

The deadline is January 15, midnight.

1. Consider the equality symmetry. Prove that for every orbit-finite nominal sets  $X$  and  $Y$ , for some  $n \in \mathbb{N}$  there is an equivariant partial surjective function

$$f : X^n \rightarrow Y$$

2. Consider a nondeterministic two-way automaton in the equality symmetry. That is, the transition relation is a subset

$$\delta \subseteq (Q \cup \{\vdash, \dashv\}) \times A \times (Q \times \{\vdash, \dashv\}) \times \{-1, 1\}$$

where  $\vdash$  and  $\dashv$  stand for start/end markers. For  $k \in \mathbb{N}$ , an automaton is called  $k$ -reversal bounded if on every input, every run (accepting or not) makes at most  $k$  changes of direction. (A change of direction is a  $+1$  transition followed by a  $-1$  transition, or the other way round.) Prove that the following question is undecidable:

Is there some  $k \in \mathbb{N}$  such that the automaton is  $k$ -reversal bounded?

3. Prove that the following problem is decidable: given a nondeterministic two-way automaton (as in the previous exercise) and  $k \in \mathbb{N}$ , decide if some word is accepted by a run with  $k$  reversals.
4. Prove that the symmetry for the Fraïssé class of all graphs has least supports.