n \geq 6$.

Conjecture B: *A finite group* G *admits a smooth one-fixed-point action on a sphere of some dimension if and only if* G *is an Oliver group (i.e.,* $G \notin \mathcal{G}_p^q$ *for any* p *and* q*).*

In the talk, these are affirmatively answered as Theorem A (with Bak) and Theorem B (with Laitinen), respectively.

$$\text{manifold with one fixed point} \quad \xrightarrow{G\text{-surgery}} \quad \text{sphere with one fixed point}$$

Dietrich Notbohm

Lie group theory for finite loop spaces

Classical Lie group theory is a well understood subject. There even exists a classification of compact connected Lie groups. Following an old idea of Rector, i.e. passing from G to BG, allows to express notions of Lie group theory in terms of classifying spaces and to extend them to finite loop spaces and p-compact groups.

A loop space is a triple $X := (BX, X, e)$ where $e : \Omega BX \to X$ is a homotopy equivalence. Then a homomorphism $f : X \to Y$ between loop spaces is a map $Bf : BX \to BY$.

X is called finite if X is a finite CW-complex and called a p-compact group if BX is p-complete and if $H^*(X; \mathbf{F}_p)$ is finite. Examples of p-compact groups are given by the completion of the finite loop space (BG, G, e) associated to a compact connected Lie group G.

p-compact groups enjoy much of the properties of compact Lie groups. For example Dwyer and Wilkerson showed recently that every p-compact group has a maximal torus. This is not true for finite loop spaces.

In the lecture we will give a survey how far Lie group theory for p-compact groups is developed.

Wiesław J. Olędzki

Low dimensional projective spaces detected by spectral invariants

This paper deals with sooth homotopy projective spaces of dimensions 4, 5 and 6 or, equivalently, smooth fixed point free involutions of homotopy spheres of these dimensions. We give a simple and transparent construction of diffeomorphism everything smooth fixed point free involutions of the spheres S^5 ans S^6. Then we show that the one-fold sooth suspension of the Cappell-Shaneson's exotic involution T^4_{CS} of S^4 and the Fintushel-Stern's exotic involution T^4_{FS} of S^4 are smoothly conjugated and both are smoothly conjugated to one of our involutions of S^5 which we precisely identify. Next we use a topological formula for the eta-invariant of certain Dirac-type operators to detect exotic homotopy projective spaces of dimension 4 (see Theorem 2 below).

Let $c_2 : S^2 \to SO(3)$ be the clutching map for the tangent bundle to S^3. Let G (resp. Γ) : $S^2 \times S^2 \to S^2 \times S^2$ be given by the formula $G(x, y) = (x, c_2(x)y)$ (resp. $\Gamma(x, y) = (c_2(x)y, x)$), let $h = \Gamma \circ G$ and let h_n be just the n-th power of h. Let $\mathbf{Z}_2$ act on $D^3 \times S^2$, $S^2 \times D^3$ and $S^2 \times S^2$ by the usual "antipodal" action. Note that all the maps G, Γ and h are $\mathbf{Z}_2$-equivariant. Now let us form $\mathbf{Z}_2$-manifolds $(\Sigma^5_n, T^5_n) = (D^3 \times S^2 \cup_{h_n} S^2 \times D^3, Ant \cup_{h_n} Ant)$

Our main results raed as follows.

Theorem 1: *Any smooth involution T^5 of a smooth homotopy sphere Σ^5 is smoothly conjugated to,precisely one of our involutions (Σ^5_n, T^5_n) , $n = 0, 1, 2, 3$. Any smooth involution T^6 of a smooth homotopy shere Σ^6 is smoothly conjugated to the one-fold smooth suspension of precisely one of our involutions (Σ^5_n, T^5_n) , $n = 0, 1, 2, 3$. The one-fold smooth suspension ΣT^4_{CS} of the Cappell-Shaneson's exotic involution T^4_{CS} is smoothly conjugated to the one-fold smooth suspension ΣT^4_{FS} of the Fintushel-Stern's exotic involution T^4_{FS} and both these involutions are smoothly conjugated to our involution T^5_3*

