Automata and Algebras for Infinite Words and Trees

Mikołaj Bojańczyk*

University of Warsaw

Abstract. Regular languages can be studied not just for finite words, but also for finite trees, infinite words and infinite trees. Almost all of the theory of regular languages, such as closure under boolean operations, works also for these extensions, but the constructions are significantly more challenging and mathematically interesting. For instance, automata for infinite words can be determinised under a suitable choice of acceptance condition, but the proof requires an intricate combinatorial construction.

In the first part of my talk, I will describe the now classical results on automata recognising regular languages of infinite words and infinite trees. I will mention the connection of automata with monadic second-order logic, discovered by Büchi and Rabin, which has been one of the main sources of inspiration in the theory of automata for infinite objects. More on these topics can be found in the general survey [4] or in the collection of more specialised surveys [1].

The second part of my talk will be on the algebraic approach to regular languages. For finite words, the algebraic approach is to use semigroups instead of automata. The beauty of the algebraic approach is that it uncovers connections between classical mathematical concepts and formal language theory. For instance, a celebrated theorem of Schützenberger says that a regular language of finite words can be defined by a regular expression without the star (but with complementation) if and only if the language can be recognised by a semigroup that does not contain any nontrivial group. There is also a well understood algebraic theory for infinite words, which uses variants of semigroups. The algebraic theory of for regular languages of finite words is described in the book [3], and its extensions to infinite words are described in the book [2].

However, the algebraic theory of infinite trees, or even finite trees, is currently not well understood, and seems to be a very challenging problem. I will end my talk by describing this problem, and why it might be interesting for people studying algebra and coalgebra.

References

1. Erich Grädel, Wolfgang Thomas, and Thomas Wilke, editors. *Automata, Logics, and Infinite Games: A Guide to Current Research [outcome of a Dagstuhl seminar, February 2001]*, volume 2500 of *Lecture Notes in Computer Science*. Springer, 2002.

* Author supported by ERC Starting Grant "Sosna"

2. Dominique Perrin and Jean-Éric Pin. *Infinite Words.* Elsevier, 2004.
3. Howard Straubing. *Finite Automata, Formal Languages, and Circuit Complexity.* Birkhäuser, 1994.
4. Wolfgang Thomas. *Languages, automata, and logic.* In G. Rozenberg and A. Salomaa, editors, Handbook of Formal Languages, volume III, pages 389-455. Springer, New York, 1997.