Let M^4 be a closed non-orientable Pin^+ (resp. Pin^c) manifold. Let

$\tilde{M}^4$ be the orientation covering of M^4 nad let T^4 be the obvious involution of $\tilde{M}^4$. Let $\tilde{M}^3 \subset \tilde{M}^4$ be a characteristic submanifold for T^4 and let $T^3 = T^4|\tilde{M}^4$, $M^3 = \tilde{M}/T^3$. Fix a Pin^+ (resp. Pin^c) structure Ψ (resp. Φ) on M^4 and let F (resp. $\tilde{F}$) be a ($\mathbf{Z}^2$-equivariant) stable almost framing of M^3 (resp. $\tilde{M}^3$) "connected" with Ψ (resp. Φ). Let $\mu(M^3; F)$, resp. $\alpha(\tilde{M}^3, T^3)$, resp. $\rho(M^3, \mathbf{Z}_2)$ be the Milnor, resp. Browder-Livesay, resp. Wall inwariant.

Theorem 2: *Let M^4, M^3 etc. be as described above. Then*
a. $\eta(M^4, \Psi) = \pm 1/8(\mu(M^3; F) - 1/2\rho(M^3, \mathbf{Z}_2) + \mu(\tilde{M}^3; \tilde{F}))mod\ \mathbf{Z}$
b. $\eta(M^4, \Phi) = \pm 1/8(\mu(M^3; F) - 1/2\rho(M^3, \mathbf{Z}_2) + \mu(\tilde{M}^3; \tilde{F}))mod\ 1/2\mathbf{Z}$

Under certain additional onditions these formulas can be improved to *mod* $2\mathbf{Z}$, resp. *mod* $\mathbf{Z}$ form, and this is the case for FRP^4_{CS} and FRP^4_{FS}. Using these improved formulas we get $\eta(FRP^4_{CS}, \Psi') = \pm\eta(FRP^4_{FS}, \Psi'') = \pm 7/8 mod\ 2\mathbf{Z}$.

Teimuraz Pirashvili

Leibniz algebras and homology

Leibniz algebras and its homologies were introduced by J.L.Loday. Let $\mathbf{k}$ be a field. Leibniz algebra $\mathbf{g}$ over $\mathbf{k}$ is a $\mathbf{k}$-vector space equipped with a bilinear map

$$[-,-] : \mathbf{g} \times \mathbf{g} \to \mathbf{g}$$

which satisfies the identity

$$[x, [y, z]] = [[x, y], z] - [[x, z], y] \tag{L}$$

Every Lie algebra is a Leibniz algebra. A representation of $\mathbf{g}$ is a $\mathbf{k}$-vector space $\mathcal{M}$, equipped with a bilinear maps $[-,-] : \mathcal{M} \otimes \mathbf{g} \to \mathbf{g}$ and $[-,-] : \mathbf{g} \otimes \mathcal{M} \to \mathbf{g}$ satisfying (L) for all possible situations. A representation is symmetric if $[m, x] + [x, m] = 0$ and is antisymmetric if $[x, m] = 0$. The category of all representations is isomorphic to the category of right modules over some ring $UL(\mathbf{g})$. Let $\mathbf{g}_{Lie} = \mathbf{g}/\{[x, x] = 0\}$. Then one can define

$$HL^*(\mathbf{g}, M) = Ext^*_{UL(\mathbf{g})}(U(\mathbf{g}_{Lie}), M),$$

$$HL_*(\mathbf{g}, A) = Tor^*_{UL(\mathbf{g})}(U(\mathbf{g}_{Lie}), A).$$

Proposition 1 *Let char* $\mathbf{k} = 0$ *and* $\mathbf{g}$ *is a finite dimensional, semisimple Lie algebra and* $\mathcal{M}$ *be a finite dimensional representation. Then*
$$HL^i(\mathbf{g}, \mathcal{M}) = 0 \text{ for } i > 0.$$

Corollary *Such Lie algebras are rigid and projective in the category of all finite dimensional Leibniz algebras.*

Proposition 2 *Let* $0 \to \mathbf{a} \to \mathbf{h} \to \mathbf{g} \to 0$ *be a split exact sequence with* $[g, a] = 0$ *for any* $a \in \mathbf{a}$ *and* $g \in \mathbf{h}$. *Then there exists a spectral sequence*

$$E^p_{pq} = HL_p(\mathbf{g}, \mathbf{a}) \otimes HL_q(\mathbf{h}) \Longrightarrow HL_{p+q+1}(\mathbf{h})/HL_{p+q+1}(\mathbf{g})$$

Corollary *If char* $\mathbf{k} = 0$ *and* $\mathbf{g}$ *is a finite dimensional Leibniz algebra with* $\mathbf{h} = \mathbf{g}_{Lie}$ *a semisimple Lie algebra. Then*

$$HL_n(\mathbf{g}) = \otimes^n(\mathbf{a_h}),$$

where $\mathbf{a} = ker(\mathbf{g} \to \mathbf{g}_{Lie})$.

Stewart Priddy

A counterexample to the homomorphism question for classifying spaces

(joint work with John Martins)

For finite groups we study the p-completion of BG, how this space is related to G, and how maps arise between such spaces. The main result is that $BG^\wedge_p$ is determined by the Quillen category of p-subgroups of G with maps induced by conjugations.

There are maps $BG^\wedge_p \to BG'^\wedge_p$ not induced by homomorphisms. This is true even for equivalences, consequently several people have raised the question as to whether a homotopy equivalence $BG^\wedge_p \simeq BG'^\wedge_p$ can be realized by an alternating chain of homomorphisms each inducing an equivalence. We give examples shoving this is false.

Józef H. Przytycki

Vassiliev invariants of knots
- survey from the combinatorial point of view

We introduce the notion of skein modules of 3-manifolds and explain how Vassiliev invariants can be described in this setting.

Skein modules are quotients of modules over isotopy classes of links in 3-manifold by a properly chosen local (skein) relations.

Example. Let M be a 3-manifold, R a commutative ring with unit $\mathcal{L}$ the set of all oriented links in M, up to umbient isotopy, and $R\mathcal{L}$ the free R-module generated by $\mathcal{L}$. Let S_2 be the submodule generated by skein expressions

$$L \qquad - L$$

The skein module $\mathcal{S}_2(M)$ is defined as $\mathcal{S}_2(M) = R\mathcal{L}/S_2$. Fact: $\mathcal{S}_2(M) = RH_1(M, \mathbf{Z})$

Example. Vassiliev - Gusarov skein modules.
Let $\mathcal{K}$ be the set of oriented knots in M and $\mathcal{K}^{sg}$ be the set of singular knots (double points have chosen orientation in the neighbourhood). Let $\sim$ be the "resolving singularity" relation:

$$=$$

(double point with orientation
chosen in the neighbourhood)

Of course $R\mathcal{K} = R\mathcal{K}^{sg}/\sim$. Let $R\mathcal{K} = C_0 \supset C_1 \supset ... \supset C_1 \supset ...$ be a filtration of $R\mathcal{K}$ where C_i is generated by knots with i double points. $W_i = R\mathcal{K}/C_{i+1}$ is called the Vassiliev - Gusarov skein module of degree i. The Vassiliev invariant of degree i is an element of the dual space $V^i = W_i^* = Hom(W_i, R)$.

John Rognes

$K_4(\mathbf{Z})$ is the trivial group.

We prove that the fourth algebraic K-group of the rational integers is the trivial group.

The method is first to approximate the K-theory spectrum $\underline{\underline{K}}(\mathbf{Z})$ by a spectrum level rank filtration

$$* \simeq F_0\underline{\underline{K}}(\mathbf{Z}) \to ... \to F_k\underline{\underline{K}}(\mathbf{Z}) \to ... \to \underline{\underline{K}}(\mathbf{Z})$$

where $F_k\underline{\underline{K}}(\mathbf{Z})$ is built from $\mathbf{Z}$-modules of rank $\leq k$.

Next we compute the third stage:

$$H_*^{spec}(F_3\underline{\underline{K}}(\mathbf{Z})) \cong (\mathbf{Z}, 0, 0, \mathbf{Z}/2, 0, ...)$$

Finally we prove that $F_3\underline{\underline{K}}(\mathbf{Z}) \to \underline{\underline{K}}(\mathbf{Z})$ is four-connected, which determines $H_*^{spec}\underline{\underline{K}}(\mathbf{Z})$ through degree four. The Atiyah - Hirzebruch spectral sequence for stable homotopy then allows a computation of $\underline{\underline{K}}(\mathbf{Z})$ through degree four.

Yuli Rudyak

On the orientability with respect to Morava K- theories

An obstruction theory for $K(n)$-orientability is considered. One of typical results is the following.

Theorem: *Let $K(n)$ be a Morava K-theory for $p = 2$ and given n.*
a) If $m > n$, then there exists a vector bundle which is $K(m)$-orientable and not $K(n)$-orientable.
b) If $m < n$, then there exists a spherical fibration which is $K(m)$-orientable and not $K(n)$-orientable.

(The proof of (a) is trivial: the universal vector bundle over $BO^{(2^n)}$ gives an example).

It would be good to replace spherical fibrations in (b) with vector bundles, but I can't do it.

Similar result (with PL bundles instead of vector bundles) holds for $p > 2$.

Michał Sadowski

Homotopical invariants of homotopically trivial actions on closed aspherical manifolds and their applications

The organisation of the talk is as follows:

1) Summary of known results for connected compact Lie group actions.

Let G be a torus. The map $ev : G \ni g \mapsto g*$ induces the evaluation homomorphism $ev_* : \pi(G, 1) \to \pi(M, *)$. It is possible to describe, in terms of ev_*, when:

(a) M (a finite covering space of M) is G homeomorphic (G diffeomorphic if G acts smoothly) to $G \times M/G$.
(b) G acts freely (effectively).
(c) two free actions of G on appropriate aspherical manifolds are topologically conjugate.

2) The construction of homotopical invariants of homotopically trivial actions. The notion of π_1-conjugacy.

3) Applications to free, smooth, homotopically trivial actions on closed aspherical manifolds.
(a) π_1-conjugacy is the same as equivariant homotopy equivalence.
(b) π_1-conjugacy and topological conjugacy of homotopically trivial actions on closed nonpositively curved manifolds and infrasolvmanifolds.
(c) An estimate of the number of topological conjugacy classes of of homotopically trivial finite groups actions on manifolds from (b).
(d) Other results and comments.

Andras Szucs

Smooth maps with local and global restrictions

How to use singularity theory to the computation of homotopy groups of Thom spaces? Cobordism groups of singular maps with arbitrary global and simple local restrictions are considered. Analogues of the Thom spaces are constructed for these types of cobordisms. These spaces give models for loop spaces of Thom spaces (analogous to the James product model for ΩSX). An information about the unstable homotopy groups of Thom spaces is obtained.

Marek Szyjewski

An invariant of quadratic forms over schemes

If the dualization functor $\wedge$ is exact (e.g. on finitely generated projectives or coherent locally free sheaves), then it induces a natural involution $\wedge$ on appropriate K-theory, namely the Adams operation Ψ^{-1}. Resulting action of the two element group $\mathbf{Z}_2$ on various subgroups ot the Grothendieck group $K_0(X)$, and its Tate cohomology groups

$$EX = Ker(1-\wedge)/Im(1+\wedge) \quad E^-X = Ker(1+\wedge)/Im(1-\wedge)$$

appeared in Ranicki's works. For $1 \pm \wedge : K_0(X) \to K_0(X)$ both EX and E^-X are functorial, EX is a ring and E^-X is an EX-module. There is a natural ring homomorphism $e^0 : W(X) \to EX$ induced by a forgetful functor, and a $W(X)$-module homomorphism $W^{-1}(X) \to EX$. Moreover, there is a $W(X)$-module homomorphism

$$L_1(X) \oplus L_3(X) \to E^-X$$

$$(\mu, \beta; A, B) \mapsto [A] - [B] \ mod \ Im(1-\wedge).$$

The author's attention is focused on the Witt ring $W(X)$ of symmetric bilinear forms on a variety X over a field F, $char\ F \neq 2$. The point is that EX, E^-X may be actually computed.

1. If X is a regular curve (e.g. a spectrum of a Dedekind ring), then

$$EX = \mathbf{Z}_2 \oplus {}_2Pic(X), \quad E^-X = Pic(X)/2Pic(X),$$

$$e^0 : W(X) \to EX \text{ is surjective.}$$

Note that for Dedekind ring R, $L_1(R) = Pic(X)/2Pic(X)$. In particular, if X is a smooth, complete curve over an algebraically closed field, then $EX = {\mathbf{Z}_2}^{1+2g}$ end e^0 is an isomorphism.

2. If $X = Spec\ R[X_1, ...X_n]$, the affine space over a regular ring R, then $W(X) = W(R)$ (Karoubi's theorem), $EX = ER$, $E^-X = E^-R$.

3. If $X = Proj F[X_0, X_1, ..., X_d]$, the projective space of dimension d over a field F, $char\ F \neq 2$, then

$$W(X) = W(F) \text{ (Arason's theorem)}, \quad EX = \mathbf{Z}_2, \quad E^-X = \begin{cases} 0 & \text{for } d \text{ odd} \\ \mathbf{Z}_2 & \text{for } d \text{ even} \end{cases}.$$

4. If $X = \mathbf{P}^d_F \times Y$, then

$$EX = (E\mathbf{P}^d_F \otimes EY) \oplus (E^-\mathbf{P}^d_F \otimes E^-Y)$$

$$E^-X = (E\mathbf{P}^d_F \otimes E^-Y) \oplus (E^-\mathbf{P}^d_F \otimes EY).$$

For instance for $X = \mathbf{P}^1_F \times \mathbf{P}^1_F$ $W(X) = W(F)$, $EX = \mathbf{Z}_2 \oplus \mathbf{Z}_2$ and e^0 is not surjective.

5. X - a projective quadric of dimension $d = 2m+1$, defined by a quadratic form of maximal index over a field F, $char\ F \neq 2$. Then

$$EX = \mathbf{Z}_2, \qquad E^-X = \mathbf{Z}_2.$$

6. X - a projective quadric of dimension $d = 2m$, defined by a quadratic form of maximal index over a field F, $char\ F \neq 2$. Then

$$EX = \mathbf{Z}_2 \oplus \mathbf{Z}_2, \qquad E^-X = \begin{cases} 0 & m \text{ even} \\ \mathbf{Z}_2 \oplus \mathbf{Z}_2 & m \text{ odd} \end{cases}.$$

Application of 6. Take $M = End_{\mathcal{O}_X}(V)$ where V is an indecomposable component of the Swan sheaf. Let β = trace of product. Then $e^0(M,\beta)$ is the second generator of EX for $m > 1$ (and $m = 0$ too). So e^0 is surjective. Moreover, $W(X) \neq W(F)$!
In particular for $m = 2$ the quadric X is the Grassman manifold $Gr_2(4)$. So for Grassmannians different to projective spaces there is no generalization of Arason's theorem, mentioned in 3. above.

Alexander Tralle

A formula for Hochschild cohomology of topological spaces with cohomology algebra $H^*(X,k) = k[X_1,...,X_n]/(f_1,...,f_n)$

We give an explicit formula for Hochschild (co)homology (in Goodwillie's sense) $HH^*(X)$ of any topological space with cohomology of the form

$$H^*(X,k) = k[X_1,...,X_n]/(f_1,...,f_n) \tag{I}$$

where $k[X_1, ..., X_n]$ is a polynomial algebra of variables $X_1, ..., X_n$ over a field k $(char(k) = 0)$ and polynomials $f_1, ..., f_n$ constitute a regular sequence.

The first explicit calculation of $HH^*(X)$ in the case (I) was done by D.Burghelea and M.Vigué, namely a formula for Poincaré series $P_{HH^*(X)}(t)$ was obtained for any simply connected toplogical space X with cohomoly algebras either of the form $k[X_1]/(X_1^{n+1})$ or $\Lambda(v)$.

In the present note we prove an intersting formula for arbitrary space whose cohomology algebra is of the form (I) and show by examples how to apply it. Our result can be applied to a wide class of topological spaces (e.g. homogeneous spaces of compact Lie groups G/H, $rank(G) = rank(H)$). When the multiplicative structure onf $H^*(X)$ can be described explicitely by generators and relations, our formula is of particular interest. It should be mentioned that T.Goodwillie's isomorphism

$$HH^*(X) = H^*(\Lambda X)$$

(ΛX being a free loop space over X) reveals the importance of various explicit formulae fore $HH^*(X)$. Besides that our result reveals the exact relations between $H^*(X)$ and $HH^*(X)$. Some formulae were obtained in the author's papers in the case od $H^*(X)$ having two generators as well as in the case of homogeneous spaces.

Everywhere below we consider graded algebras and denote degrees of elements by deg.

Theorem: *Let X be any simply connected toplogical space with cohomology algebra of the form (I) satisfying assumptions:*

1) $f_1, ..., f_n$ form a regular sequence of polynomials in $k[X_1, ..., X_n]$
2) all f_i are indecomposable, that is f_i are polynomials only of variables X_j satisfying the inequality $deg(X_j) < deg(f_i) - 1$.

Then the following isomorphism of graded algebras is valid:

$$HH^*(X) = H^*(X) \otimes \Lambda(x_1, , x_n)/(\Sigma_{j=1}^n \frac{\partial f_1}{\partial X_j} x_j, ..., \Sigma_{j=1}^n \frac{\partial f_n}{\partial X_j} x_j) = \oplus$$

$$\oplus \Sigma_{s=1}^n \Sigma_{i_1 < ... < i_s} (Ann_{H^*(X)\otimes\Lambda(x_1,...,x_n)}(\Sigma_{j=1}^n \frac{\partial f_{i_1}}{\partial X_j} x_j, ..., \Sigma_{j=1}^n \frac{\partial f_{i_n}}{\partial X_j} x_j)/$$

$$/(\Sigma_{j=1}^n \frac{\partial f_1}{\partial X_j} x_j, ..., \Sigma_{j=1}^n \frac{\partial f_n}{\partial X_j} x_j)) \otimes k^+[Y_{i_1}, ..., Y_{i_s}]$$

where $\Lambda(x_1, ..., x_n)$ is the exterior algebra generated by free variables of odd degrees x_i, $deg(x_i) = deg(X_i) + 1$ and $k[Y_1, ..., Y_n]$ is a polynomial algebra of free variables $Y_1, ..., Y_n$ such that $deg(Y_i) + deg(f_i)$ for all i.

Remark 1: In fact the assumption 2) in the formulation of the theorem is not restrictive, as far as if $H^*(X)$ can be represented in the form (I) with the regularity condition, it can be also represented in the form, satisfying both assumptions.

Remark 2: We use the following notations: for any ring A the symbol $(a_1, ..., a_n)$ denotes the ideal generated by $a_1, ..., a_n$. The following agreement is accepted: if $I \supset A$ is an ideal, the factor ring $I/I \cap (a_1, ..., a_n)$ is simply denoted by $I/(a_1, ..., a_n)$. If $k[Y_1, ..., Y_n]$ is a polynomial algebra $k^+[Y_1, ..., Y_n]$ denotes the subalgebra generated by monomials $Y_1^{k_1} \cdots Y_n^{k_n}$, $k_i > 0$ for all i.

Georgia Triantafillou

Diffeomorphisms of Manifolds with Finite Fundamental Group

Let M be a closed smooth manifold and let $Diff(M)$ be the group of diffeomorphisms of M. The space $Diff(M)$ is in general very complicated and its homotopy type is known only in some special cases. In particular the component group $\pi_0(Diff(M))$ can be quite large.

The first general result about the nature of $\pi_0(Diff(M))$ for simply connected manifolds is due to D. Sullivan who showed that if M is a smooth, closed, orientable, simply connected manifold of dim ≥ 5 then $\pi_0(Diff(M))$ is commensurable to on arithmetic group and therefore it is finitely presented.

On the other hand $\pi_0(Diff(M))$ can be very bad if M is not simply connected. For instance for the torus of dim 5 $\pi_0(Diff(T^5))$ is infinitely generated. The question remains how bad is $\pi_0(Diff(M))$ if $\pi_1(M)$ is finite. We show

Theorem 1: *If M is a smooth closed orientable manifold of dim ≥ 5 with finite fundamental group then $\pi_0(Diff(M))$ is a finitely presented group.*

This result follows from the following theorem. Let $\tilde{\pi}_0(Diff(M))$ denote the group of pseudoisotopy classes of diffeomorphisms of M.

Theorem 2: *Under the same assumptions as above $\tilde{\pi}_0(Diff(M))$ is commensurable to an arithmetic group.*

The methods involve surgery, the Atiyah - Singer G signature theorem and minimal model theory.

Rainer Vogt

The equivalence of topological Hochschild homology and MacLane homology

In 1990 T.Pirashvili and F.Waldhausen proved the surprising fact that for discrete rings R topological Hochschild homology of R, $THH(R)$ defined 1985 by M.Bokstedt, is equivalent to MacLane homology $H^{ML}(R)$ of R, defined almost 30 years earlier by Maclane. The result follows from the verification for THH of a set of axioms which characterizes the homology of the category of finitely generated projective R-modules with coefficient in the Hom-functor. In an earlier paper M.Jibladze and T.Pirashvili proved the same for MacLane homology.

We show that from the point of view of "homological algebra in stable homotopy theory" (cf. P.May's lecture) the equivalence of $THH(R)$ and $H^{ML}(R)$ is conceptually clear. It simply is a change of rings argument. The conceptual proof relies on the validity of a conjecture that two canonical multiplications on the spectrum $\underline{\underline{R}} \wedge \underline{\underline{\mathbf{Z}}}$ are equivalent. Here $\underline{\underline{R}}$ denotes the Eilenberg - MacLane spectrum of R. A proof of this conjecture would require fiddling with the coherence structure of smash products of spectra and A_∞ ring spectrum structures. To avoid this we reformulate the conceptual ideas into the machinery used by Bokstedt in his definition of $THH(R)$, where the analogue of the conjecture can be proved more easily.

Andrzej Weber

De Rham isomorphism for L_p-cohomology and intersection homology

The purpose of this talk is to present a connection between the intersection homology theory and L_p-cohomology. We investigate the simplest case: riemannian pseudomanifolds. These are piecewise linear singular spaces with a riemannian metric on the nonsingular part which is concordant with a triangulation. We show that there exists a perversity (the maximal smaller then $\frac{i}{p}$) such that intersection homology with respect to this perversity and L_p-cohomology coincide. It's a version of the de Rahm isomorphism. The condition of L_p-growth in the neighborhood of singularities for forms corresponds to restrictions for intersections of chains with singular strata. Next we consider an important abstract functional condition of a negligible boundary. For example it allows us to define a pairing between L_p- and L_q-cohomology. It turns out that in our case it is a purely topological condition concerning the structure of singularities. The negligible boundary condition is equivalent to vanishing of intersection homology of links in certain dimensions. The spaces with this property for $p = 2$ were discovered by Cheeger [C2] and independently by Goresky and MacPherson [GM2], [S] and called Witt spaces. In the end we show that the negligible boundary condition is Equivalent to the duality in the derived category between the sheaves of L_p- and L_q-cohomology. It seems that pseudomanifolds with a negligible boundary are suitable objects to perform analysis and sheaf theory.

Mariusz Wodzicki

Beyond the index invariant - the higher algebraic K-theory of operator ideals

Let J be a two-sided ideal in the ring $\mathcal{B} = \mathcal{B}(H)$ of bounded linear operators on the standard separable Hilbert space H. The index of a Fredholm operator identifies $K_0(\mathcal{B}, J)$ with $\mathbf{Z}$. Higher relative K-groups $K_i(\mathcal{B}, J)$, $i > 0$ play the role of receptors of higher index invariants. For $J = \mathcal{F}$, the ideal of finite rank operators, one has $K_*(\mathcal{B}, \mathcal{F}) \simeq K_*(\mathbf{C})$. For $J = \mathcal{K}$, the ideal of compact operators, Karoubi's Conjecture (1978) predicted that $K_*(\mathcal{B}, \mathcal{K}) \simeq K_*^{top}(\mathbf{C})$ (proved by Suslin and the author, 1990).

In the present talk we prove that $K_*(\mathcal{B}, J)$ is, for a Banach ideal J, determined by a series of short exact sequences

$$0 \to HC^{\mathbf{Z}}_{2j-1}(\mathcal{B}, J) \to K_{2j}(\mathcal{B}, J) \to \mathbf{Z} \to HC^{\mathbf{Z}}_{2j-2}(\mathcal{B}, J) \to K_{2j-1}(\mathcal{B}, J) \to 0$$

($j \in \mathbf{Z}$). Among others, a new interpretation of Beilinson's regulators is given.